INTERMEDIATE ELECTRICITY

INTERMEDIATE ELECTRICITY

BY

ROBERT W. HUTCHINSON, M.Sc.

AUTHOR OF "ADVANCED TEXTBOOK OF ELECTRICITY AND MAGNETISM"
"ELEMENTARY ELECTRICITY AND MAGNETISM"; "ELEMENTARY
TECHNICAL ELECTRICITY"; "WIRELESS, ITS PRINCIPLES AND
PRACTICE"; "TELEVISION UP-TO-DATE," ETC.
JOINT AUTHOR OF "TECHNICAL ELECTRICITY"
FORMERLY PRINCIPAL, MUNICIPAL TECHNICAL COLLEGE, SMETHWICK

LONDON

UNIVERSITY TUTORIAL PRESS LTD.

Clifton House, Euston Road, N.W. 1

BOOK
PRODUCTION
WAR ECONOMY
STANDARD

This book is produced in complete conformity with the authorised economy standards

Published 1941
Reprinted 1942, 1944

PRINTED IN GREAT BRITAIN BY UNIVERSITY TUTORIAL PRESS LTD, FOXTON
NEAR CAMBRIDGE

PREFACE

THE present book under its title *Intermediate Electricity* is intended to replace *Intermediate Textbook of Electricity and Magnetism* which, with its various slight revisions at successive periods, has continued to be a favourite with teachers and students alike for over twenty years.

The text has been completely rewritten on thoroughly modern lines, a considerable amount of new matter incorporated, and a large number of new diagrams and illustrations included. Broadly speaking, the same *general* order of treatment has been followed, for I am convinced, as the result of very many years experience with students on both the pure and applied sides, that this best leads to a logical development of the subject and its units, and sound theory and definition; and it eliminates excessive make-shift explanations to be corrected or discarded at a later stage. The actual detailed treatment itself is completely modern from the outset, and all explanations of observed facts and standard experiments are dealt with in terms of the science as we know it *to-day*. The fact is stressed that magnetism itself is merely a phase of electricity—electricity in motion—and current effects are introduced from the beginning: the perfect *unity* of currents, charges, magnetism is repeatedly emphasised: the natural coherence of the subject as a whole irrespective of the historical dates of discovery of isolated facts and principles is kept in the foreground.

The Introductory chapter includes a much fuller account than is usual at this early stage of the modern theory of electricity and electrical phenomena. I have, however, given these early facts in as simple and as " homely " a way as is possible so that even a beginner can readily understand them and apply them *from the outset* to the explanation of the facts and experiments he encounters in his studies: fuller details follow throughout succeeding pages. Incidentally, certain practical and applied features are also introduced and explained at earlier stages than is usual if they are of such an important and " everyday " nature that the student is constantly encountering them in his work, in his studies or in his general reading.

Practical testing and practical applications and theory run hand in hand in the book. Special attention has been devoted to the various units and systems of units, and their relationships indicated, so that students may experience no difficulty in connexion with modern research work or general problems necessitating

a change from one system to another: in fact the care which has been devoted to units throughout will, it is hoped, banish the student's troubles in this direction. The book contains numerous fully worked examples illustrating important principles and applications, and a number of problems of a similar character to be worked by the student. Over 580 diagrams and illustrations of a kind best suited to aiding the student to a clear understanding of the principles and methods involved are inserted. The more important symbols recommended by the International Electro-technical Commission have been adopted.

Though the mathematics involved is confessedly more or less elementary, use has been made of the notation and first principles of the Calculus. In the earlier portions of the book such applications usually appear in the form of neater alternative investigations, solutions, and proofs, but the methods are occasionally employed if it appears that a distinct advantage is gained thereby. Students of Physics cannot realise too early that a knowledge of the rudiments of the Calculus is an essential part of their mental equipment. Little is actually required at this stage, and the elements at least are now generally taught in schools: in any case, however, the necessary knowledge can be acquired by a very slight expenditure of time.

Although I have in no way restricted myself by the bounds of any particular examination syllabus, the book, as its title suggests, thoroughly covers—more than covers—the requirements of candidates reading for the Intermediate Science examinations of the Universities. It also meets the needs of candidates for University Scholarship and Higher School Certificate, and of those entering for appointments in the Army, Navy, Air Force, and Civil Service, or aiming at others of the professional examinations in which "Electricity" plays a part. It is hoped, too, that it may be of some service to students and workers in applied electricity, particularly in electrical engineering, telegraphy, telephony and radio, who wish to acquire the necessary background of theory, and to keep in touch with the wonderful developments, possibilities and anticipations in the electrical domain of Physics.

My best thanks are due to certain manufacturers and friends (mentioned in the text) for the kind loan of some of the illustrations, and I am greatly indebted to the Senate of London University for permission to use certain examination questions: some are also included, by permission, from the examination papers of other Universities.

R. W. HUTCHINSON.

CAMBRIDGE

ILLUSTRATIONS

CONTENTS

INTERMEDIATE TEXTBOOK OF ELECTRICITY

CHAPTER I

INTRODUCTION

THE STARTING POINTS OF OUR KNOWLEDGE OF ELECTRICITY—WITH A GLANCE AT SOME OLD AND NEW IDEAS

WE gather from the writings of Thales of Miletus that so far back as 600 B.C. the Greeks knew that pieces of amber possessed, when rubbed, the property of attracting light bodies, and it is from the Greek name *elektron* (amber) that our word *electricity* has come. Nature, however (although really providing us with " electricity "), does not give many examples of electrical forces *which man is aware of* in his everyday life, as she does in the case of heat, light, sound, etc.—the occasional thunder-storm on the violent side, and, on the gentler side, the little crackling when we brush our hair or stroke the cat, being the main examples. In consequence man did not develop an electrical sense, and about 2000 years elapsed before his knowledge of the subject began definitely to expand.

1. Positive and Negative Electric Charges

In 1600 A.D., Dr. Gilbert, physician to Queen Elizabeth, showed that many other substances behaved in the same way as the amber of the Greeks, and it is now known that, with proper precautions, most substances suitably rubbed will attract to some extent such light articles as pieces of paper, bran, cork, pith, etc.: glass rubbed with silk, sealing-wax with flannel, and vulcanite with fur show the effect very well. A substance showing this property was said to be *electrified*, to be *charged with electricity*, to possess a *charge of electricity*, or to be in a *state of electrification*, and the " agent " which caused this was called electricity. A substance showing no signs of electrification was said to be electrically *neutral*.

It was at first thought that metals could not be electrified, for on holding them in the hand and using various rubbers they showed no signs of electrification. Later it was found that if a metal rod

was fitted with a handle of glass or vulcanite it could be electrified in this way. The explanation is that metals (and the human body) are **conductors** of electricity, and therefore the "charges" flow along them to the hand and to earth. *The exact explanation of this "flow" is given later.* Substances like glass, vulcanite, sealing-wax, etc., do not allow electricity to flow through them and leak to earth, and are called **insulators.**

Further investigation showed that two electrified bodies exerted a certain force on each other: in some cases they attracted, and in some cases they repelled. Whether the *electric force* (as it was called) between them was one of attraction or of repulsion depended on the materials of the two bodies, and on what they were rubbed with. Thus if a vulcanite rod which has been rubbed with fur be suspended from a stand by a *dry silk* thread (Fig. 1), and a second vulcanite rod also rubbed with fur be brought near, the suspended one will be repelled; but a glass rod rubbed with silk will attract it. Similarly, if the electrified glass be suspended, a second glass rod rubbed with silk will repel it, but the vulcanite rod rubbed with fur will produce attraction. If the experiment be repeated with various bodies and rubbers it will be found that if two electrified bodies A and B *both* attract or *both* repel an electrified body C, then A and B will repel each other: but if A attracts and B repels C, or if A repels and B attracts C, then A and B will attract each other.

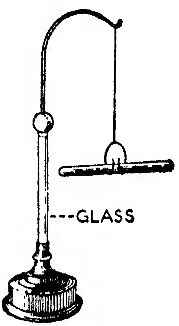


FIG. 1.

Numerous experiments like the above led the early scientists to the conclusions that (1) there were two states of electrification; (2) bodies in similar states of electrification repelled; (3) bodies in unlike states of electrification attracted. The two states are that shown by glass rubbed with silk and that by vulcanite rubbed with fur. Sealing-wax rubbed with flannel gives the same electrification as vulcanite rubbed with fur, for they repel. Paper rubbed with india-rubber repels glass rubbed with silk, so that its electrification is like that of the glass.

The early investigators then decided on names for the two electrifications. Partly for reasons which will be seen presently, they finally agreed to call the state of electrification shown by glass rubbed with silk **positive,** and to say that any body showing this electrification was *positively charged.* They further agreed to name the state of electrification shown by vulcanite rubbed with fur

negative, and to say that any body showing this electrification was *negatively charged.* Hence:—(1) Positively charged bodies repel each other. (2) Negatively charged bodies repel each other. (3) A positively charged body and a negatively charged one attract.

As mentioned above, the kind of electrification depended on the rubber as well as on the body rubbed. Glass rubbed with silk becomes positively electrified, but the same glass warmed and rubbed with fur becomes negatively electrified: rough or ground glass rubbed with silk becomes negative. Vulcanite rubbed with fur becomes negative, but when rubbed with india-rubber it becomes positive. For experimental purposes, it is useful to remember that, in general, common metals rubbed with fur become negative, but if rubbed with india-rubber they become positive.

Experiment also showed that when, with proper precautions, glass, for example, was rubbed with silk, not only was the glass positively electrified, but at the same time the silk was negatively electrified and by an equal amount. This applied in all cases, i.e. *when bodies are electrified by rubbing them, both states of electrification are always produced at the same time and in equal amounts, the one on the body rubbed, the other on the rubber.*

The choice by the early scientists of the names *positive* and *negative* was justified, for the properties of positive and negative electrifications are somewhat similar, in some respects, to the properties of the signs + and − in Algebra, *e.g.* equal positive and negative electrifications are capable of neutralising or "cancelling" each other. At the same time it must be remembered that the names were *allotted* more or less arbitrarily: they *agreed* to say the glass was positive and the vulcanite negative. In view of what modern work has proved, it would have been better if they had allotted the names the other way about, *i.e.* called the glass negative, vulcanite positive.

It will be seen later that it is not the friction of the rubbing process, but the *contact of the dissimilar substances* which is the true cause of the electrifications in the experiments considered. If a rod of paraffin-wax be merely dipped in water contained in a glass vessel and then be taken out, the wax is negative and the water positive: if two different metals are merely put in contact, both become slightly electrified—one positive, the other negative. In the case of glass, vulcanite, etc., intimate contact at a large number of points along the surface is necessary, since they do not allow "electrification" to spread over them from one or two points of contact, and this is secured by rubbing, the function of which is to obtain an extended close contact.

It may be noted, too, that in all these rubbing experiments of the early days, and even in all the great electrical developments of later days, we never *make* "electricity"—we merely cause certain changes in what Nature has already provided. We have, however, so increased our knowledge, and so harnessed "electricity" to our needs, that it is one of the most important

factors in our industrial, domestic, and social lives. And we have gone further than that. We have discovered that the chemical and most of the physical properties of everything are really electrical properties. We have discovered that the actual rock-bottom " make-up " of everything is electrical —the things we see around us, all kinds of matter, whether on this earth or on the other heavenly bodies, whether alive or dead. Electricity is, in fact, the fundamental brick which builds up everything.

A further step by the early scientists was to find some explanation, some theory of electricity and electrification, to fit the knowledge they had gathered together. Now in the old days when anything mysterious occurred it was the custom to put it down to the action of some " spirit " or " fairy " or " demon," or " black vapour," or " fluid," etc. Thus heat was at one time looked upon as a fluid, and similarly the two electrifications were put down as being due to the presence in a body of *two imponderable fluids*. When the two fluids were present in equal amounts the body was neutral: if one of the fluids was in excess of the other the body showed one state of electrification: if the other fluid was in excess, the body showed the other state: the two fluids attracted each other, but fluid repelled fluid of the same kind. This *two-fluid theory* was propounded by Symmer.

Another theory was that electricity was a *single imponderable fluid*, and that every body in its normal or neutral condition contained its definite or stock-amount of this fluid: if a body contained more than its stock amount, *i.e.* had an *excess* of electric fluid, it was positively charged, and if it contained less than its stock amount, *i.e.* was *deficient* in fluid, it was negatively charged. Thus when glass was rubbed with silk some of this fluid was supposed to pass from the silk to the glass, so that the glass had an excess of fluid and was positive, and the silk had an equal deficit and was (equally) negative. This electric fluid was assumed to attract ordinary matter but to repel electric fluid. The *one fluid theory*, due in its original form to Benjamin Franklin, became quite a favourite and its influence is still felt to-day.

Fluid theories have, of course, been abandoned, and brilliant research in recent years has carried us at least one vital step forward in the search for " explanation "—for " fundamental causes." Now experience indicates that at the outset of his electrical studies a student *can* appreciate and understand many *elementary* facts in connexion with the modern idea, and apply them to the explanation of many electrical phenomena and standard experiments he encounters in his reading. In the next section we

give, therefore, in simple language, a few points about the modern conception—just sufficient to enable the student, *from the beginning*, to view matters, wherever possible, on the lines on which they are viewed to-day and not as they were regarded years ago. Further details are given in subsequent chapters as they are required.

2. Protons and Electrons

As is well known, an *element* is a substance which cannot be decomposed into other substances, and there are just about ninety different kinds known to us. Theory and experiment indicate that there should be ninety-two (and the existence of a ninety-third has been *suggested*). Most substances we encounter are not elements, but they are all made up of elements, although their properties may be quite distinct from those of the elements composing them.

Consider, now, any particular element, say copper. Common experience shows that a sheet of copper can be divided into smaller and smaller pieces, and it might be surmised that, given suitable mechanical and chemical means, this division could be carried on indefinitely. Further investigation showed, however, that there was a limit to the process, that the copper was, in fact, composed of particles beyond which it seemed impossible to go, and the same applied to all the other elements. These smallest particles of the elements which behave generally as if they could not be further divided, are known as **atoms,** so that there are just about ninety different kinds of atoms known to us. All are excessively small: it would take of the order a hundred million side by side to make an inch: it takes 6×10^{23} atoms of hydrogen to make up about 1 gramme of hydrogen. The chemist's precise definition of the atom is given later.

To be exact, the smallest particle of a substance which *can exist by itself as that substance* is a molecule of the substance. But a molecule is made up of one or more atoms, and in the case of the element copper the molecule is simply one atom. This, however, is a detail at present.

Now modern work in Physics has shown that an atom, small though it is, is neither fundamental nor indivisible, but has a complex internal structure of its own. Theories of atomic structure are still being investigated, and certain modifications may still be made, but latest work supports the conception that, internally, an atom consists of a central core or **nucleus** surrounded by a number of tiny particles in very rapid motion, spinning, and also travelling all round the nucleus. The actual volume of the nucleus *plus* that of

all the moving particles is, however, small compared with the volume which the entire atom occupies—the nucleus and the moving particles are so small that the structure of the atom is, comparatively, very open. The tiny particles in constant movement are called **electrons,** and they show the same electrification as a vulcanite rod which has been rubbed with fur, *i.e.* they are *negative.* The nucleus has *on the whole* a *positive* charge due to the presence in it of positive particles called **protons.** When the atom is neutral the total (*free*) positive charge at the nucleus is equal to the total negative charge of all the electrons outside the nucleus.

Electrons have been detached from all sorts of atoms and carefully investigated, and in all cases they are identical. They cannot be split up into anything simpler, and are the lightest "things" known, their mass (which is probably purely "electricity") being always about $\frac{1}{1850}$ of the mass of the hydrogen atom which is the lightest atom known. Moreover, each always shows the same amount of electricity, and this is the smallest amount or "charge" of electricity it is possible to obtain: hence the charge is often spoken of as a *natural unit of electricity.* They are inconceivably small: it would take of the order sixteen million times a million of them in a row touching each other to make an inch, so that even the tiny atom is of the order four thousand million, million times the very tiny electron. As stated, the electrons have a spinning motion, and they also travel round the nucleus somewhere about 10^{19} times per second, but the *exact* nature of the movement all round the nucleus is not known. The positive charge of a proton is equal to the negative charge of an electron. The mass of a proton is, however, much greater than that of an electron: it is practically the same as that of a hydrogen atom, *i.e.* about 1850 times the mass of an electron. It follows from this that the total mass of an atom is due almost entirely to the nucleus where the protons are.

Now modern work indicates that *every* element has its atom made up in the way indicated above, and that it is only the number and arrangement of the "charges" which determine *what the substance is*: in other words, the *distinguishing chemical properties* of the element depend upon the number and arrangement of the electrons outside the nucleus (remember their total negative is *numerically* the same as the *free* positive at the nucleus). The chemist's Periodic Table, found in any Chemistry book, gives a list of all the elements arranged in the order of increasing atomic weight. Taking the first ten on this list, viz. hydrogen, helium, lithium,

beryllium, boron, carbon, nitrogen, oxygen, fluorine, neon, a neutral atom of hydrogen (the lightest atom) consists of one moving electron (negative) outside the nucleus and a nucleus which is an equal positive charge or proton; an atom of helium has two electrons outside the nucleus, and the latter has two equal *free* positive charges; lithium has three electrons outside the nucleus and a nucleus with three equal *free* positive charges; beryllium has four, boron five, carbon six, nitrogen seven, oxygen eight, fluorine nine, neon ten, and so on up to the heaviest element, uranium, with ninety-two. As a further illustration, copper, the twenty-ninth element on the list, has an atom with 29 electrons outside the nucleus which means a negative charge outside of 29 units, and the *free* positive charge at the nucleus is also 29 units, *i.e.* a positive charge equal to that of 29 protons. By a unit charge is meant here the charge of an electron (or proton).

The (negative) electrons are kept in the atom by the attraction exerted on them by the positive nucleus. It is clear that the

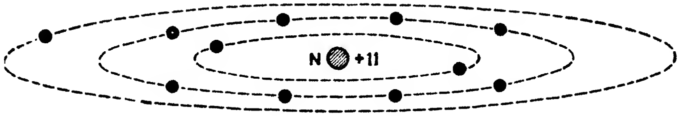


FIG. 2. Sodium Atom.

electrons near the nucleus will be more firmly held and more difficult to pull out of the atom than those further from the nucleus, the electrons near the surface requiring the least expenditure of energy to detach them, for apart from being at the greatest distance from the attracting nucleus they are subject to the repulsion of the electrons in between. These *and other more exact energy considerations* which cannot be given at this stage have led to the conclusion that the electrons outside the nucleus can be regarded as arranged and travelling in successive *shells* or *layers* or *levels* all round the nucleus, the electrons in each level being associated with a definite amount of energy (within certain limits) and necessitating the expenditure of a definite amount of energy for their removal. These **energy levels** are denoted by the letters K, L, M, N, O, P, etc., the K level being nearest the nucleus. Thus the sodium atom has a *free* positive charge of 11 at the nucleus, and 11 electrons outside arranged and moving in three shells round the nucleus, 2 electrons moving in the innermost or K energy level, 8 in the L level, and 1 in the M level. This atom is *roughly* indicated in Fig. 2.

A Warning Note.—It is impossible to *really* represent an atom in a sketch, and Fig. 2 must not be taken too literally: it must not be taken to mean that the electrons are all in one plane, nor that electrons travel round the nucleus in *simple curves* like the orbits of the solar system. This latter planetary analogy has often been used in the past, but it is misleading, and the word "orbit" is now ceasing to be used to any extent by physicists in speaking of atomic structure. The dotted curves in the figure are merely to be taken as roughly indicating that in the sodium atom there are these three energy shells or levels *all round* the nucleus, and that the 11 electrons are housed in the three shells as shown.

To further illustrate the point, details of the first twenty elements of the Periodic Table are given below. The one electron of the

No.	Element	Free Charge at Nucleus	Electrons (– ve) in the Levels:—			
			K	L	M	N
1	Hydrogen	+ 1	1			
2	Helium	+ 2	2			
3	*Lithium*	+ 3	2	1		
4	Beryllium	+ 4	2	2		
5	Boron	+ 5	2	3		
6	Carbon	+ 6	2	4		
7	Nitrogen	+ 7	2	5		
8	Oxygen	+ 8	2	6		
9	Fluorine	+ 9	2	7		
10	Neon	+ 10	2	8		
11	*Sodium*	+ 11	2	8	1	
12	Magnesium	+ 12	2	8	2	
13	Aluminium	+ 13	2	8	3	
14	Silicon	+ 14	2	8	4	
15	Phosphorus	+ 15	2	8	5	
16	Sulphur	+ 16	2	8	6	
17	Chlorine	+ 17	2	8	7	
18	Argon	+ 18	2	8	8	
19	*Potassium*	+ 19	2	8	8	1
20	Calcium	+ 20	2	8	8	2

hydrogen atom and the two of the helium atom move in the K level, but this level will take no more than two, so that the third electron of lithium forms a new level L. This continues until neon is reached with its ten electrons arranged two in the K level and eight in the L; but this level will take no more than eight, so that the eleventh electron of sodium forms a new level M. This again continues until argon is reached with eight of its electrons in the M level; but *when this is an outermost level* it will take no more than eight, and the additional electron of potassium forms a new level N. This is repeated *although the general arrangement becomes more complicated further down the list.* Note that the greatest number of electrons an *outermost level* will take is eight.

Now just a few words at this stage about the nucleus which, unlike an electron, has a certain internal structure of its own. We have seen that the nucleus has *on the whole* a positive charge (*free* + charge) due to the protons in it, equal to the negative charge of all the electrons outside: but, hydrogen excepted, there are electron charges in the nucleus also. Some at least of the electrons and protons in the nucleus are *grouped or combined in various ways*, but we will neglect this at present. Thus, taking the case of an atom of carbon (which, as stated above, has a nucleus with, *on the whole*, a charge + 6 and six electrons outside), the arrangement is as shown in Fig. 3, *i.e.* the nucleus has a charge + 12, for it contains *twelve* protons, but there are also *six* electron charges each − 1, in the nucleus, so that the *free* positive charge is + 6, as stated, balancing the six electrons outside. Note that the position of carbon in the Periodic Table is No. 6, and the chemist's *atomic weight* of carbon (taking hydrogen = 1) is *about* 12.

As another example take the case of helium. This atom has four protons in the nucleus (charge + 4), but there are two electron charges also (charge − 2) so that the *free* positive charge is + 2, as stated, balancing the two electrons outside. Note that the position of helium in the Periodic Table is No. 2, and the atomic weight is *about* 4 (Fig. 4).

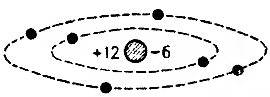


FIG. 3.

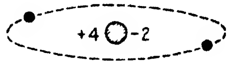


FIG. 4.

As a final example take the heaviest element, uranium. Our present knowledge suggests that the nucleus of the atom contains 238 protons (charge + 238) and 146 electron charges (charge − 146), so that the *free* positive charge is + 92, as stated, balancing the 92 electrons outside. Note that the position of uranium in the Periodic Table is No. 92, and that its atomic weight is *about* 238.

And so with the atoms of the other elements. The position number of the element in the Periodic Table—the **atomic number** as it is called—is the same as the number of electrons outside the nucleus (or, which is the same, the *free* positive charge at the nucleus): the number giving the **atomic weight** is *approximately* the same as the *total* number of protons in the nucleus. This last statement should really be discussed in greater detail, but it would take us too far at this stage. (Atomic weights, for example, are *not always exact whole* numbers: helium, for instance, is 3·97, not 4, but this is dealt with later, and the student need not worry about

it now.) As indicated above, and as will be seen presently, some of the protons and electrons in the nucleus are now taken to be grouped or combined in various ways in order to fit in with certain well-known facts and many recently discovered ones.

So far reference has been made only to the elements, but most ordinary substances are not elements. They are, however, composed of elements, so that when we get down to the rock-bottom make-up of all matter we have the protons and electrons, positive and negative electricity. We have gone from " fluids in bodies " to the " ultimate particles " of all matter, but we have not yet attempted to answer the beginner's perhaps futile question—What is electricity? All we can say so far about this is that **electricity is some fundamental property of the " ultimate small particles " of which all matter is made up which causes them to exert forces of attraction and repulsion on each other.** And after all, *every* definition merely refers the thing defined to something simpler, more fundamental—and there must be some limit to the process. Nature can, and sometimes does, put " Finis " to man's inquisitiveness.

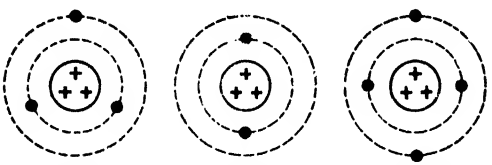


FIG. 5.

In various ways one or more of the *electrons outside the nucleus* can be detached, temporarily, from the atom, in which case the atom exhibits a positive charge since the *free* positive due to the protons at the nucleus now exceeds the negative of the electrons outside the nucleus. Similarly an atom may capture one or more electrons, temporarily, in which case the atom exhibits a negative charge since this exceeds the free positive at the nucleus. An atom (or group of atoms) which has lost electrons is called a **positive ion,** and one which has gained electrons a **negative ion** (Fig. 5). It is clear that it will be the outermost electrons, those most easily detached, which will enter into ordinary experiments of this nature.

When vulcanite is rubbed with fur some of the outer electrons of the fur atoms are transferred to the vulcanite atoms, for the outer electrons of the fur atoms are not so strongly held in the atoms as they are in the case of vulcanite. The vulcanite has therefore an excess of electrons and is negatively charged; the fur is equally deficient in electrons so that the positive centres of the

atoms of the fur predominate and the fur is equally positively charged. When glass is rubbed with silk some of the outer electrons of the glass atoms, being less firmly held than those of the silk, pass to the silk atoms, so that the glass is positively charged and the silk equally negatively charged. Note that *it is the (outer) negative electrons which move in these experiments*—the nucleus is more or less fixed in the atom. The attraction between two oppositely charged bodies is the attraction between the predominating positive nuclei charges of the atoms of one body and the surplus electrons of the atoms of the other.

Reference has been made to *conductors* and *insulators*. The outer electrons of the atoms of any solid substance are not much further from the nuclei of adjacent atoms than they are from their own nucleus, and are under some attraction from them. Now in a conductor these electrons are not held rigidly to their atoms—they are free to wander about in the spaces between the atoms, so that when any additional influence acts tending to move them in any direction they readily drift through the spaces in that particular direction. In a perfect insulator there are no electrons free to move in this way, each nucleus holding on firmly to its own electrons. Of course there is no sharp dividing line between the two: practical conductors oppose the movement in varying degrees, and insulators permit the movement to various extents.

3. A Few More Interesting Points in Modern Theory

The preceding is all that the student need trouble about at present in connexion with the structure of atoms. It is instructive, however, to briefly note one or two other general points which will be dealt with more fully in subsequent chapters: they may be omitted at this stage, but it will add interest to the study of the subject from the outset if the student notes them now. We put them as simply as possible.

The chemical properties of an element and most of its physical properties depend, as has been mentioned, on the number of electrons outside the nucleus of its atom, which, of course, is numerically the same as the free positive charge at the nucleus, or as the " atomic number " (the *mass* of the atom is governed by the *total* protons in the nucleus). It is the number of electrons in the *outermost* level which determines the " valency " of the element, *i.e.* the number of hydrogen atoms with which one atom of the element can combine. These outer-level electrons are in consequence now usually referred to in Chemistry as **valency electrons,** and the ordinary types of

chemical change really consist in the *transfer or sharing of valency electrons* between two or more atoms. A few examples might be quoted:—

If the *outermost* level has its greatest number of electrons (eight), the element is chemically inert, *i.e.* its atom is incapable of combining with other atoms to form chemical compounds of the normal types. This is the case with *neon* and *argon* in the table on page 8, and with certain elements further down the table: it is also so in the special case of *helium* in the table, for the outermost level here is the K for which the maximum is two electrons.

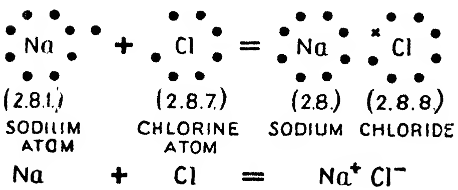


FIG. 6. Na + Cl = NaCl (Molecule of sodium chloride). Only the electrons in the two outer levels of sodium and in the outer level of chlorine are diagrammatically indicated.

If the outermost level contains only one electron (*e.g.* sodium), that electron may not be very strongly held, for the attraction by the nucleus will be nearly cancelled by the repulsion of the electrons in between: hence the electron will more or less readily leave the atom which then becomes a *positive ion*. Similarly an atom with only one electron less than the maximum (*e.g.* chlorine with *seven* in the outer level) will tend to take up an electron to fill the level and the atom becomes a *negative ion*. Atoms of the first kind are said to be *electropositive*, and of the second kind *electronegative*.

Thus sodium (Na) readily combines with chlorine (Cl) to form the compound sodium chloride or common salt (NaCl), and in the combination the solitary outer electron passes from the sodium atom to the chlorine atom so that one is positive, the other negative, and the "molecule" of NaCl is maintained by their mutual attraction (Fig. 6). In the solid state the "*molecule*" of the sodium chloride is neutral, for the *total* + charge still balances the *total* — charge, but when dissolved in water the molecule dissociates or splits up into a sodium atom which has lost an electron, *i.e.* a *positive sodium ion*, and a chlorine atom which has gained an electron, *i.e.* a *negative chlorine ion*. More will be said about the sodium chloride molecule later (page 377).

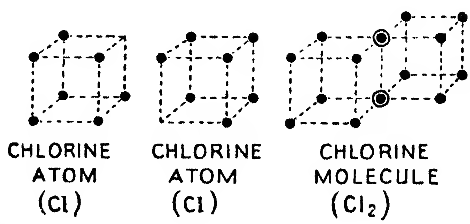


FIG. 7. Cl + Cl = Cl_2 (Molecule of chlorine). Only the electrons in the outer level of the chlorine atom are indicated.

In some cases compounds are not formed in this way, *i.e.* by the *transfer* of electrons, but by the *sharing* of electrons. Such sharing is also seen when

atoms of certain elements join to form molecules. Thus an atom of chlorine has seven electrons in its outer level, and a molecule of chlorine is made up of two atoms, the molecule being formed by the sharing of a pair of electrons (Fig. 7).

It is a well known fact that there are cases where the atoms of any one element are ***not*** exactly alike although the chemical properties are the same. We find, for example, chlorine atoms differing in weight but otherwise having identical properties: such are known as **isotopes.** There are atoms of chlorine (the seventeenth element in the table) the nuclei of which are explained by 37 protons and 20 electrons: the atomic weight is thus about 37, the *free* positive charge at the nucleus is 17 (atomic number) and there are 17 electrons outside. There are also chlorine atoms accounted

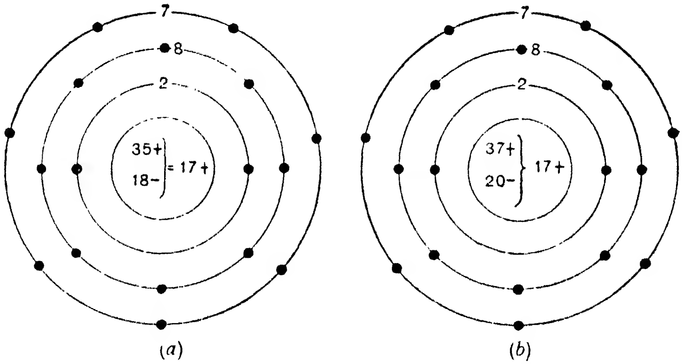


FIG. 8. The two isotopes of chlorine. Atomic weight of (*a*) = 35. Atomic weight of (*b*) = 37.

for by 35 protons and 18 electrons in the nucleus: the atomic weight is thus about 35, the *free* positive at the nucleus is 17 (atomic number), and there are 17 electrons outside. As the chemical properties depend on the electrons outside or on the atomic number, and this is 17 in both cases, the atoms have identical properties although the atomic weight is about 37 in one case and 35 in the other: *they are both chlorine* (Fig. 8). The chlorine encountered in the laboratory contains both kinds of atoms, but more of the second type (slightly more than two-thirds), and the usual atomic weight determination gives the resultant value of about 35·5.

The recently found substance **heavy hydrogen** or **diplogen** is an isotope of hydrogen: its nucleus, called **diplon,** can be accounted for by 2 protons and 1 electron, for its atomic weight is about 2

(hydrogen = 1), but the *free* positive charge at the nucleus is 1, and there is 1 electron outside just as with ordinary hydrogen.

In both the cases considered above, however, it is now generally taken that there are certain "groupings" of the protons and electrons *in the nucleus* as already mentioned: this is referred to again presently.

It is extremely difficult to break up the nucleus of an atom in the laboratory, but it has been accomplished in many cases. However, there are some *heavy atoms* in which the nuclei are not so stable: in fact their nuclei are gradually undergoing a breaking-up process *without any effort on our part* (it seems that the atomic number must exceed 82 for this *natural, spontaneous* breaking-up to happen). These atoms are said to be **radio-active,** and to be undergoing "radio-active disintegration." The best example is **radium,** but other radio-active bodies have been discovered. It must be remembered, then, that *radio-active phenomena are due to the spontaneous disintegration, or splitting up, of atomic nuclei.*

It is clear that the breaking up of the nucleus of an atom (either by Nature or by man), the expulsion from it of protons and electrons *singly* or *grouped*—and they have been ejected both singly and combined—may result in changing the atom into an entirely different atom. Thus if one proton and one electron were ejected from a nucleus, the atomic weight would decrease by 1 owing to the lost proton, but the atomic number, *i.e.* the *free* positive at the nucleus and the number of electrons balanced outside on which the chemical properties of the substance depend, would remain the same—they would simply be isotopes. But if, say, four protons and two electrons were ejected, the atomic weight would decrease by 4 and the atomic number and balanced electrons outside by 2, so that two electrons outside would leave the atom: the atom would therefore be a new atom—a *new substance* with new chemical properties. Radium comes in this way from the heaviest element uranium, by a series of successive radio-active transformations, and our lead is a product of still further successive transformations. Note in passing that, whilst radium is being gradually made from its parent uranium by these transformations, uranium is not being made from any other element: uranium, our heaviest element is therefore slowly—but *very, very slowly*—dying.

A quite usual ejection from the nucleus of the radio-active atom is what is called an **alpha particle,** which has been found to be *the nucleus of an atom of helium* (another frequent ejection from the

nucleus is called a **beta particle,** which has been found to be an *electron*). But the nucleus of a helium atom has been explained as being built up of 4 protons and 2 electrons, so that this is the case referred to above. From this *and other considerations* it may be surmised, as has been often mentioned in preceding pages, that *in the nucleus* of most atoms—not only the heavy radio-active atoms—some at least of the protons and electrons are grouped or combined, and this is supported by the results of laboratory experiments on the breaking up of atomic nuclei.

Thus take the case of, say, the nitrogen nucleus. The atomic weight of nitrogen is approximately 14, and its atomic number is 7 (its *free* charge at the nucleus is + 7 and there are 7 electrons outside), so that its nucleus can be regarded as made up of:—

14 protons + 7 electrons.

But the nucleus would also be accounted for by the following:—

3α particles + 2 protons + 1 electron,

giving again an atomic weight about 14, and an atomic number 7, *i.e.* a *free* charge + 7 to balance 7 electrons outside: the three α particles account for 12 protons and 6 electrons.

Again take the case of the two chlorine isotopes mentioned on page 13. The nucleus of the one of weight 37 can be accounted for by 37 protons + 20 electrons as indicated. It can also be accounted for by:—

8α particles + 5 protons + 4 electrons,

giving an atomic weight about 37 and a free positive charge 17 to balance the 17 electrons outside, and the removal of 2 protons and 2 electrons from this nucleus would give the other isotope of weight 35.

Again, in experiments on the *light nucleus* of the beryllium atom and on that of boron, it was found that very small particles were ejected which had *the same mass as a proton but no charge:* such a particle is called a **neutron,** and is apparently a proton and electron combined.

Take the case of boron. This has an atomic weight about 11 and atomic number 5 (and 5 electrons outside): its nucleus can therefore be regarded as made up of 11 protons + 6 electrons; but it would also be accounted for by:—

2α particles + 1 proton + 2 neutrons,

the 2α particles accounting for 8 protons and 4 electrons and the 2 neutrons for 2 protons and 2 electrons.

As another example, the *diplon* which, as stated on page 13, can be explained by 2 protons + 1 electron giving an atomic weight 2 and an atomic number 1, would also be accounted for by:—

1 proton + 1 neutron,

and quite recently the diplon has been broken up into a proton and a neutron.

The α particle itself which was explained by 4 protons + 2 electrons (helium nucleus) would also be accounted for by:—

2 protons + 2 neutrons,

and this is now generally favoured as the make-up of the α particle. The above three examples are roughly depicted in Fig. 9.

In some quite recent investigations a particle having the same positive charge as a proton but of mass equal to that of an electron has been noted, and it is called a **positron.** (Do not call it a " positive electron," as is sometimes done.) This peculiar case is very elusive, and very rarely met with: its possible explanation is referred to later.

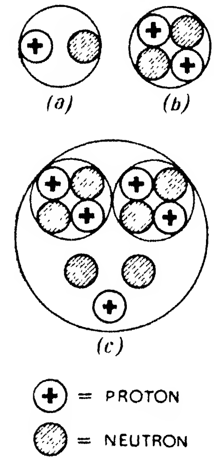


FIG. 9.
Atomic Nuclei. (*a*) = Nucleus of atom of heavy hydrogen (diplon). (*b*) Nucleus of helium atom (α particle). (*c*) Nucleus of boron atom.

The whole subject of the breaking-up of atomic nuclei and the transformation from one kind of atom to another both by nature (as in *natural* radio-active phenomena) and in the laboratory (where, in the process of breaking up atomic nuclei, small amounts of ovei two hundred different forms of "radio-active" matter have been produced) is dealt with later.

In view of the publicity of a *popular* character given to the matter quite recently, one more point may be briefly referred to here for it seems to have aroused some interest even in the general public. In normal radio-activity and in laboratory work on the disintegration of atomic nuclei the particles ejected from the nuclei are all light particles—hydrogen and helium nuclei (protons and α particles), electrons, and neutrons (proton *plus* electron): in other words only light pieces are knocked out. Very recent experiments in France and America seem to indicate that, by bombarding uranium atoms by very fast moving neutrons, the uranium nucleus has been broken up into *rubidium* and *caesium*, *i.e.* into nearly two halves. This "atom splitting" is accomplished by means of a very powerful machine known as the *cyclotron* (another type of machine has just been developed at Cornell University). Now normally, uranium and its successive products go on ejecting α particles, etc., for some thousands of years before even our heavy lead appears, and at each ejection energy is liberated, so that the total energy liberated during the gradual process is very great indeed. It would appear, then, that in these very recent experiments a tremendous amount of energy will be liberated at *one single explosion*—energy estimated by Joliot (France) as equivalent to that represented by 200,000,000 volts (the real meaning of this will be understood later)—and it

is this aspect of the matter which seems to have aroused the public interest. In connexion with it, however, it must be remembered that *energy has to be applied* to bring about the explosion, and that many millions, probably, of bombarding projectiles are necessary to get one really effective hit: input energy must be taken into account in considering output, and in operations and appliances in general, input exceeds output. The matter is referred to again later.

4. Continuous or Direct Electric Currents (D.C.)

It is well known that the expression " electric current " means a flow of electricity. Now, at first sight there seems little connexion between the electrification produced in a glass rod rubbed with silk or a vulcanite rod rubbed with fur, which causes them to attract, say, a piece of paper, and the *electric current* from a battery or dynamo, but, so far as " electricity " is concerned, there is no difference. We can electrify a brass ball by rubbing it, then join it to another brass ball by a wire, and we get a flow of electricity in the wire, *i.e.* an electric current: but such a current is of little use for studying the effects of a flow of electricity, for the quantity of electricity involved is small, the flow is not steady, and it only lasts for a moment. The first production of an electric current, as the expression is now understood, came about in a somewhat different way.

In 1780, Galvani, an Italian professor, was dissecting frogs on a table on which an electrical machine (Chapter V.) was working, and he found that when he touched the main nerve of the frog's leg with his dissecting knife the dead frog kicked. Convinced that this was due to some electrical action, he decided to try the effect of a thunderstorm, and proceeded to fasten his dead frogs by brass hooks to an iron fence in readiness for the next storm. The day was fine, no thunder was about, yet whenever he fixed the hook attached to the frog's leg on to the fence the frog kicked: he therefore concluded that the frog contained the electricity which was set free in some way by the hook and the fence.

Volta (1745-1827), another Italian professor, thought that the electricity was liberated owing to the *brass* hook, the *iron* fence, and the *natural liquid* of the frog's leg all being in contact: in fact he thought a frog was not necessary, and that all that was wanted was two different metals and a suitable liquid. Consequently he started experimenting on these lines, and in 1800 produced what was called the *voltaic pile.* This consisted of several discs of zinc and silver, together with discs of paper soaked in salt water, the various discs being arranged as shown in Fig. 10. On placing one

hand on the top disc and the other on the bottom disc a small electric shock was felt as long as the hands were kept there. Later, instead of using paper soaked in salt water, he used cups containing salt water, and arranged his apparatus as shown in Fig. 11; this came to be known as the *crown of cups*. If a wire joined the ends of the crown of cups a *continuous* flow of electricity took place through the wire and through the cups. His device was the first battery, and the current it gives is spoken of as a **continuous** or **direct current** (D.C.).

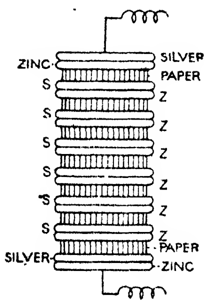


FIG. 10.

We can now examine this question of the electric current in a little more detail and in quite an elementary way, *but in the light of modern views.*

It is well known that water flows from a high water level to a low water level. Thus, if the two vessels of Fig. 12 contain water as indicated, then on opening the stop-cock water will flow from A to B, and the flow will continue until the two come to the same level. The actual quantity of water in B may be considerably greater than the quantity in A; *it is the difference in level which settles the direction in which the water will flow.*

Similarly, heat flows from a body at a high temperature to a body at a lower temperature: and air flows from a place where the air pressure is high to a place where the air pressure is lower. In the first case it is the *difference in temperature* which settles the

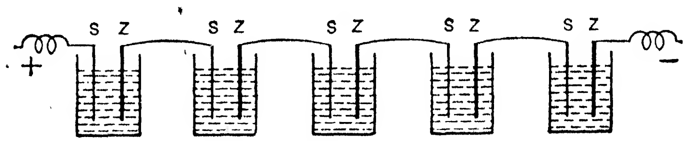


FIG. 11.

direction in which the heat flows. In the second case it is the *difference in air pressure* which settles the direction in which the air flows. Now in the study of electricity, **electrical potential** (or *electric pressure*) is used in a somewhat similar way to temperature, level, and pressure in the above cases (the *exact* meaning of electrical potential will be given later): but a little " snag " arises here owing to the old fluid ideas conflicting with modern views.

Consider a positively charged brass ball which is therefore deficient in electrons. If it be joined to the earth by a wire it becomes neutral, its " charge " disappears, so that there must have been a *flow of electrons in the direction earth to ball* which has made up the deficit of electrons. But electricity *flowing* is an electric current: hence a current has passed in the direction earth to positive ball. This is spoken of as the **electronic current**: it is the *true* current for it is the electrons (negative) which actually *move* along the wire.

But the early pioneers reasoned in this way. The positive ball, they said, has a surplus of electric fluid: when joined to earth this surplus fluid flows to earth, and it does so because the electric pressure (or potential) of the ball is higher than the electric pressure (or potential) of the earth. *The electric current,* they said, *flowed from the higher potential ball to the lower potential earth.* Further, the earth was taken as the zero of potential (just as the sea is taken as zero " level "), and the ball was said to be at a positive (above zero) potential. As this direction of flow is still spoken of we call it the **conventional current** to distinguish it from the true electronic current which flows the other way, *i.e.* from what they had called the lower to the higher potential.

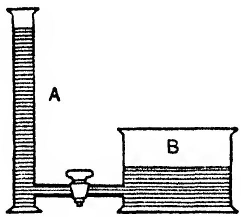


FIG. 12.

Again, if a negatively charged ball, *i.e.* one with an excess of electrons, be joined to the earth it also becomes neutral so that there must have been a flow of electrons in the direction negative ball to earth, *i.e.* an electronic current has flowed in this direction. But the early pioneers reasoned in this way. The negative ball is deficient in electric fluid: when joined to earth electric fluid flows from the earth to make up the deficit, and it does so because the electric pressure (or potential) of the ball is lower than the electric pressure (or potential) of the earth. *The electric current,* they said, *flowed from the higher potential earth to the lower potential ball,* and as the earth was taken as zero potential, the ball was said to be at a negative (below zero) potential: this current must again be called the conventional current to distinguish it from the true electronic current which flows the other way, *i.e.* from negative ball to earth, or from what they had called the lower to the higher potential.

If a plate of zinc and a plate of copper be immersed in dilute sulphuric acid (Fig. 13), the zinc acquires a negative charge and the copper a positive charge. There is, therefore, excess electrons

at the zinc, positive ions at the copper, and a **potential difference** (P.D.) is set up between the plates, the copper being at a higher potential than the zinc. Such an arrangement is a **simple voltaic cell,** the copper being called the positive pole and the zinc the negative pole of the cell. On joining the poles by a wire BA, *electrons set off along it* in the direction B to A, *i.e.* from zinc to copper, the electrons drifting in the spaces between the atoms, colliding with each other and with outer electrons of the atoms, driving some of the latter out and taking their place; and at the end A the evicted electrons are taken in by the positive ions of the copper waiting as it were for them. One would expect the flow to last only for a short time, for electrons leaving the zinc would tend to raise the potential of the zinc (make it *less negative*), and electrons entering the copper would tend to lower the potential of the copper (make it *less positive*), *i.e.* the flow of electrons would tend to equalise the potentials of the plates in which case the flow would stop; but the chemical action in the cell, as will be seen in Chapter IX., keeps up the P.D. between the plates so that a continuous steady flow is maintained.

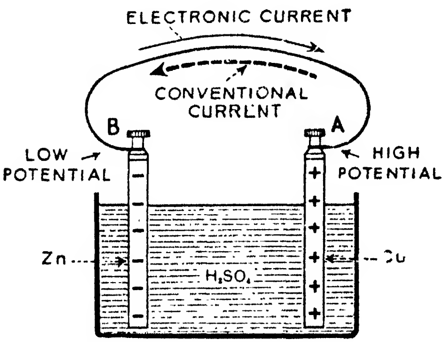


FIG. 13. A Simple Cell.

This *movement of electrons* in the general direction B to A of the copper wire, *i.e.* from the negative pole to the positive pole of the battery, or from what had been called the lower to the higher potential, is the *electric current*, and the greater the number of electrons passing any section of the wire per second the greater is said to be the **current strength.** This is, of course, the true movement for any ions in the wire are fixed. It is still often said, however, that " the positive pole is at a higher potential than the negative pole, and this P.D. drives a current from the positive to the negative pole through the wire," *i.e.* the conventional current direction is often referred to. In many cases it does not matter which view is taken for current direction, but in several modern practical applications of electricity, and in most modern research work, it is necessary to consider the modern view for the old ideas cannot explain the facts.

A Warning Digression. A word of warning to the beginner is necessary here. Consider a long pipe full of water, and that water is being made to flow steadily along it. As water is forced in at one end an equal amount leaves the other end *practically instantaneously*, but any particular particle of water entering at one end may take a fairly long time to reach the other end and be ejected. It is somewhat similar with the electric current. Electrons are drained from the zinc into the end B of the wire (Fig. 13), practically instantaneously an equal number of electrons are ejected from the end A into the copper, and there is a " movement," an " easing off " of electrons at every part of the wire in the direction zinc to copper: but it would take any particular electron entering B a long time to reach A and be ejected. In fact if the wire AB was one square millimetre in cross-section and the current strength was one *ampere* (the ampere is the unit of current, and is defined later), the drift velocity of the electrons would only be about ·004 centimetre per second: but although each electron only moves a short distanee, there is a general drift of electrons at every part of the wire in the direction zinc to copper. The rate at which the " electron disturbance " is propagated along the wire is very great—practically instantaneous—nearly the same as light.

It has been emphasised that in metallic conductors—wires and cables—*the electrons only are the " carriers " of electricity, i.e.* the current consists of a movement of electrons in the general direction lower to higher potential. In certain liquids called electrolytes positive *ions* and negative *ions* move in opposite directions, the negative ions from low to high potential and the positive ions from high to low potential. In gases, electrons and positive ions move in opposite directions. We are mainly concerned at present, however, with currents in metallic conductors, and in these cases the movement is that of electrons only—the electronic current.

5. Alternating Electric Currents (A.C.)

The current from a battery flows continuously in one direction (D.C.). There are, however, currents which do not flow in this way but regularly reverse, flowing for a certain time in one direction, then reversing, and flowing for the same time in the opposite direction; such are called **alternating currents** (A.C.).

In practice, alternating currents vary periodically both in strength and direction, *i.e.* not only do the electrons flow first one way then the other in the wire, but the number actually moving varies from instant to instant, and this variation in strength and direction is conveniently represented by a curve as shown in Fig. 14, where *current strength* is marked off along the vertical and *time in seconds* along the horizontal. If the curve is above the horizontal line the current is flowing in one direction in the conducting wire

(say to the right), and if the curve is below the horizontal the current is flowing in the other direction (to the left). Thus the current strength increases from zero at O to a maximum at P, and then decreases to zero at Q, and during this time, represented by OQ, it has been flowing from left to right in the wire: then the current comes on in the opposite direction (right to left in the wire), and the strength increases from zero at Q to a maximum at R and then decreases to zero at S: then the current reverses, comes on in the first direction, and the variations are repeated.

The time which elapses, say, from the instant when the current is a *maximum* in one direction to the *next* instant when it is *a maximum and flowing in the same direction* is called the **period** of the alternating current, and the number of such time intervals in one second is called the **frequency**: thus in Fig. 14 the time taken from the condition P to the condition T (or from O to S, or Q to V, or

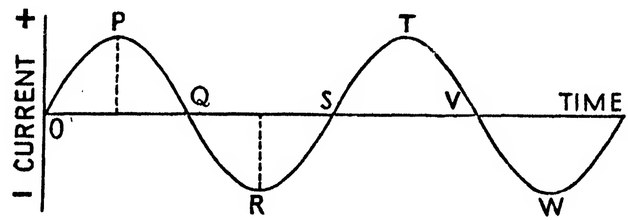


FIG. 14.

R to W) is the *period*, and the number of these time intervals in one second is the *frequency*. In doing a double surge, say to the right and back to the left, the current goes through all its values in both directions, and the process of doing so is called a **cycle of current.** Thus in Fig. 14 the curve from O to S represents a cycle: hence *the frequency is evidently given by the number of cycles executed in one second.*

The frequency in common use for lighting and power is 50 cycles per second (period therefore $\frac{1}{50}$ second). Thus if Fig. 14 represents such a current, it increases from zero at O to a maximum at P in $\frac{1}{200}$ second, then falls to zero at Q in the next $\frac{1}{200}$ second, and so on: the time from O to S is the period, viz. $\frac{1}{50}$ second. When the frequency rises to the order 100,000 cycles per second the current is generally called a **high frequency (H.F.) oscillatory current,** or a *high frequency electrical oscillation.* Such H.F. currents—of the order 1,000,000 cycles and more per second—are used in wireless,

and even higher frequencies are met with in television—60,000,000 cycles or more per second.

Alternating currents are dealt with in detail in Chapter XVIII.

6. Magnets and Magnetism

The name " magnet " was first applied to pieces of a black mineral found near the town of *Magnes* (now Manissa), which is not far from Smyrna in Asia Minor, and it was known to the Greeks so far back as 470 B.C. This *natural magnet* or " magnesian stone " which possessed the power of attracting iron, is now known as magnetite or magnetic oxide of iron (Fe_3O_4). As some pieces of magnesian stone, suitably shaped and suspended, were found to set in a definite direction nearly north and south, it came to be known as *lodestone* or lead-the-way stone: it led the way in navigation—hence the ship's compass.

As ordinarily understood, a magnet (*artificial magnet*) is a piece of iron or steel (generally horse-shoe shaped or in the form of a bar) which, like the natural lodestone, attracts other pieces of iron or steel, and, when suspended, comes to rest in a definite, nearly north and south direction, the one end always pointing to the north and the other to the south: and further, if a piece of iron or steel be rubbed with a magnet the iron or steel becomes a magnet. Only iron and steel can be made to show this *magnetism* to any marked extent: nickel, cobalt, and manganese can show magnetic properties, but they are weak compared with iron and steel. The force which a magnet exerts on iron and steel is called a *magnetic force.*

Now, here again, at first sight there seems little connexion between magnetism and either electric charges on bodies or electric currents, but just as the latter two are fundamentally the same, save that in one case the electric charge is " at rest " (so called) on the body and in the other case it is moving through the conductor, so magnetism is closely connected with electricity: magnetism is really due to electricity in motion.

Thus, whenever charged bodies are set in motion magnetic forces are produced. If there are two charged bodies a short distance apart there will be a certain electric force between them: if they are set in motion along parallel paths, the electric force between them becomes less owing to the setting up of magnetic force which partly cancels the electric force. If a current of electricity is set flowing in a wire, magnetic force immediately appears in the space about the wire. If a wire be coiled round a bar of iron and a current be started in the wire, the bar becomes a magnet. Similarly,

the magnetic properties of the natural lodestone, and in fact of all magnets and magnetic substances, are really due to the movement of electric charges—the electrons—inside the atoms, for the spins and movements round the nucleus will evidently have magnetic effects similar to the currents mentioned above. The total magnetic effect of an atom will be the resultant of the effects of the various electrons: it will be great if the movements are in the same direction so that the individual effects help each other: it will be nil if some directions are opposite to others so as to completely balance each other.

In the early days, and, in fact, almost into comparatively recent times, magnetism was studied in connexion with *masses of material*, rods of iron, etc. In quite recent years experiments in connexion with single atoms moving in spaces where magnetic force is acting, and with the effects of magnetic force on radiation from atoms, have led to consideration of the *magnetic properties of single atoms*, and the conclusions are those mentioned above—that magnetism is due to the movements of the electrons inside the atoms of the material.

CHAPTER II

GENERAL PRINCIPLES OF MAGNETISM

REFERENCE has been made (page 23) to the lodestone or *natural magnet* and to *artificial magnets* of iron or steel; and we have seen that the most striking features about a magnet are: (1) it attracts other pieces of iron and steel; (2) if suspended it comes to rest in a definite direction nearly north and south, the one end always pointing north and the other end south; (3) by means of it other pieces of iron and steel can be made into magnets. It has also been mentioned that very few substances can be made to show "magnetic properties" to any extent—iron, steel, nickel, cobalt being the chief examples, iron and steel standing out far ahead of the others.

In 1845 Faraday concluded, however, that all substances were to a more or less degree *affected* by a magnet, a few being attracted but most repelled. Further investigation led to the classification of all substances into three groups, viz. (1) *Ferromagnetic substances*, a small group definitely attracted by the end of a magnet, consisting of iron, steel, nickel, cobalt. (2) *Paramagnetic substances* which are feebly attracted, consisting of manganese, crown glass, platinum, aluminium, oxygen, and a few other substances. (3) *Diamagnetic substances* which are repelled: this is the largest group, the best examples being bismuth, phosphorus, and antimony, but all diamagnetic effects are *very* feeble. The distinguishing characteristics of ferromagnetics, paramagnetics, and diamagnetics will be given later (pages 49-55, 95).

In practical electrical work the ferromagnetics and paramagnetics are sometimes spoken of as *magnetic* substances, and all others as *non-magnetic*.

Now it was stated in Chapter I. that magnetism was a phenomenon due to *electricity in motion*, that the magnetic properties of lodestone and artificial magnets were due to the movement of electrons inside the atoms, and the same applies fundamentally to other substances. It was also stated that when a current flowed in a wire magnetic effects similar to those shown by a magnet immediately appeared. Thus if a current flows in a coil such as is shown in Fig. 15 the coil behaves in the same way as a bar magnet.

A piece of iron hung by a thread and brought near either end is attracted. If the coil be suitably suspended (*e.g.* as shown) so that it can move in a horizontal plane it comes to rest nearly north and south. (The end at which the electrons of the current are moving clockwise—*i.e.* conventional current direction *counterclockwise*—points north, and the other end where the conventional direction is *clockwise* points south.) If a bar of iron be put inside the coil the iron is converted into a magnet. As another example, if a coil of wire, a small dry battery, and a large cork be arranged as in Fig. 16, and the whole floated at the centre of a large vessel of water, the arrangement will turn so that the face of the coil at which the conventional current is counterclockwise is towards the north.

Since then all "magnetism" is due to electricity in motion it would perhaps be more logical to commence the study of magnetism

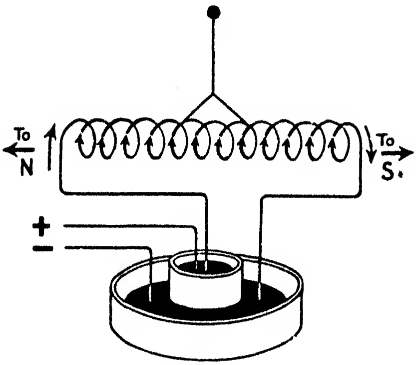


FIG. 15.

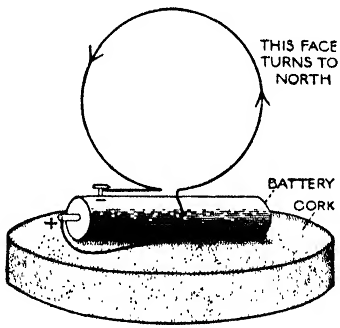


FIG. 16.

In Figs. 15 and 16 the *conventional* current is indicated by arrows.

from the point of view of electric currents. This method of approach is, however, more difficult for the beginner and the necessary experiments more complicated. Whilst therefore dealing with these current effects as required and emphasising the unity of electric charges, currents and magnetism, we will begin with magnetic properties as shown by magnetic material which lends itself to quite simple experiment.

1. Magnetic Poles

If a bar magnet be placed in iron filings the latter mainly adhere to *regions* near the ends, none clinging to the centre. *The strongest parts of a magnet are therefore near the ends*, and these are called

the **poles,** the one which points towards the north when the magnet is suspended being called the " north pole," and the other the " south pole " of the magnet.

We can roughly show how the power of attraction (or, as it is usually, but more loosely, worded, the distribution of " magnetism ") varies along a bar magnet thus:—Lay a magnet on a sheet of paper and pencil-mark round it. Hang a small ball of soft iron from a spring balance, let the ball rest on the magnet at different points, and note at each point the " pull " (force) necessary to detach the ball. Erect perpendiculars on the paper at the various points, and mark off along them distances representing these forces. By joining the points a " curve of distribution " is obtained as in Fig. 17. It is usual to draw the curves for the two halves at opposite sides.

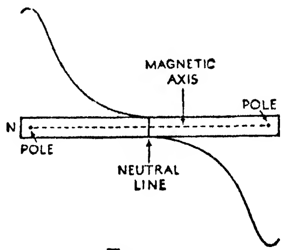


FIG. 17.

It might be thought from Fig. 17 that if the magnet were broken at the centre one half would show " north pole properties," and the other half " south," but this is not the case. Every separate piece of a broken magnet is a complete magnet with its two poles, and, in fact, **it is impossible to produce a magnet with only one pole.** At each " break " a north pole appears at one side and a south pole at the other (Fig. 18).

Like positive and negative charges, north and south poles, equal in strength, appear at the same time when a magnet is made, but whilst the two charges can be separated or isolated, the positive charge existing on one body and the negative on another, the two poles cannot be isolated, *i.e.* a north pole never exists without its corresponding south pole on the same

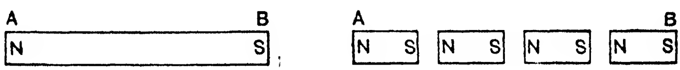


FIG. 18. Breaking the Magnet AB.

body. The nearest approach in practice to, say, an isolated north pole is the north pole at the end of a *very long and thin* magnet.

Again, we have seen that a bar magnet exhibits magnetic attraction in varying degree along its surface, from zero at the centre to a maximum in *regions* near the ends, and, strictly, we cannot regard its " magnetism " as concentrated at two *points* or " poles " within the bar. However, as will be seen later, when a bar magnet is suspended the earth exerts magnetic forces upon every part of it. On the north half these are a set of *parallel* forces acting towards the north, such forces increasing in magnitude towards the

end, and they have a resultant force passing through a point near the end, called in Mechanics the *centre* of the system of the parallel forces. Similarly, for the south half, the forces being in the opposite direction, towards the south, and the centre of the system being a point near the end of this half. These two points may be regarded as the poles. Now it will be seen in Art. 4 that the earth is surrounded by what is called a " magnetic field " which is of uniform strength over extended areas, and in which the magnetic forces due to it are parallel, and *strictly* it is only in respect of the action between an ordinary magnet *and a uniform field* that the " poles " may be regarded as representing the actual magnet. In many investigations, however, we take it that our magnets *are* acted upon by the same forces as they would be if they had equal north and south properties concentrated at two " poles."

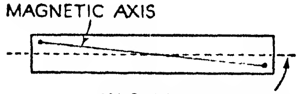


FIG. 19.

The longer the bar in comparison with its thickness (*i.e.* the greater its *dimension ratio*), the nearer the poles are to the ends. An indefinitely thin magnet would exhibit no lateral magnetism, and the poles would be at the ends; such a magnet is called a *simple magnet.* The Robison *ball-ended magnet* consists of a magnetised steel rod on the ends of which balls of steel or iron are screwed, and this acts almost like a simple magnet with poles at the centres of the balls. The straight line joining the poles is called the **magnetic axis** of the magnet, and the line at right angles to this, midway between the poles, is called the **neutral line** of the magnet (see Fig. 17). Note that the magnetic axis of a magnet does not always coincide with its " axis of symmetry " (Fig. 19).

2. Attraction and Repulsion between Poles

In experiments it is often more convenient to use a *compass needle* (Fig. 20) than a suspended magnet: this is a small magnet supported on a fine vertical pivot so that *it can move in a horizontal plane.*

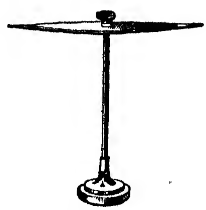


FIG. 20.

Suspend a magnet in a stirrup by a thread, or use a compass. Bring the north pole of a second magnet *gradually* near the north pole of the suspended one: *repulsion* ensues. Bring the south pole of the second near the north pole of the suspended one: *attraction ensues.* Similarly, bring the north pole of the second near the south pole of the first and the result is *attraction.* Bring the south poles *gradually* together and the result is *repulsion.* Thus we have the fundamental law, viz. **like poles repel and unlike poles attract.**

Again, if we have two coils of wire A and B (of which A, say, is suspended) carrying currents, and we present the end of B at which the current is flowing counterclockwise to the end of A at which the current is also counterclockwise, there will be repulsion, but the counterclockwise current end of B will attract the clockwise current end of A. Further, the S pole of a magnet will repel the clockwise (conventional) current end of a coil, but it will attract the counterclockwise (conventional) current end (Fig. 21).

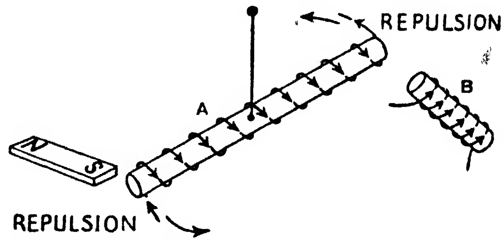


FIG. 21. The coils are wound on cardboard tubes.

The setting of a suspended magnet nearly north and south is another illustration. The earth itself is a magnet having its magnetic " poles " not so very far (practically speaking) from its geographical poles. The magnetic pole in the northern hemisphere is called the " north magnetic pole of the earth," but as it attracts the *north* pole of a magnet the two are unlike poles, i.e. *the north magnetic pole of the earth is like the south pole of a magnet.* Similarly, the magnetic pole in the southern hemisphere is called the " south magnetic pole of the earth "; but it attracts the *south* pole of a magnet, and therefore *the south magnetic pole of the earth is like the north pole of a magnet.* The vertical plane in which the magnetic axis of a suspended magnet or a compass comes to rest is called the **magnetic meridian** *of the earth* at that particular place. In this country the magnetic and geographical meridians are roughly inclined to each other as shown in Fig. 22, *i.e.* the north pole of a compass points a little to the west of the true north: the angle between the two meridians is called the **declination** or *variation.*

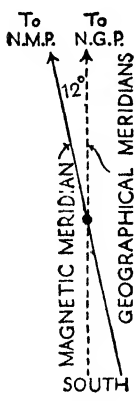


FIG. 22.

3. Magnetic Material near, or in contact with, a Magnet

Support a magnet vertically in a clamp. Bring a short piece of steel near the bottom end: it clings to the magnet. Bring another piece near the first: it clings to the first (Fig. 23). After a time, detach the upper piece from the magnet. The two pieces of steel

still cling together, and on separating and testing them with a compass each will be found to be a magnet with poles as shown. Modify the experiment as indicated in Fig. 24 using bars of soft iron lying on the table but not in contact. The bars are magnetised with the polarity as shown.

These poles produced in a piece of iron or steel or other magnetic material by the influence of another magnet, in contact or at a distance, are called **induced poles,** and the phenomena is sometimes referred to as *magnetic induction.* (As this latter expression is used in a special—though related—sense later, it is perhaps better to use here the expression *inductive influence* of the magnet.) The rule for the induced polarity is clear from the figures, viz. *the end of the iron nearest the magnet pole has polarity opposite to that of the magnet pole*: the N. pole of a magnet induces a S. pole in the near end of an iron bar, and the S. pole induces a N. pole. The experiment also shows why a magnet attracts a piece of iron. When the magnet pole is brought near the iron, the latter is magnetised, and the result is attraction between the magnet pole and the unlike pole induced in the part of the iron nearest to it. (Had the material been diamagnetic its induced polarity would have been the other way about, *i.e.* the near end would have been *like* the magnet pole and there would have been repulsion.)

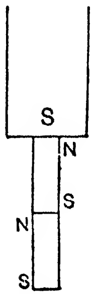


FIG. 23.

The earth, being itself a magnet, also tends to magnetise magnetic material in this way. Thus a vertical bar of iron in the northern hemisphere will be magnetised by the earth's inductive influence, *the bottom end being a north pole* and the top a south. A bar of iron lying horizontally in the meridian will be magnetised, *the end pointing northwards being a north pole.* A bar of iron held in the meridian but inclined (in this country) at an angle of about 67° to the horizontal, the lower end towards the north, will be even more strongly magnetised, *the lower end pointing northwards being a north pole* (the magnetisation will be assisted by tapping the bar with a wooden mallet). All this agrees with the law of induced polarity. The north magnetic pole of the earth is like the south pole of a magnet and therefore induces a north pole in the end of the bar towards

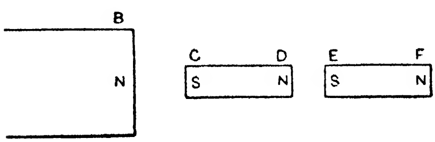


FIG. 24.

it. In the southern hemisphere the vertical bar, for example, would be magnetised *with the bottom end a south pole.*

In this country the *total* magnetic force due to the earth acts in the magnetic meridian, towards the north, and inclined at an angle of about 67° to the horizontal, this angle being called the dip or *inclination.* This is indicated in Fig. 25 where the vertical plane LMM′L′ represents the magnetic meridian, LN the direction of the earth's *total* force T, and $\theta°$ the dip (67° approx.): it is clear that the iron will be best magnetised when placed in the position LN. If LN represents the total force T in magnitude as well as direction, then T can be resolved into a horizontal component H represented by LP and a vertical component V represented by LQ: H is effective in magnetising the horizontal bar, and V in magnetising the vertical bar.

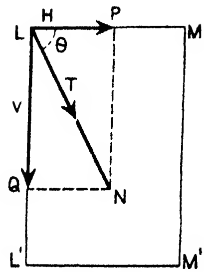


FIG. 25.

The preceding explains to a certain extent *the demagnetising or weakening effect of the poles of a magnet on itself.* Thus consider the material of the bar magnet at O (Fig. 26). Both end poles of the magnet are tending to magnetise the material at O so that *northern polarity is towards the right and southern polarity towards the left,* and this is opposite to the condition of the magnet: thus the end poles are tending to weaken the magnet. It is clear that a short bar magnet will be subject to this self-demagnetisation more than a long thin bar magnet. This self-weakening effect in the case of permanent magnets is reduced by arranging bar magnets in pairs when not in use as shown in Fig. 27, with soft iron "keepers" across the poles. The north pole of a magnet, for example, induces a south pole in the part of the keeper in contact with it, and the effect of this at any point of the magnet is opposite and nearly equal to the demagnetising effect of the magnet pole. This explanation will do for the present, but the full function of keepers will be better understood later (Art. 10).

FIG. 26.

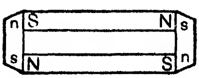


FIG. 27.

4. Magnetic Fields. Lines and Tubes of Force and Induction

The "magnetic field" of a magnet is the space round about the magnet (i.e. *on all sides of it*) *where magnetic effects due to it are felt,* i.e. *the whole space round about where magnetic material will be acted on by magnetic force due to the magnet. A similar definition applies to the magnetic field of a current in a wire.*

Now consider a point P in the field of a bar magnet NS (Fig. 28). It is impossible to get a pole by itself, but *imagine* an isolated north pole placed at P and that it is perfectly free to move. It will be attracted by S in the direction PS, and repelled by N in the direction NP. Let PQ and PR represent these forces: then PT will represent the resultant force on the north pole at P. The pole will be urged in the direction of PT, but as it changes its position the direction of the resultant force changes, and the direction of motion of the pole changes also. Thus the free north pole will be urged along a curve starting from the north end N and finishing at the south end S of the magnet (Fig. 29). *The line (or curve) along which an isolated north pole would travel if free to move in a magnetic field is called a "line of magnetic force."*

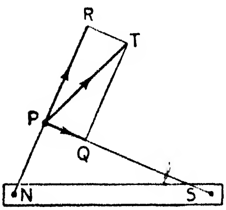


FIG. 28.

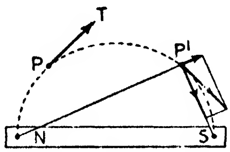


FIG. 29.

Again, consider a small compass *ns* placed at P (Fig. 30). The resultant force on the north pole *n* is represented by the diagonal *a*V of the parallelogram on the right, whilst the resultant force on *s* is represented by the diagonal *b*T on the left. The pole *s* tends to move in the direction of *b*T, whilst the pole *n* tends to move in the direction of *a*V. The net result is that the compass sets with its magnetic axis in a definite direction, and if it is small, *this direction is a tangent to the line of force through the small compass.* Clearly, if the lines of force in a field are straight the compass sets with its magnetic axis along the lines. Further, if the needle is a very small one, or if it is a piece of iron filing magnetised by the inductive influence of the magnet, *we may assume that it lies on and along the line of force whether straight or curved,* and this gives a means of tracing the lines (see below).

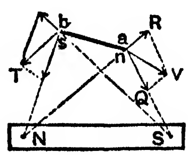


FIG. 30.

To summarise: **a magnetic line of force is a line along which an isolated north pole would travel if free to move in a magnetic field, and it is such that the tangent at any point gives the direction of the resultant force at that point.** Figs. 31-38 show different cases of lines of magnetic force in the case of magnets. The following points should be noted:—

(1) Lines of force cannot cross each other, for this would mean that at the point of intersection the resultant force would be in two different directions, which is impossible. (2) The direction in which

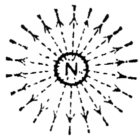


FIG. 31.

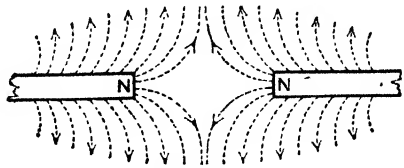


FIG. 32.

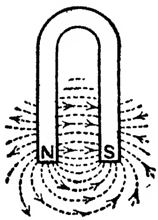


FIG. 33.

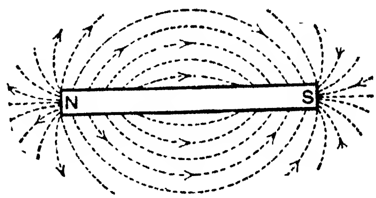


FIG. 34.

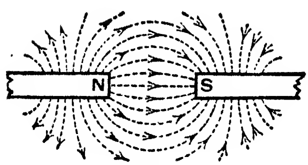


FIG. 35.

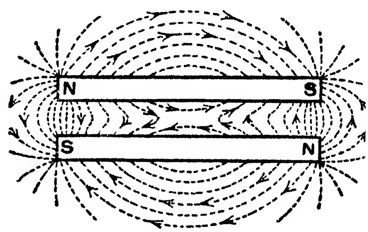


FIG. 36.

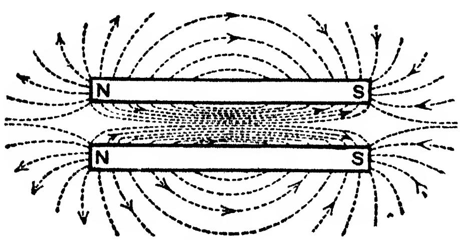


FIG. 37.

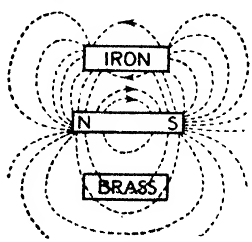


FIG. 38.

a free *north* pole would move along a line is called the positive direction or *the* direction of the line, or the direction of the field: thus the direction is *from N to S* in the figures and is indicated by

arrows. (3) The field of a magnet exists on all sides of it, and is not confined to one plane as the diagrams must be. (4) *In the figures the lines of force start at a N. pole, curve round, and end at a S. pole* (*e.g.* Fig. 35): lines associated with two like poles seem to turn away from each other (*e.g.* Fig. 32). (5) From the diagrams it is seen that in a general way the lines are crowded together near the poles where the field is strongest and are spread out further away from the magnet where the field is weaker: in a *uniform field*, *i.e.* a field which has the same strength and direction at all points, the lines of force would be straight and parallel. (The earth's magnetic field over fairly extended areas is practically uniform.)

Lines of force indicate the *directions* of the forces in a magnetic field, and when we " map the lines in a field " we merely use some magnetic device (see below) for indicating to the eye the general *directions* of the forces, just as a chain, for example, supported at one end will hang vertically and indicate the *direction* of the force of gravitation. Faraday, however, directed his attention to the "medium" in between, say, the poles of two magnets rather than on the action of the poles on each other at a distance, and contended that this medium played an important part in the attraction or repulsion between the poles. In fact, he assumed magnetic lines to be, so to speak, *real connexions* in the medium between the poles rather like stretched elastic threads, and that each thread or " line " tended to contract in the direction of its length, whilst lines proceeding in the same direction tended to repel each other laterally. Thus longitudinal contraction would account for the attraction between unlike poles, for it would pull them together (*e.g.* examine Figs. 35, 36), whilst lateral repulsion would account for the repulsion between like poles, for it would push them apart (Figs. 32, 37).

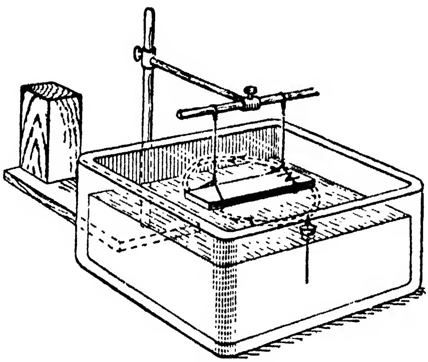


FIG. 39.

There are three simple ways in which we can show to the eye the directions of the magnetic lines in the field of a magnet.

(1) Fix a magnetised knitting needle vertically in a cork and float in a trough of water, the south pole of the needle being downwards and the north pole projecting a little above the cork. Support a bar magnet as shown (Fig. 39) and bring the cork near the north end of it. The south pole of the needle is so far away from the magnet compared with the north pole that we can ignore it. The floating cork and needle will move as indicated, the north pole of the needle travelling along a *curved* path which represents the line of force. Repeat, commencing from various positions, and a series of curved paths will be observed like the lines of force of Fig. 34.

(2) Place a sheet of cardboard (or glass) over a magnet, and dust iron filings over it. The filings come under the inductive action of the magnet, and each, becoming a magnet, sets itself parallel to the direction of the resultant force at the point. On gently tapping the card, the filings arrange themselves in curves, and roughly map out the magnetic lines.

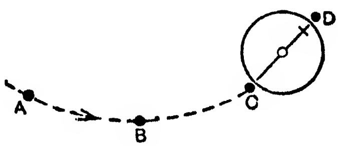


FIG. 40.

(3) Place a *small* compass in the field, and make a mark (A and B) opposite each end, A being opposite its south pole, say, and B opposite its north. Then move the compass along until its south pole is over B, and mark the new position C of its north pole (Fig. 40). Repeat this, and finally draw a curve through the various points; this is the line of force required, and the whole field can be mapped by starting the compass from different positions.

Strictly, *every* point in a field has its line of force, for magnetic force acts at *every* point, so that the number of lines coming from a pole is infinite. In the mathematics of lines of force it is usual, however, to conceive them gathered together in such a way as to form tubular spaces, such tubes touching each other laterally and filling the entire field. These are called **tubes of force,** and when conceived on a definite plan (to be given later—page 58) so that a definite number are assumed to emanate from any given pole they are called **unit tubes of force.** The stronger the pole the more unit tubes of force are conceived to start from it, and the stronger a magnetic field the more unit tubes are conceived to pass through a given area. Further, if these tubes be endowed with the property that they tend to contract in the direction of their length and to

expand laterally, we have practically the Faraday explanation of the fundamental facts in magnetism.

Often the expression " lines " is used in *quantitative work* where " unit tubes," *conceived on a definite plan* to be given later, is really intended, and in electrical engineering it is, in fact, the custom. In this book " lines " is used in a general sense, and " unit tubes " whenever numerical relations are referred to.

In this section we have been concerned solely with the magnetic fields of *permanent magnets in air* and with the *lines* (*and tubes*) *of force* in the fields. Air is (practically) non-magnetic, so that there is no magnetisation of the medium round about our magnets. If, however, part of the medium consists of magnetic material, matters become more complex. This is dealt with fully in Chapter III., but one point may be briefly referred to here for it has some bearing on the case of our permanent magnets.

If a piece of iron or other magnetic material be placed in a magnetic field, *e.g.* the iron CD in the field off N (Fig. 24) or the iron in the field of NS (Fig. 38), it becomes, of course, part of the medium surrounding the magnet: but it undergoes certain magnetic changes itself, whilst at the same time it modifies the original field. This is due to the fact that the iron being magnetic material is *magnetised by inductive influence, i.e.* by the magnetic forces of the field, and develops end poles. From the point of view of magnetic lines, we have now to consider not only the magnetic lines of the original field but also lines due to the magnetisation of the iron including the demagnetising effect of the end poles. So far as the *iron part* of the medium is concerned the net result is, we find, that there are many more lines passing through the iron than passed through the same air space: these resultant magnetic lines are not called lines of force but **lines of induction,** a line of induction being a line in the direction of the " induction." So far as the other part of the medium (air) in the vicinity of the iron is concerned, we find the field is altered, being stronger in some parts and weaker in others.

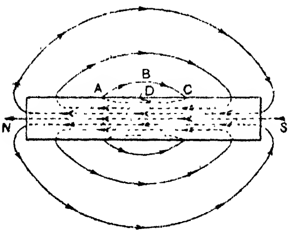


FIG. 41.

Now consider again the special case of our permanent magnet in air (ignoring the earth's field in which the magnet lies): *it has, of course, been previously magnetised " by induction "* by being subjected to the magnetic force in some magnetic field, and has retained the magnetic properties when taken out of the magnetising field. So far we have dealt only with the space outside it—the " external field "—and have seen that lines of force start at the north pole, pass through the (air) field, and end at the south pole. The magnet itself constitutes what is sometimes called the " internal field," and the lines reaching the south pole are continued through the substance of the magnet itself, finally reaching the north pole and forming *closed curves* (Fig. 41). The internal lines are, of course, not lines of force but *lines of*

induction. Thus each line forms a *closed circuit, i.e.* when a line of induction such as CDA leaves the magnet at A, it is continued as a line of force ABC in the air, and when this reaches C it is continued by the line of induction. Air is almost non-magnetic (and is always taken to be so), so that "field or force" and "induction" are practically the same for air, and *lines of force in air are also lines of induction in air.* Strictly, then, *the closed curves are lines of induction throughout,* although the parts in air are also lines of force. As a further example, Fig. 42 shows the steel permanent magnet (with cylindrical soft iron core between its poles) of a moving coil ammeter (page 341): the closed curves (dotted) are lines of induction, but the parts in air from N. to S. poles, viz. PQ, TU, and VW are also lines of force.

Lines (and tubes) of force and lines (and tubes) of induction are really concerned with different ways of looking at the actions in a field. Lines of force give the direction of the magnetic force in a field, and as will be seen later, the **strength** (H) **of the field** is, by convention, measured by the *number of unit tubes of force passing through unit area* taken at right angles to the field direction: in the magnet diagrams we have had (Figs. 31-38, 41) our lines of force have all started at N. poles and ended at S poles—started and terminated at boundaries between iron and air When we think of induction we are concerned more with some *alteration in the medium* due to the magnetic force, and the magnetic lines are further grouped into "tubes of induction" so that the **induction** (B) is measured by the *number of unit tubes of induction per unit area:* tubes of induction do not start or stop anywhere but form closed curves. For any material subjected to magnet force (*e.g.* iron in a magnetic field) $B = \mu H$ where μ is a number depending on the material (and certain other factors). For magnetic material μ is greater than unity and B is greater than H: thus in our iron the magnetic tubes are not tubes of force, for the number of tubes per unit area does not measure the field strength or magnetising force (H): the number is greater and measures B—the tubes are tubes of induction. For air (strictly a *vacuum*) μ = unity, B = H, and lines (or tubes) of induction and lines (or tubes) of force are identical. The full distinction between lines (or tubes) of force and induction will, however, be understood after reading Chapter III.

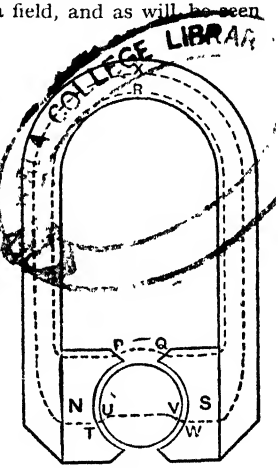


FIG. 42.

So far, then, the idea of a magnetic field and magnetic lines (and tubes) is simple: but the early Physicists went much further, as has probably been gathered from the conception of Faraday already referred to. They could not tolerate the idea, for example, of one magnet attracting or repelling another, or magnetising magnetic material "at a distance" with no connecting link between them. They could not accept "action at a distance" as a definite fact of

Nature, but preferred to think of some physical "contact." They preferred to think, in fact, that the medium in between played an important part in the actions, and as magnets (and charges and currents) acted on each other *in a vacuum*, the all-important medium in question was said to be the **aether** or **ether**—a thin, weightless medium which the earlier scientists contended (and many modern scientists also say) fills all space, and penetrates and fills all matter. Finally they decided that this medium was in a state of *strain* due to the magnet—a strain such as would be explained by a tension along the lines and a pressure at right angles to them—and that when another pole or magnet was put in the field, what happened was due to the interaction of the original strain and the strain due to the second magnet, etc. The "strains in the aether" were the important connecting links: the magnetic lines were, in fact, *strain lines in the aether.* Note again Faraday's explanation of attraction and repulsion—page 34—which is, of course, associated with this "strain" idea.

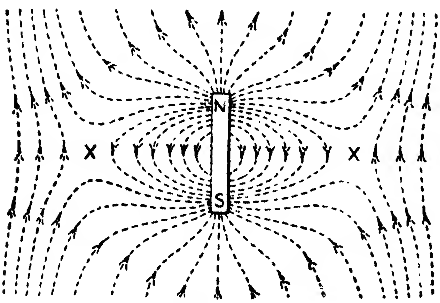


FIG. 43.

As has been indicated, if a piece of iron be placed in a magnetic field (air) we find more "magnetic lines" passing through the iron than passed through the same air space—the strain effect is increased or multiplied. Evidently it must be taken that it is easier to set up magnetic strain in aether when it is contained in iron than when it is contained in air or glass or other non-magnetic substance. Further, the iron is magnetised, *i.e.* when the aether in iron is strained magnetically a physical change takes place in the iron which creates new magnetic strains, and thus multiplies the strain effect.

The above conception of aether and aether strains presents several serious difficulties, and many scientists are now inclined to get away from it altogether; but it has played an important part in the historical development of the subject, and to "ignore the aether," to "put it out of existence" necessitates in some cases alternative explanations of certain scientific facts which are not quite "beyond suspicion." For the present the student must keep an open mind on the question: further details appear in subsequent chapters.

5. Combined Field of the Earth and a Magnet

It should be noted in connexion with the magnetic field of a magnet, that the earth's magnetic field is always present, so that when we "map the field of a magnet" we are, to speak exactly, mapping the *combined* field of the earth and magnet. Near the magnet, the magnet's influence is much the greater of the two, so that the earth's effect can be neglected in comparison. Note, however, the following cases:—

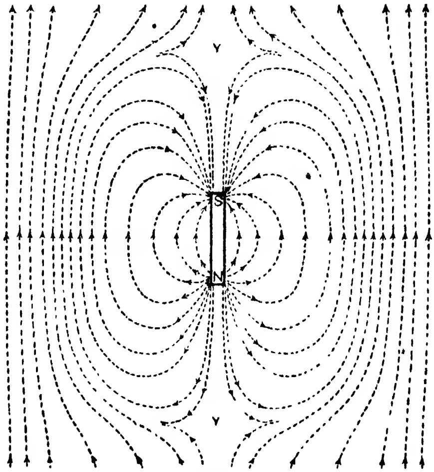


FIG. 44.

(*a*) Using a large sheet of paper, place a bar magnet on the middle of it, with its magnetic axis in the magnetic meridian, *its north pole pointing northwards*, and trace the combined field of the earth and the magnet by the compass method. Fig. 43 shows the result. At points near the magnet the forces due to the magnet predominate and the lines obtained are similar to Fig. 34. At more remote points the earth's field predominates and we get the earth's lines (south to north) distorted somewhat by the presence of the magnet. Due magnetic east of the magnet the earth's field and the magnet's field *are in opposite directions*; hence at a certain point X, the two fields are equal and opposite, and the resultant field is zero.

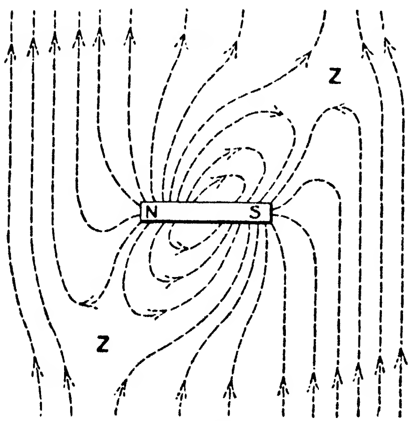


FIG. 45.

X is called a **null or neutral point.** Between X on the right and the magnet a compass obeys the magnet and sets with its north

pole southwards towards the south pole of the magnet. On the right of X the compass obeys the earth and sets with its north pole northwards. At X the compass sets in any position. There is a second null point at X on the other side of the magnet.

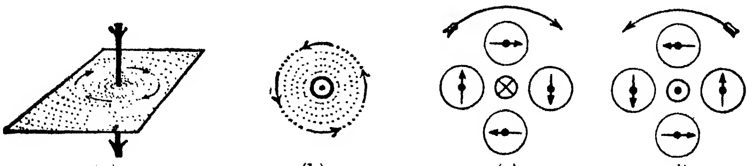


FIG. 46. Current (conventional) towards the reader is shown by a dot, away from the reader by a *cross*.

(*b*) Repeat with the magnet lying with *its south pole pointing northwards*. The null points Y are off the ends of the magnet due magnetic north and south of it (*Fig.* 44).

(*c*) Repeat with the magnet lying east and west at right angles to the magnetic meridian. Fig. 45 gives the result when the S. pole is to the east. Z are the null points. Repeat with the N. pole to the east.

6. The Magnetic Field of the Current in a Wire

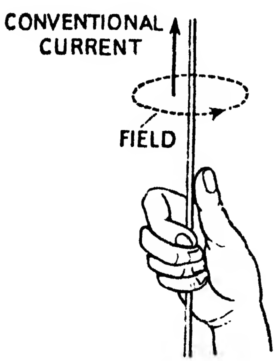


FIG. 47.

We will briefly glance at a few simple cases of the magnetic fields due to currents in wires. Now the direction of the field depends on the direction of the current in the wire, and it has been emphasised in Chapter I. that the *real* direction of flow (the electronic current) is opposite to what has always been taken in the past (the conventional current). However, for reasons given in Art. 7, the conventional current direction is assumed here.

(1) Fix a piece of cardboard horizontally, pass a wire vertically through it, and let a strong current flow through the wire. Sprinkle iron filings and gently tap. The filings arrange themselves in concentric circles round the wire; these indicate the magnetic lines in the magnetic field due to the current (Fig. 46). Move a small compass needle round the wire and note in which

direction the north pole points. If the conventional direction of the current is downwards, the north pole points as indicated by Fig. 46, *a*, *i.e* the .positive direction of the lines is *clockwise*. If the conventional direction of the current is upwards, the positive direction of the lines is *counter-clockwise* (Fig. 46, *b*). See also Figs. 46, *c*, *d*. A handy rule for giving the positive direction of the lines in the case of a current in a straight wire is as follows: —*Clasp the wire in the right hand so that the thumb points in the direction of the conventional current: the fingers curl round the wire in the direction of the magnetic lines* (Fig. 47).

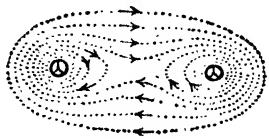


FIG. 48.

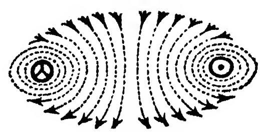


FIG. 49.

(2) Fix two wires vertical and parallel as above, pass equal currents in the wires, and by the compass-needle method map the *combined field*. Fig. 48 gives the lines when the currents are in the same direction in the wires (conventional direction downwards), and Fig. 49 when they are in opposite directions. Since lines tend to contract in the direction of their length and to repel each other laterally, it is clear that there will be a tendency for the wires to move nearer together in Fig. 48 and further apart in Fig. 49. Ampere showed that *two parallel currents always attract each other when they flow in the same direction, and repel when they flow in opposite directions* (Figs. 50 *a*, *b*).

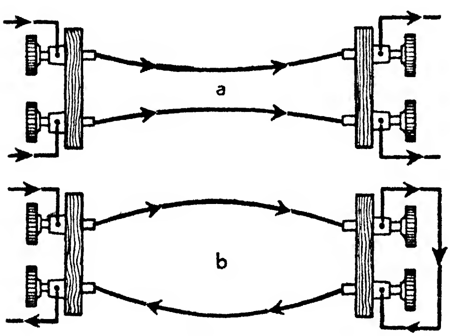


FIG. 50.

(3) Fig. 51 gives the combined field of the earth and a current in a vertical wire (conventional direction downwards). X is a *null point*. Compare with Fig. 43. As lines tend to contract in the direction of their length and to repel laterally, there will be a tendency for the wire to move towards the right. As an exercise, map the case with the current flowing up the wire.

(4) Fig. 52 shows the field in the case of a coil of wire (called a *solenoid*) carrying a current, the conventional current direction being again indicated. Note how the solenoid resembles a bar magnet, the end on the left where the conventional current direction is *counter-clockwise* corresponding to the N. pole of the magnet, and the end on the right where the direction is *clockwise* corresponding to the S. pole. The case of a single turn of wire is shown in Fig. 53: the face of the coil towards the reader at which the magnetic lines enter and the current (conventional) is *clockwise* is a "south face."

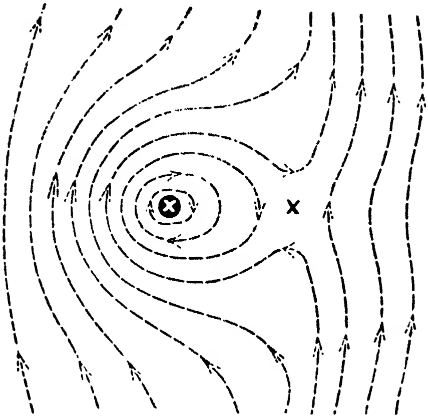


FIG. 51.

7. Methods of Making Magnets

Given one magnet, any number can be made by using the property of inductive influence. Thus the bar of iron CD (Fig. 24) was converted into a magnet merely by being placed in the magnetic field of the magnet N, its length being along the magnetic lines off the end of N. For a permanent magnet hard steel must be used, and better results will

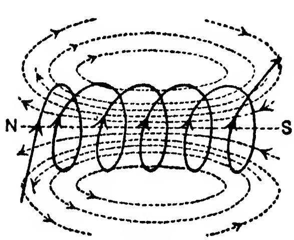


FIG. 52.

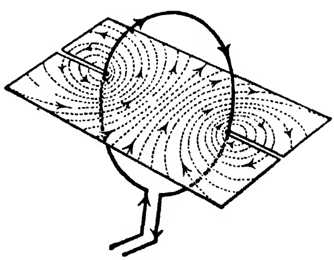


FIG. 53.

be obtained by the method shown in Fig. 54. Place the bar of steel AB on the table and, holding a bar magnet NS as shown, rub AB from end to end several times. Repeat this on the other side of the bar, which will then be magnetised as shown. Note that

the end of the bar where the rubbing magnet leaves it (viz. B) is of opposite polarity to the rubbing pole.

There are modifications of the above, but modern methods use the electric current. Make a coil of insulated copper wire. Place the rod to be magnetised in the coil (Fig. 55) and join the ends of the coil to a battery. Gently tap the bar with a wooden mallet during the passage of the current. The bar will be magnetised, one end being a north pole the other a south pole. More powerful magnetising effects will be obtained by winding the wire on a cylinder of cardboard (into which the bar is placed), the successive turns being close together forming what is called a closely wound solenoid, and, better still, if several layers of winding are employed, for the magnetisation produced is proportional to the current *and to the number of turns in the coil* (only if the bar is not completely magnetised, *i.e.* is not *saturated*).

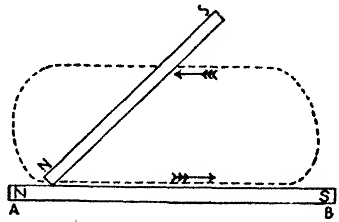


FIG. 54.

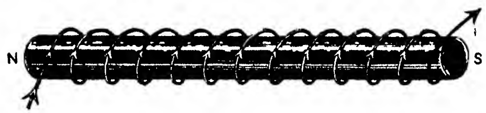


FIG. 55.

If the magnet to be made is horse-shoe shaped, wind the coil on the limbs as shown in Fig. 56, being careful after winding one limb to " cross over " in commencing to wind the other limb, so that the ends may be north and south poles. (See below.)

The polarity of the bar in these methods depends, as already mentioned, on the direction of the current in the coil, and many " handy rules " were devised in the past for determining this polarity. These experimenters all used, however, the old conventional current direction, and as these rules are still used, often under the name of the original propounder, and we are going to *quote* some of them here, we shall assume the conventional current direction.

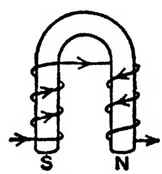


FIG. 56.

(1) END RULE.—Look at the end of the bar: **if the (conventional) current in the coil is " clockwise " in direction that end of the bar is a south pole: if it is " counter-clockwise " that end is a north pole (Fig. 57).**

(2) CORKSCREW RULE.—Imagine a corkscrew lying along the solenoid and turning in the direction of the (conventional) current: the point of the corkscrew will be moving towards the *north* pole. This is practically another form of the preceding rule.

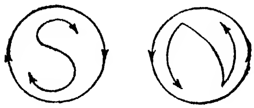


FIG. 57.

(3) AMPÈRE'S SWIMMING RULE.—Imagine a man swimming in the circuit in the direction of the (conventional) current with his face towards the bar: his *left hand* will point towards the *north* pole of the bar.

(4) RIGHT-HAND RULE.—Hold the thumb of the right hand at right angles to the fingers. Place the hand on the coil with the fingers in the direction of the (conventional) current and the palm

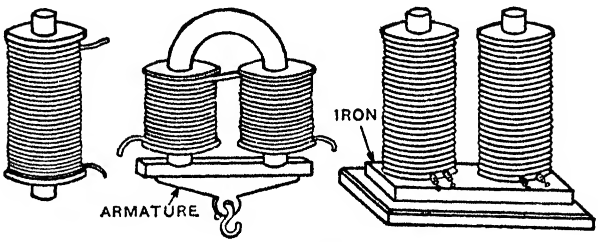


FIG. 58.

facing the bar: the *thumb* will be towards the *north* pole of the bar. This is practically a modified form of the swimming rule.

Electromagnets (Fig. 58) consist of cores of *soft iron* wrapped round with coils of insulated copper wire. The cores become powerful magnets while the current is passing, and bars of steel can be converted into magnets by rubbing them over the poles of these electromagnets: mere contact, in fact, with a pole of a powerful electromagnet will convert a bar of iron or steel into a strong magnet. Electromagnets can be made very powerful —capable of lifting 40 tons and more.

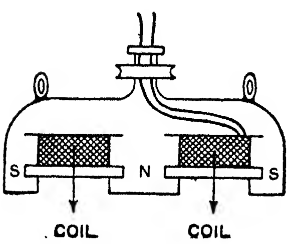


FIG. 59.

This marked holding and lifting power of these magnets is turned to practical use in many engineering workshops. To take one example only, instead of using hooks for lifting large sheets of

iron when moving them from one machine to another, an electromagnet hanging from a crane is used. This is magnetised by a current when required, and thus a large thin sheet, which is a difficult thing to grip in the ordinary way, is easily handled. The appliance is also used for lifting heavy masses of pig-iron, steel ingots, steel scrap, etc., or for raising, say, a heavy iron ball (10 tons or more) to a height and then dropping it on to scrap iron for the purpose of breaking up the scrap. Fig. 59 gives the principle of the Witton-Kramer *lifting magnet*, and Fig. 60 the actual appliance. The magnet is of the " pot " type, *i.e.* one pole is in the centre and the other is in the form of a ring surrounding it. The magnetising coil is held in position by a shield of non-magnetisable material (the shield is not shown).

FIG. 60.

Electromagnets of various types play an important part in the construction and working of dynamos, motors, transformers, electrical measuring instruments, electric bells, telephones, and other electrical appliances: many of these are dealt with in subsequent chapters.

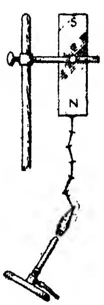


FIG. 61.

8. Effect of Heat and Vibration, Lamination, Consequent Poles

Several simple experiments can be devised to show the effect of heat on magnets and magnetic material.

(*a*) Note the amount of deflection of a compass produced by a small magnet at a suitable distance. Now heat the magnet to a temperature of about 100° C., and bring the compass to the same distance from it as before: the magnet will be found to be weaker. Heat to a bright red heat, and test: it will have lost its magnetic properties.

(*b*) Suspend an iron ball by a copper wire and heat it to a bright red. Hang it near the end of a magnet: it is not attracted, but as it cools down it becomes more and more attracted.

(*c*) Fix a magnet vertically and hang on to it a number of small iron nails (Fig. 61): each nail is, of course, a magnet. Heat the lowest nail with a Bunsen flame: when it becomes red hot it falls off. Repeat with the next nail, and so on, and the same thing happens.

Iron, then, seems to lose its magnetic properties—more exactly, it changes from a ferromagnetic to a paramagnetic—when heated above a certain temperature: this temperature is called the **critical temperature** and is round about 800° C., varying according to the quality of the iron. The critical temperature for nickel is about 350° C., for cobalt about 1100° C.

We have seen that *tapping a bar of iron while it is being magnetised* assists the magnetisation (pages 30, 43): this effect of vibration is also seen in the strong magnetisation acquired by steel

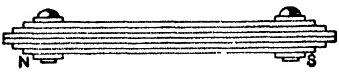


FIG. 62.

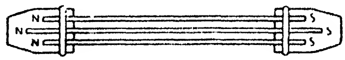


FIG. 63.

drills particularly when drilling cast iron, and in the magnetism acquired by steel ships during construction. On the other hand, *hammering a magnet after it is removed from the magnetising influence, e.g.* knocking a permanent bar magnet about, weakens it.

It was found that if a thick magnet were put in strong nitric acid and the outer surface eaten away, it showed no signs of magnetisation, indicating that only the surface layers had been "magnetised." Since it is difficult, therefore, to magnetise the inside of a thick bar, large permanent magnets are often made up of thin strips separately magnetised and then bolted together, all their north poles at one end and their south poles at the other. These are known as **laminated magnets** (Fig. 62). The laminated construction is also often employed in electromagnets; the cores are not solid but are built up of thin soft iron sheets or wires. Transformers, chokes, induction coils, etc., are practical forms of the laminated construction. Another method often used to obtain a strong permanent magnet is to fix the like poles of two or more thin magnets in blocks or pole pieces of soft iron, forming what is often called a *compound magnet* (Fig. 63).

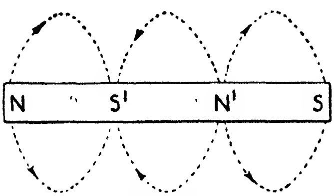


FIG. 64.

Although it is impossible to obtain a magnet with only *one* pole, it is quite possible to obtain a magnet with *more than two* poles, the extra poles being known as **consequent poles.** Fig. 64 shows a magnet with consequent poles at N′ and S′. By touching a bar of

steel at several points with the poles of a strong magnet, consequent poles are produced at the points touched. Again, in Fig. 65 we have the electrical method of magnetising a bar of steel so that there is a north pole at each end and a south pole at the middle.

9. Iron and Steel and Magnetic Alloys

In some of the preceding experiments soft iron has been specified, and in others hard steel. The magnetic difference between the two is that *soft iron is more readily magnetised than hard steel, and for the time being is stronger, but it loses its magnetic properties much more quickly than hard steel.* Both these points may be shown by the following simple experiments:—

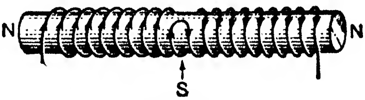


FIG. 65.

(*a*) Repeat the experiment (page 30) of hanging small pieces of hard steel on the magnet, and continue until no more pieces can be supported. Repeat again, using pieces of soft iron of equal weight to the pieces of steel. More pieces of iron than of steel can be supported. This shows that *the iron is more readily magnetised than the steel.*

(*b*) Hang the steel pieces on again and then detach the upper piece from the magnet: the pieces still hold together. Repeat with the iron: on detaching the upper piece the irons fall apart—at any rate with the least shake. This shows that *the iron loses its properties more readily than the steel.*

(*c*) Place two bar magnets AB and CD on opposite sides of one pole of a compass, as shown in Fig. 66. AB attracts the S. pole of the compass towards

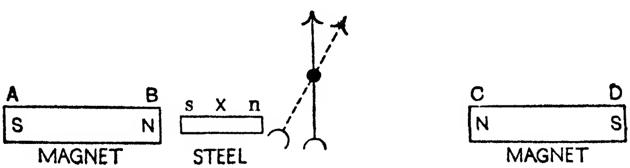


FIG. 66.

the left, and CD attracts it towards the right. Adjust the distances of AB and CD until the compass is not deflected. Place a bar of hard steel at X; it is magnetised as shown, and attracts the S. pole of the compass. Note the deflection of the compass. Remove the steel, place a bar of soft iron of the same size in the same position, and note the deflection. The deflection with the iron is greater, *showing that the iron is more strongly magnetised.* Remove the iron and magnets. Knock the iron and steel about a little, then replace the steel (in the same position as before) and note the deflection. Remove the steel, substitute the iron, and note the deflection. The deflection with the steel is greater, *showing that the steel has kept its induced magnetic properties better than the iron.*

The stronger the poles developed in a material when under a given inductive influence or magnetising force the greater is said to be the **susceptibility** of the material. *The susceptibility of iron is greater therefore than that of steel.*

The power of retaining magnetic properties after the magnetising force is removed, is called **retentivity,** and the " magnetism " so retained is called *residual magnetism.* In some of the simpler laboratory experiments it may appear that hard steel has greater retentivity than soft iron, but as a matter of fact, soft iron shows more residual magnetism and therefore greater retentivity than hard steel *if it is protected from any disturbing influence*; but the least disturbing influence (vibration, careless handling, etc.) will wipe it out. The power of retaining the magnetic properties in spite of such subsequent treatment is called **coercivity.** *The coercivity of hard steel is greater, therefore, than that of soft iron.*

For permanent magnets we require a material which shows as much residual magnetism as possible, *i.e.* has high retentivity, *but it must also have great coercivity* (this latter being of chief importance), and the material employed is hard steel. Now steels differ very much in this property of hardness: thus what is known as *mild steel* is nearly as soft as iron, whilst *glass-hard steel* is very hard indeed. To glass-harden steel, it is heated to a bright red heat and then plunged *suddenly* into cold water (or, for a greater degree of hardness, into mercury or vegetable oils). Further, if the permanent magnet is to be used in an electrical instrument where its magnetic properties must remain constant, the magnet is artificially *aged.* One method of doing this is to immerse the magnet in steam for several hours: this somewhat weakens it, but it results in greater permanence and *constancy* of the magnetic properties.

For electromagnets which have to quickly take up strong magnetic properties and quickly lose them, we require a material of large susceptibility and small coercivity, and the material employed is soft iron—often pure Swedish iron thoroughly annealed. Irons differ very much in hardness: thus cast-iron is much harder and more brittle than wrought iron. We have seen above that *suddenly* cooling the hot metal hardened it: *gradually* cooling would, on the other hand, soften it. The process of annealing, for example, is performed by allowing the hot metal to cool slowly.

The addition of other substances to iron or steel greatly affects the magnetic properties of the " alloy." Thus permanent magnets are improved if the steel contains 5 to 8 per cent. of *tungsten,* and a frequently used **tungsten steel** is one containing 5 per cent. of

tungsten and 0·7 per cent. of carbon: experiment indicates that about 4 per cent. of *molybdenum* is even more effective than tungsten. *Cobalt*, too, improves the permanent magnet, and **cobalt steels** are extensively used for magnetos in motor cars: a well-known cobalt steel is one containing 35 per cent. cobalt and ·085 per cent. carbon. Another alloy of recent development which has excellent properties as a permanent magnet is remarkable in that it is an alloy of *nickel* and *aluminium*, and has no iron in it. And yet another, still more recent, is made of *nickel*, *aluminum*, and *cobalt*.

In practical electrical work various magnetic alloys are employed, the nature of the alloy depending on the purpose for which it is to be used; for, of course, alloying to obtain best results so far as one property is concerned may give poorer results so far as another property is concerned. A material which is now largely used in electrical machinery is **stalloy**: it is an alloy of iron with 3 to 4 per cent. of *silicon*, acquires very strong magnetisation under *big* magnetising forces, and has other desirable features so far as alternating current work is concerned. **Permalloy** is another alloy now extensively used: it consists of 25 to 50 per cent. iron and 75 to 50 per cent. *nickel*, and acquires fairly strong magnetisation under *small* magnetising forces, although its maximum magnetisation is not so great as soft iron and certain other alloys. **Mumetal** is an alloy similar to permalloy. Alloys of the permalloy and mumetal type are very suitable for simple laboratory work owing to the fact that they are so easily magnetised (even by the earth's weak field) and demagnetised. It is interesting to note that there are alloys of steel which are *non-magnetisable* and which are required for special purposes. Thus **manganese steel** (about 15 per cent. of manganese) is non-magnetisable: another alloy, viz. about 80·2 per cent. iron, ·8 per cent. carbon, 14 per cent. nickel, and 5 per cent. manganese, is practically unmagnetisable.

10. "Molecular" Theory of Magnetism. Modern Electron Theory

As has been indicated (pages 24, 25), modern explanations of magnetism are given in terms of *atomic structure*—moving electrons inside the atoms of the substance. Many explanations of magnetism were, however, put forward in the early days, Weber's "*molecular*" *theory* being the most important: modern theory is, in fact, really an extension of Weber's conception consequent upon recent discoveries. According to Weber's theory (somewhat modified):—

(1) The "molecules" not only of a magnet but also of a piece, say of "unmagnetised" iron, are complete magnets. Now it was

surmised quite early that these molecules of Weber's theory were not the chemist's molecule—they were probably smaller than that —and it would be better to avoid the name molecule altogether. However, for simplicity, but with this warning, we will, for the present, use the name "molecular magnets" for these Weber elements—these small particles which are magnets.

(2) Before the material is magnetised these molecular magnets, acting on each other, arrange themselves with their magnetic axes in various directions so as to form "closed circuits" amongst themselves, *i.e.* the north pole of one particle comes up against the south pole of another, so that none of the poles can produce any magnetic force outside and the material shows no signs of magnetisation (Fig. 67).

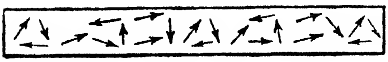


FIG. 67.

(3) The process of magnetisation consists in rotating these molecular magnets so that their magnetic axes point in the direction of the magnetising force, their north poles thus pointing one way and their south poles the other. The influence which prevents them all pointing one way on the application of the least magnetising force is due to the action of each particle on its neighbour. The conditions are now somewhat as indicated in Fig. 68, where the bar has been rubbed, say, by the S. pole of a magnet from the end B to A, so that all the north poles are pulled round to face A. Throughout the interior each pole touches an unlike pole and the two neutralise, so that there is no free polarity. At the end on the right we have a number of free north poles, and at the end on the left a number of free south poles, so that the former end of the material has north polarity, the latter end south polarity. The figure indicates the existence of lateral magnetism; Fig. 69 shows an ideal case of a bar magnetised to saturation with no lateral magnetism.

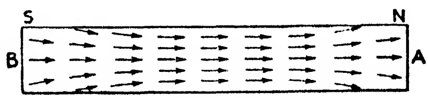


FIG. 68.

There are many facts in elementary magnetism which could be accounted for by Weber's theory: a few of these can be noted:—

(1) The result obtained by breaking a magnet (page 27).

(2) The possibility of obtaining consequent poles (page 46). Thus Fig. 70 shows the internal arrangement of the magnetic particles of Fig. 64.

(3) Surface layers only of very thick bars become magnetised (page 46), for the magnetising force is only capable of rotating the surface layer of molecular magnets.

(4) There is a limit to the magnetisation of a material beyond which it is impossible to go however great the magnetising force, and the material is then said to be saturated. On the theory this would mean the setting of all the molecular magnets in a definite direction.

FIG. 69.

(5) Heat is produced when a bar is rapidly magnetised and demagnetised. When a molecular magnet swings round it acquires kinetic energy and oscillates until that energy is converted into heat.

(6) Soft iron is more readily magnetised than hard steel, but it loses its magnetisation much more readily (page 47). This is explained by the fact that the **molecular rigidity** of hard steel *is* greater than that of soft iron: the particles of steel are therefore more difficult to move into a definite direction, but once there they are more difficult to move back again.

(7) The effect of heat on magnets and magnetic material (page 45). The heat increases the molecular motion and lessens the molecular rigidity, so that the tiny magnets obey their natural tendency, *i.e.* swing round and form circuits amongst themselves. The molecules of a red-hot iron ball are moving so rapidly that the magnetising force cannot get the particles into a definite direction.

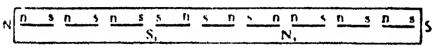


FIG. 70.

(8) If a bar be gently tapped while being magnetised the magnetisation is assisted (pages 30, 43), for the vibration (to put the matter simply) loosens the molecules and the magnetising force is better able to set the particles in the definite direction. For a somewhat like reason tapping a magnet after the magnetising force is removed or throwing it about will demagnetise it.

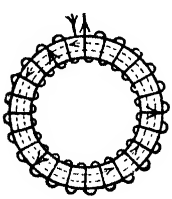


FIG. 71.

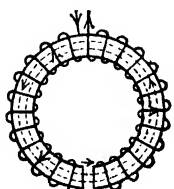


FIG. 71*a*.

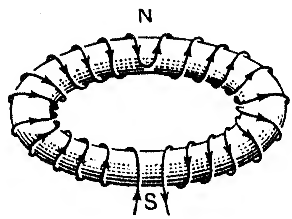


FIG. 72.

(9) A ring of iron may be magnetised *circumferentially* by drawing a magnet round it, or by an electric current (Fig. 71). The molecular magnets are set in a definite direction, but there are no poles and no signs of magnetisation. If the ring be cut, however, a north pole appears at one side and a south pole at the other (Fig. 71*a*). This is in accord with the theory. Note,

however, that an iron ring can be magnetised so as to have poles, and Fig. 72 shows one method: such a ring would set with its magnetic axis in the magnetic meridian if suitably suspended.

(10) The action of keepers (page 31). Without the keeper the end magnetic particles, owing to attractions and repulsions between them, tend to turn and form circuits amongst themselves, thus weakening the magnet. When the keeper is on, each molecular pole is faced by an opposite pole in the keeper (Fig. 27), and the two tend to hold and strengthen each other. Moreover, as stated in Art. 3, the pole of the keeper tends to cancel any inductive action of the end pole of the magnet on other parts. Further, the magnetic chains in the magnet are continued in the keeper (Fig. 73), so that we have now *closed magnetic chains* which have very little tendency to break and rearrange; hence the magnet retains its properties. A horse-shoe magnet fitted with its keeper has practically no external field.

The molecular theory accounts therefore for many facts so far as magnetic substances are concerned, i.e. *ferro-magnetics* and *paramagnetics*, in which the magnetisation is in the same direction as the magnetising field and which are attracted by a magnet pole—move from weak to strong parts of a field: it does not however explain why the particles are magnets, nor does it explain the *diamagnetics* in which the magnetisation is in the opposite direction to the field and which are repelled by a magnet pole—move from strong to weak parts of a field (page 25).

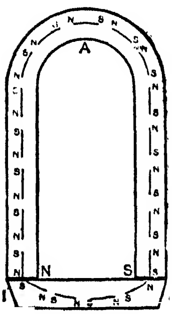


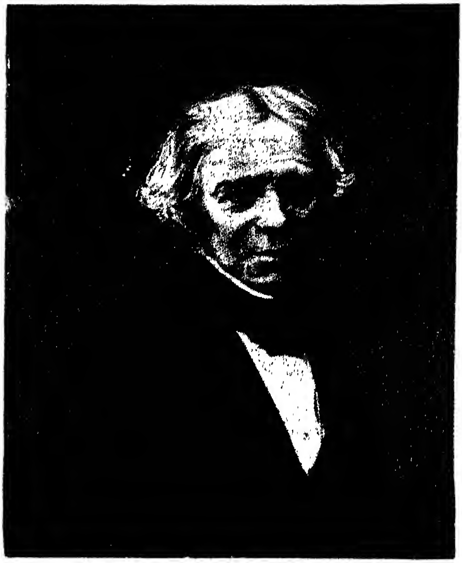
FIG. 73.

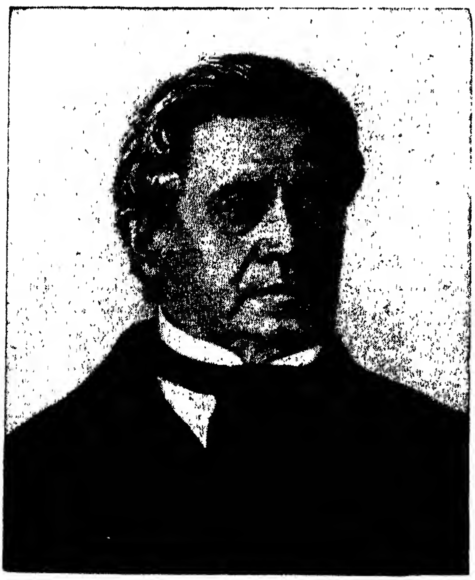
Now a current in a coil produces a magnetic field, *the direction of the field depending on the clockwise or counter-clockwise direction of the current*, and it magnetises magnetic material; and Ampère assumed that the magnetism of the Weber molecules was due to currents of electricity flowing round them in perfectly conducting channels, in which case, once started, they would continue to flow without dissipation of energy, and the particles would be magnetised as required by the theory.

The electron theory carries us a step further, for it brings the explanation down to atoms and their electrons. Instead of studying magnetism mainly from the point of view of *masses of material*—rods of iron, etc.—modern work has investigated it from the point of view of *single atoms*. Evidence of the magnetic properties of single atoms has been obtained in two ways: Gerlach and Stern noted the effect of a magnetic field on single atoms projected across it, and Zeeman noted the effect of a field on the spectrum of an atom emitting radiation. The Zeeman Effect is referred to later,

MICHAEL FARADAY,
1791-1867.

English scientist, born in London. Became Sir Humphry Davy's assistant, Royal Institution, 1813, and later Professor of Chemistry. Discovered electromagnetic induction (1831), the foundation of electrical engineering. To perpetuate his name the unit of capacitance is called the *farad*, and an important quantity of electricity in electrolysis (9650 units) the *faraday*.





JOSEPH HENRY,
1799-1878.

American scientist, born at Albany, New York. Professor of Natural Philosophy at Princetown and first secretary of the Smithsonean Institute. Had a controversy with Morse as to which of them was the inventor of the electric telegraph. Conducted important research in electromagnetism. To perpetuate his name the unit of inductance is called the *henry*.

but Gerlach and Stern's experiment may be briefly noted even at this stage.

A very high vacuum was obtained in the air-tight brass box BB (Fig. 74). Metallic silver was contained in an electric furnace F and vaporised. A stream of silver atoms was thus projected through a slit in the furnace and through two further slits in the partitions SS, finally falling on a cold glass plate G and causing a thin line deposit of silver. In its journey the stream of atoms passed between the poles of an electromagnet with specially shaped pole-pieces (one wedge-shaped, the other grooved). On switching on the magnet the silver line on G broke up into two, showing that in passing through the non-uniform field between the poles some atoms were attracted towards the N. pole and some towards the S. pole. The *full* meaning of the experiment and the conclusions which can definitely be drawn from it will be understood later: but it certainly indicates magnetic properties in single atoms.

Now for some explanation. In every atom we have electrons spinning and also travelling round the nucleus. The spins result in

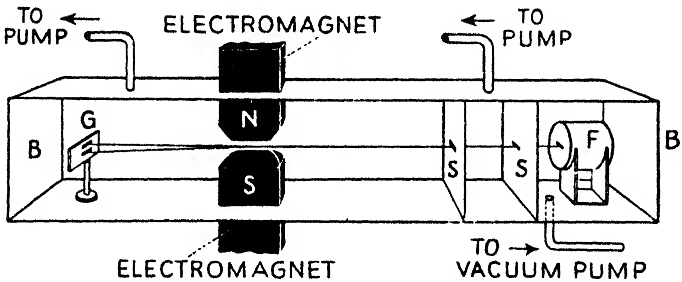


FIG. 74.

magnetic fields, and so do *some* of the rotations. (It was stated on page 8 that electrons do not travel round the nucleus in single curves in one plane: with *some*, however, the rotation is such that the effect is, as it were, concentrated more or less into a " belt " or " road " round the nucleus, and these do result in magnetic fields.) In any one atom the *directions* of the spins and of the rotations are never such that *all* the electrons help each other—there is a certain amount of neutralising or cancelling of each other's effects. If they completely neutralise each others effects the atom will have no *resultant* magnetic field. On the other hand, if the electron movements do not completely neutralise, the atom will have a *resultant* magnetic field, *i.e.* it will show magnetic properties. It may, in fact, be described as having a " magnetic axis," and when placed in a magnetic field it may turn so that this axis is in the direction of the field, *i.e.* rotate into such a position that it *assists*

the field and there is an increase in the induction: in such cases it would behave in the same way as the Weber elements. A substance whose atoms behave in this way would be *paramagnetic.*

It may be noted in passing that slight *thermal agitation* plays a part in the appearance of paramagnetic properties. The explanation of this cannot however be given at this stage.

There is, as mentioned on page 25, a small group—iron, nickel, cobalt—which have pronounced magnetic properties, and are known as *ferromagnetics.* There is as yet no really satisfactory explanation of the marked magnetic properties of ferromagnetics. It is probably due to the fact that in these few substances neighbouring atoms (all of the paramagnetic kind explained above) have such influence on each other that, by a kind of linking of certain outer electrons, they form a larger group of associated atoms which is influenced *as a whole* by the field in which the substance is placed.

The full explanation of *diamagnetism* on the electron theory is given in Chapter XVII. When a material is placed in a magnetic field the latter has really two effects. If the atom has a resultant magnetic field, one effect, as explained above, is to tend to orientate the atoms into a direction such that they *assist the field and increase the induction* giving rise to paramagnetic properties. The field, however, has another effect: it produces an *alteration in the electron movements*, a *change in the electron " orbits "* and, as will be seen in Chapter XVII., this slight change is such that the effect is to *oppose the field and therefore decrease the induction:* in simple language, it tends to produce a slight " magnetisation " effect in the *opposite* direction to the magnetising field, and the material tends to show diamagnetic properties. In the atoms considered above, *i.e.* those with resultant magnetic fields, the paramagnetic effect outweighed the diamagnetic, and the substance was paramagnetic—or ferromagnetic. In the case, however, of atoms in which the electron movements completely neutralise, the magnetising field has no directional effect on the atom, only the second effect is present—the alteration in the electron " orbits " by the field—and the substance is diamagnetic: its " polarity " will be " the other way about " as compared with magnetic substances, and it will be repelled, for example, by a magnet pole instead of attracted by it as a magnetic substance is. As stated (page 25), all diamagnetic effects are feeble.

Further particulars of ferromagnetics, paramagnetics, and diamagnetics are given in Chapters III., XVII.

CHAPTER III

MAGNETIC UNITS AND THEORY

IN Sections 1-19 of this chapter the poles and magnets will be assumed to be in air (strictly *in vacuo*), the modifications which would be brought about by the substitution of another medium being simply indicated where necessary. In the remaining sections other material media in the magnetic fields will be considered.

A—Magnetic Pole, Field, Potential, Moment, Shell

1. Pole Strength. Unit Pole

It has been agreed that *the strength of a pole be measured by the force it exerts on another given pole at a given distance,* the two being entirely free from the effect of other poles. Now early investigation showed that if two poles were imagined isolated and placed at two points, *the force between them would be directly proportional to the product of the pole strengths and inversely proportional to the square of the distance between them.* Thus if m_1 and m_2 denote the pole strengths, d the distance apart, and F the force between them:—

$$\text{F is proportional to } \frac{m_1 m_2}{d^2}; \qquad \therefore\ \text{F} = c\,\frac{m_1 m_2}{d^2},$$

where c is a factor depending only on the medium in which the poles are placed and on the units we adopt in our measurements.

The first part of the law is simple. The second part is the well-known *law of inverse squares*: if the distance between the poles were increased 2, 3, 4, . . . times the force would be reduced to $1/2^2$, $1/3^2$, $1/4^2$ . . . , *i.e.* to $\frac{1}{4}$, $\frac{1}{9}$, $\frac{1}{16}$ . . . , whereas if the distance were reduced to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ . . . the force would be increased 2^2, 3^2, 4^2, . . . , *i.e.* 4, 9, 16 . . . times.

Suppose our poles are *in vacuo* (or what is practically the same magnetically, *in air*), and that we measure F in dynes and d in centimetres. It will evidently be most convenient if we so choose our unit pole that the factor c is unity: and this will be so if **unit pole** be taken as **that pole which when placed one centimetre in vacuo (or air) from an equal pole acts on it with a force of one dyne,** for if in the equation above $m_1 = m_2 = 1$ (unit pole), $d = 1$ (cm.), and $\text{F} = 1$ (dyne), then $c = 1$. This unit pole is called the **weber** (after the scientist Weber). Hence adopting this as our unit pole,

the force between two isolated point poles of strengths m_1 and m_2 situated d cm. apart in air is:—

$$F = \frac{m_1 m_2}{d^2} \text{ dynes} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

It should be noted that the law assumes *isolated point poles.* The actual force between the like poles (say) of two ordinary bar magnets does not obey the law, because (1) the poles are not concentrated at points and (2) the poles are not isolated—the second pole of each exerts an influence; by dealing with very long thin magnets the results are approximated to very closely.

In (1) above, the two poles are in air (strictly in *vacuo*). If this is not the case then the factor c must be introduced. Now this factor, as will be seen later, is equal to $1/\mu$, where μ (mu) is what is called the *permeability* of the medium (pages 86, 91), and the expression becomes:—

$$F = \frac{1}{\mu}\frac{m_1 m_2}{d^2} \text{ dynes} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

For a vacuum μ is taken as unity, and it is practically so for air. In fact it is only when the medium is *ferromagnetic* that μ must be taken into account: thus a quite usual working value of μ for iron is of the order 2000, and it can be *very much bigger,* but for paramagnetics and diamagnetics μ has such values as, for example, 1·000036, (platinum), 0·999824 (bismuth), etc.—numbers not so very different from unity. For air $\mu = 1{\cdot}00000038$, which is practically unity.

From (1) and (2) it follows that we might define the permeability μ of a medium as being measured by the ratio of the force between two poles in air to the force between the same two poles at the same distance in the medium. (See, however, pages 86, 91.)

2. Field Strength or Intensity. Unit Field

(1) It has been agreed that **the strength of a magnetic field at any point be represented numerically by the force in dynes acting on a unit north pole put in the field at that point:** it being assumed that the unit pole itself does not disturb the field. The *direction* of the field is the direction in which the force on the *north* pole acts. Thus if the force on the unit pole be H dynes, the strength of the field is H *units of field strength.* Clearly, if the force be one dyne the strength of the field is unity: thus **unit magnetic field is that field which exerts a force of one dyne on a unit north pole put in it. This unit field is called a gauss (after Gauss).** See also page 65).

It is evident that the strength H of the field at distance d cm. from a pole of strength m units (webers) is given by:—

$$H = \frac{m}{d^2} \text{ gauss} \quad \dots\dots\dots\dots\dots\dots (3)$$

the medium being air (strictly a vacuum), for this is the force in dynes on unit pole. (If the medium is not air but one of permeability μ, the field is $m/\mu d^2$ gauss.) Further, if a pole of strength m units (webers) be put in a field of strength H units (gauss) the *force* on the pole will be mH *dynes*. Clearly, field is a *vector* quantity, *i.e.* has direction as well as magnitude: if there are several poles the field at a point is obtained by imagining a unit *north* pole to be put there, calculating the force on this due to each pole, and then finding the resultant by the parallelogram law: this resultant force in dynes gives the field strength in gauss—it gives the field both in magnitude and direction.

Note particularly that field strength *is not the force in dynes* on unit north pole, but it is *numerically represented by* this force, *i.e.* if the force on the unit pole is 2 dynes the field is 2 units of field strength, *i.e.* 2 gauss, but *not* 2 dynes.

(2) The conception of lines (and tubes) of force leads to another method which might be used in defining field intensity. We have seen in our maps of lines of force that in a general way the lines seem to be crowded together near the poles of a magnet where its field is strongest, and spread out further away where the field is weaker. We have also seen that to overcome any question of space between lines, it is convenient to imagine the lines grouped into tubes which touch each other laterally and fill the entire field, and it follows from the way the lines spread out from the magnet, that these tubes will be narrower in the strong parts of the field (*e.g.* near the poles) than in the weak parts of the field, *i.e.* more tubes will be passing through a square centimetre of the field where it is strong than where it is weak. Now, extending this last idea, it would be very convenient if we could imagine unit field as having one tube of force passing through a square centimetre (taken at right angles to the field), and a field of strength 5 as being one with 5 tubes of force per sq. cm., and so on. And we can readily find what *assumptions* must be made in order to get this to be the case.

Imagine a north (point-) pole of m units at the centre of a sphere of radius d cm., and *we will again assume the medium is air (strictly a vacuum)*. The lines of force will be passing through the sphere normal to its surface. The strength of the field at any point on the surface of this sphere is m/d^2 units. If the field strength

is to be represented by the number of tubes of force per square centimetre, m/d^2 must therefore be the number of tubes going through every square centimetre of the sphere. The total area of the sphere is $4\pi d^2$ sq. cm., and therefore the total number of tubes passing through the whole sphere would be $4\pi d^2 \times m/d^2 = 4\pi m$; this, then, is the number of tubes of force we must *assume* as coming from the pole of strength *m in air*, and therefore we must *assume* 4π tubes of force from unit pole. On this convention the tubes are strictly called **unit tubes of force**, but " unit " is often, for brevity, omitted. Hence *we assume* that 4π unit tubes of force radiate from unit pole *in air*, and $4\pi m$ unit tubes of force from a pole of strength *m in air*. Then we have **unit field is a field which has one unit tube of force per square centimetre (at right angles to the field), and the strength of any field is numerically represented by the number of unit tubes of force per square centimetre.**

Summarising, we have then the fact that if a field has a strength of H units (gauss) it will exert a force of H dynes on a unit pole (weber) put in it, and it will be said to have H unit tubes of force per square centimetre taken at right angles to the field direction.

Since $4\pi m$ unit tubes of force pass normally through the sphere the area of which is $4\pi d^2$ sq. cm., the cross-section of each tube at the sphere is $4\pi d^2/4\pi m = d^2/m$. The field at the sphere is m/d^2. The cross-section of a tube multiplied by the field strength is therefore $(d^2/m) \times (m/d^2) = 1$. Similar remarks apply to other sections: thus *the cross-section of a unit tube of force multiplied by the intensity of the field at that section is constant.* It follows then that *the strength of a field at any point is inversely proportional to the cross-section of the unit tube at that point*, e.g. if the cross-section of a tube at B is twice the cross-section at A the field at A is twice the field at B.

Note again that in the above the pole m is assumed to be *in air* (or *vacuo*) and that we have arrived at the result that $4\pi m$ *unit tubes of force* must start from it, in order that our convention, viz. that the field be measured by the number of unit tubes of force per unit area, may apply. It is really a special case of a general theorem known as **Gauss's theorem** which applies to any medium and which is stated and proved on page 86.

3. Magnetic Potential. Unit Potential

There is another way of defining a magnetic field depending on the conception of **magnetic potential.** Let N (Fig. 75) be an isolated north pole, and imagine n to be a unit north pole at an infinite distance. To bring n from infinity to A, work must evidently be done against the repulsion of N, and to bring the unit pole to B clearly more work must be done. With a given pole N, the work in these cases depends on the positions of the points in question,

and such work is taken as a measure of what is termed the *magnetic potential* at these points. Thus, if W_1 ergs of work be done in moving the unit north pole from infinity to A, then W_1 C.G.S. units is the potential at this point A; if W_2 ergs of work be expended in moving the unit north pole to B, then W_2 C.G.S. units is the potential at the point B, and the difference in magnetic potential between A and B is $W_2 - W_1$ C.G.S. units.

There is a similarity between this measuring of *potential* and that of *height* or *ground level*. If 6 *foot-pounds of work* be done in raising one pound mass from the ground to the top of a wall, the height of the wall is 6 *feet*: the work and the height are expressed by the same number, viz. 6.

If N be replaced by a south pole the same reasoning holds, but in this case the unit north pole would move up under the attractive force; in other words, work is done *against* the magnetic force in the first case, and the unit north pole is, we say, moving from points of lower to points of higher potential, whilst work is done *by* the magnetic force in the second case, and the unit north pole is moving from higher to lower potential. Further, at an infinite distance from N the force is zero; all points at infinity are, therefore, at the same potential, and in the above this is taken as the zero of potential.

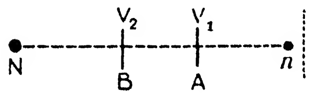


FIG. 75.

Hence *the magnetic potential at a point is defined as numerically represented by the work done, in ergs, in moving a unit north pole from infinity to that point, and the difference in magnetic potential between two points is defined as numerically represented by the work done, in ergs, in moving a unit north pole from one point to the other.* Thus the magnetic potential at a point in a field is 1 C.G.S. unit if one erg of work be done in moving a unit north pole from infinity to that point, and **the magnetic potential difference between two points is 1 C.G.S. unit if one erg of work be done in moving a unit north pole from one point to the other.** The unit magnetic potential has no specific name.

In Fig. 75 let V_1 = the potential of A and V_2 = the potential of B; the work required to move a unit north pole from A to B is, therefore, $V_2 - V_1$ ergs. Suppose A and B very near together, and let H = the intensity of the field in the small part BA; the work required to move the unit north pole from A to B is force on unit pole × distance = H × BA. Hence

$$H \times BA = V_2 - V_1; \quad \therefore H = \frac{V_2 - V_1}{BA}.$$

Now in the case of a hill the *slope* or *gradient* is obtained by taking the difference of level between two points and dividing by the horizontal distance between them. Similarly, the right-hand expression gives the *potential gradient* between A and B, hence—(*a*) **the intensity H of a magnetic field at a point is numerically equal to the magnetic potential gradient at that point**; (*b*) the intensity of a magnetic field is greatest in those regions where the potential gradient is steepest, *i.e.* where the rate of variation of potential with distance is greatest; this is evident, for given, say, a fixed value for V_1 and V_2, H is greater the smaller the distance AB. Assuming A and B very close together, writing dx for the small distance BA and dv for $V_2 - V_1$ we get the usual expression for the above, viz.

$$H = -\frac{dv}{dx} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

the insertion of the negative sign indicating, *e.g.* that the potential v *diminishes* as the distance x from the north pole N *increases*.

Summarising, we can say: (1) The magnetic potential at a point is that quantity whose rate of change with distance in any direction is numerically equal to the strength of the field in that direction. (2) The magnetic potential at a point is represented numerically by the work done in ergs in bringing unit N. pole from infinity to the point. (3) The magnetic potential difference between two points is represented numerically by the work done in ergs in moving unit N. pole from one point to the other.

Potential is a *scalar* quantity; thus the potential at a point due to a number of magnetic poles is simply the algebraic sum of the potentials due to each.

From the definition the reader must *not* conclude that " magnetic potential " and " work " are identical in nature. What is meant is that if the work done in moving unit pole from one point to another is W ergs, the magnetic potential difference between the two points is W C.G.S. units of potential, *not W ergs*.

4. Magnet in a Uniform Field

It was stated in Chapter II. that the poles of a magnet are of equal strength. Thus if a bar magnet be floated on a cork at the centre of a large vessel of water the cork and magnet will *rotate* until the magnetic axis of the magnet is in the magnetic meridian, but will not move as a whole towards the sides of the vessel: the effect of the earth's *uniform* field on the magnet is *directive* but not

translatory. If H be the strength of the earth's uniform horizontal field, m_1 the strength of the N. pole of the magnet, and m_2 the strength of the S. pole, then when the magnet is at rest in the meridian, the force on the N. pole is m_1H towards the north, and on the S. pole m_2H towards the south. But these are balancing since the magnet does not move either way: hence $m_1H = m_2H$; $\therefore m_1 = m_2$.

Again suspend a bar of steel vertically from a spring balance and note the weight. Magnetise it, suspend as before, and note the weight. It is the same. Now in the second case two *additional* forces are acting, viz. m_1V downwards on the N. pole and m_2V upwards on the S. pole, where V is the earth's uniform vertical field. As the weight is the same these are balancing: hence $m_1V = m_2V$; $\therefore m_1 = m_2$.

Now consider a uniform field of strength H and imagine a magnet suspended in the field as shown in Fig. 76 (*a*) (*i.e.* at right angles to the field at the start). The force on each pole is mH. These two equal, opposite, parallel forces form a "couple," the effect of which is to rotate the magnet until finally it lies as at (*c*). In this position it is at rest for the two equal forces mH acting in the same straight line and in opposite directions cancel each other. Thus *a magnet placed in a uniform field is acted on by a "couple" which rotates it until it lies with its axis parallel to the field, but there is no force on it tending to move it bodily.*

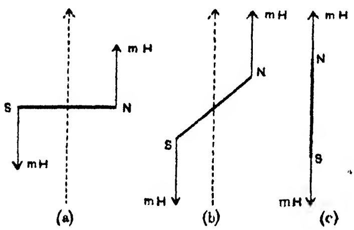


FIG. 76.

An expression for the turning effect or *moment of the couple* or *torque* on a magnet when it is at any angle, say α, with the direction of a uniform field H is readily obtained. The moment of the couple (Fig. 77) is given by the product of one force and the perpendicular distance between them. Hence:

$$\text{Moment of couple on NS} = mH \times ST = mH \times SN \sin \alpha.$$

The distance between the poles of a magnet (viz. SN) is usually denoted by l. The product ($m \times SN$) of the pole strength and the distance between the poles is called the **magnetic moment of the magnet** and is denoted by M. Hence:—

$$\text{Couple on NS} = mlH \sin \alpha = MH \sin \alpha \quad \dots\dots\dots (5)$$

The couple is greatest when $\alpha = 90°$ for then $\sin \alpha = 1$ and the couple is MH (Fig. 76*a*): it is least when $\alpha = 0°$ for then $\sin \alpha = 0$ and the couple is zero. Note that "moment of a couple" is often expressed, for brevity, as simply "couple."

Examples.—(1) *A magnet suspended by a wire hangs in the magnetic meridian when the wire is untwisted. If on turning the upper end of the wire half round the magnet is deflected 30° from the meridian, show how much the upper end must be twisted to deflect the magnet 45° from the meridian.*

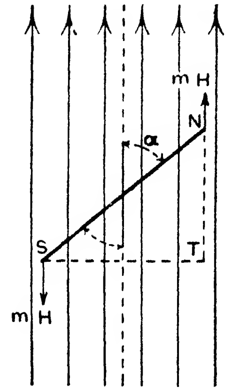


FIG. 77.

When the magnet is deflected through any angle α it is in equilibrium under the action of two couples, viz. (1) a restoring couple whose moment is MH sin α tending to bring it back into the meridian, (2) a deflecting couple due to the torsion on the wire (whose moment is, within limits, proportional to the angle of torsion) tending to turn it out of the meridian; hence—

Case 1. The upper end of the wire has to be turned through 180° to turn the bottom end through 30° in the same direction; $\therefore$ "twist" on wire $= 180° - 30° = 150°$. Now:—

$$\text{Angle of torsion} \propto \text{MH} \sin 30°; \quad \therefore 150 \propto \text{MH} \sin 30.$$

Case 2. If $\theta°$ = the required angle, the twist on the wire will be $\theta° - 45°$: hence as before $\theta° - 45° \propto \text{MH} \sin 45$. Thus we have:—

$$\frac{\theta° - 45°}{150°} = \frac{\sin 45°}{\sin 30°} = \frac{\sqrt{2}}{1} \quad \therefore \theta° = (150\sqrt{2} + 45)° = 257°.$$

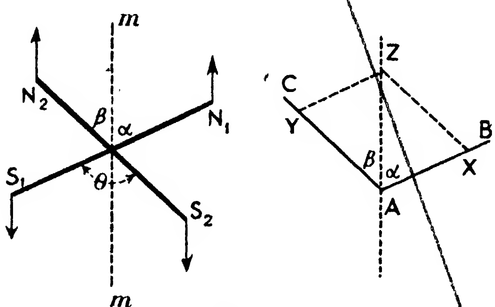


FIG. 78.

(2) *Two magnets whose moments are M_1 and M_2 are rigidly fixed together at their centres so that their axes make an angle θ with each other, and the combination is floated on a cork in water. Find the position of equilibrium with respect to the magnetic meridian.*

Let Fig. 78 represent the position of rest, the line *mm* denoting the magnetic meridian. The couple on N_1S_1 is $M_1H \sin \alpha$ and tends to rotate the combination *counterclockwise* into the meridian. The couple on N_2S_2 is $M_2H \sin \beta$ and tends to rotate the combination *clockwise* into the meridian. Since there is equilibrium

$$M_1H \sin \alpha = M_2H \sin \beta; \quad \therefore \frac{\sin \alpha}{\sin \beta} = \frac{M_2}{M_1}.$$

Thus the combination comes to rest so that the sines of the angles which the magnets make with the meridian are inversely as their magnetic moments. If $M_1 = M_2$, $\alpha = \beta$.

To get the right position graphically, draw the two lines AB and AC enclosing an angle θ. On AB mark off AX representing M_1, and on AC mark off AY representing M_2. The diagonal AZ of the parallelogram represents the meridian. From the figure $\sin \alpha/\sin \beta = XZ/AX = M_2/M_1$ as above.

So far we have assumed the field to be uniform, and have seen that there is no *translatory force* on a magnet put in it. If the field is not uniform, however, there *is* a translatory force on the magnet. In Fig. 30 the small magnet *ab* is in the non-uniform field of NS. It is pivoted and cannot move bodily, but it is clear by examining the forces *b*T and *a*V that if it had been free to move it would have tended to do so—towards the magnet. In such cases it can be proved that the forces on the poles can be resolved into a directive couple and a single translatory force, the combined effect of which is to tend to make the magnet not only take up a definite position, but move bodily. (See Art. 19.)

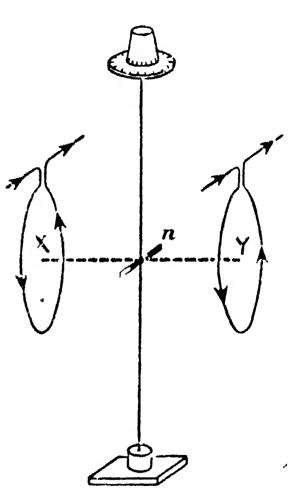


FIG. 79.

5. Magnetic Moment and Intensity of Magnetisation

The **magnetic moment** (M) of a magnet has been defined as given by its pole strength multiplied by the distance between its poles. Further, if a magnet is at right angles to a uniform field of strength H, the "couple" acting on it has its greatest value (Art. 4), which is MH: if the field has unit strength the *moment of the couple* numerically equals M, the *magnetic moment* of the magnet. Hence we can say *the "magnetic moment of a magnet" is measured by the product of the pole strength and the distance between the poles (in cm.), and it is numerically equal to the "moment of the couple" acting on the magnet when it is placed at right angles to a field of unit strength.*

We can arrive at the same result experimentally as follows. A current in a coil produces a magnetic field at right angles to the plane of the coil (Fig. 53). In Fig. 79, *n* is a small magnet suspended from a torsion head (which can be rotated) at the top, and attached

to a fixed spring at the bottom. At the commencement of the experiment bring n into its rest position in the earth's field, *i.e.* into the magnetic meridian. Now place a current-carrying coil X in the position shown so that n is opposite its centre. The coil's field is from left to right, and may be taken to be uniform over the place occupied by n: the magnet is deflected. Turn the torsion head in the opposite direction until n is brought back to its zero position, *i.e.* in the meridian and at right angles to the coil's field. Let $\theta°$ be the angle the top is turned through. Remove X, place a similar current-coil Y as shown, and adjust its position so that n remains in its zero position: Y is therefore producing the same field as X did (and in the same direction). Replace X: the field is now doubled and n is deflected. Turn the torsion head until n is again brought back to the zero position: the total angle through which the top is now turned will be found to be $2\theta°$ (about).

Thus we see that the couple (which is proportional to the angle of torsion—Example 1, page 63) necessary to maintain n at right angles to the uniform field of the coils is proportional to the field strength (and of course it depends on the magnet). Hence:—

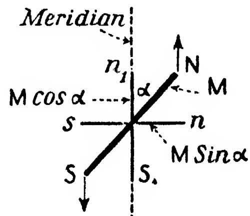


FIG. 80.

$$\text{Couple} \propto \text{H} \quad \text{or} \quad \text{Couple} = \text{MH},$$

where M is a constant for the magnet which we call its "magnetic moment." If H = 1, couple = M. Hence *the magnetic moment of a magnet is numerically equal to the moment of the couple which is required to keep the magnet at right angles to a field of unit strength.* Compare this with the previous definition.

Note that from the above if M = 1, the couple = H, and if H be unity, the couple is of unit value: hence we might say, *a field has unit intensity if a couple of unit moment is required to keep a magnet of unit magnetic moment at right angles to the field.*

A further point should be noted. Referring to Fig. 80, let us imagine that the *magnetic moment* M *of the magnet* can be resolved into two components, one M cos α along the meridian and the other M sin α at right angles to it. The couple on the first component is zero for it is along the field. The couple on the second component is given by its moment multiplied by the field, for it is at right angles to the field, *i.e.* the couple is M sin $\alpha \times$ H. Thus the total couple for the components is MH sin α, which is the same as for the

original magnet. Thus *the magnetic moment of a magnet can be treated as a vector quantity and resolved into components.* Of course, this fact is apparent, for magnetic moment has *direction* as well as magnitude.

The **intensity of magnetisation** (I) *of a magnet is measured by the ratio of the magnetic moment to the volume.* Thus if M be the magnetic moment, *m* the pole strength, V the volume (c.cm.), *l* the length (cm.), and *a* the cross-sectional area (sq. cm.), we have for the intensity of magnetisation of the magnet

$$I = \frac{M}{V} = \frac{ml}{al} = \frac{m}{a}; \quad \therefore \ M = IV \quad \text{and} \quad m = Ia \ \ldots . (6)$$

and the intensity of magnetisation is therefore sometimes defined as measured by *the magnetic moment per unit volume* (M/V) or by *the pole strength per unit area of the pole face* (*m*/*a*).

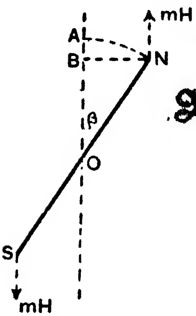


FIG. 81.

6. Work Done in Deflecting a Magnet

Work must evidently be done in deflecting, say, a suspended magnet from its position of rest in the earth's field. An expression for this work can be deduced from first principles as follows:— When the deflection is β (Fig. 81) the pole N has moved a distance AB along the direction of the field, and the work done on the pole N is measured by the product of the force and the distance, *i.e.* by $m\text{H} \times \text{AB}$; the work on the pole S is equal to this. Hence if $2l$ be the length of the magnet,

$$W = \text{Total work} = 2m\text{H} \times \text{AB} = 2m\text{H} \times (\text{AO} - \text{BO});$$

$$\therefore \ W = 2m\text{H} \times (l - l \cos \beta) = 2ml\text{H}\,(1 - \cos \beta),$$

$$\textit{i.e.}\ W = \text{MH}\,(1 - \cos \beta) \ \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

When $\beta = 90°$, work = MH; when $\beta = 180°$, work = 2MH.

This is required in some problems, as will be seen later.

7. Magnetic Shell. Strength of a Shell

In simple language a magnetic shell is a *thin* sheet of magnetic material (say iron) magnetised so that all one face exhibits north "magnetism" and all the other face south "magnetism," but the usual definition is "A magnetic shell is a *thin* sheet of magnetic material, magnetised at every point in a direction normal to the

shell at the point considered." A shell may be of any shape provided it complies with the above conditions: thus it may be a flat circular disc, a flat sheet of irregular shape, a hemispherical surface, etc.

Taking any point on a shell, *the product of the intensity of magnetisation and the thickness of the shell at that point measures what is called* **the strength of the shell** *at the point.* Thus if I be the intensity of magnetisation (pole per unit area), t the thickness, and ϕ the strength:—

$$\text{Strength of shell} = \phi = \mathrm{I}t \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Further, if the strength be the same at all points of the shell, *i.e.* if it is a *uniform magnetic shell,* and if M be the magnetic moment of the whole shell, A, its face area, and V its volume, then:—

$$\phi = \mathrm{I}t = \frac{\mathrm{M}}{\mathrm{V}}t = \frac{\mathrm{M}}{\mathrm{A}t}t = \frac{\mathrm{M}}{\mathrm{A}}; \quad \therefore \ \mathrm{M} = \phi\mathrm{A}$$

so that for such a shell we can say *the strength of the shell is measured by the magnetic moment per unit face area* (M/A).

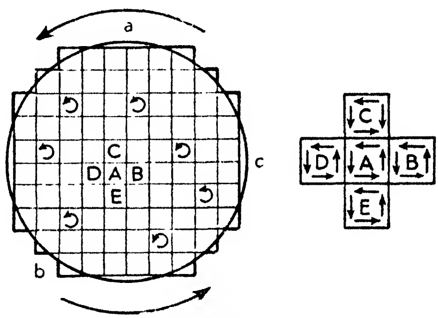


FIG. 82.

Magnetic shells are never met with as actual pieces of apparatus. Their main importance lies in the fact that the absolute or C.G.S. electromagnetic unit (e.m. unit) of current strength (Chapter X.) is so chosen that a closed circuit carrying a current i units produces the same magnetic field at any point as would be produced by a magnetic shell of the same contour (*i.e.* whose boundary coincides with the current circuit), if the strength of the shell is *numerically* equal to the strength of the current, viz. i units. If the current-carrying coil is in a medium of permeability μ, the strength of the equivalent shell must be μi units.

The fact that a shell can be equivalent to a current circuit may be seen thus:—Let *abc* (Fig. 82) be a closed circuit in which a current i is flowing counter-clockwise as indicated. Now imagine the circuit filled with small squares and that a current i flows counter-clockwise in each square. Considering any one square A the magnetic effect of the current going *up* its right-hand side will be cancelled by the magnetic effect of the equal current going *down*

in square B. Similarly, the currents in the other three sides of A will be cancelled by the equal opposite currents of squares C, D, and E. This applies to all the inner lines, leaving, as the only un-cancelled current, a counter-clockwise current i in the outer boundary of the figure formed by the squares. By taking a sufficiently large number of squares this outer boundary, in the limit, coincides with the circuit *abc*, and the magnetic effect of the current-carrying squares is the same as that of the current i in *abc*.

But each tiny square is equivalent to a small magnet of definite moment with its N. pole towards the reader, or a small shell of definite strength, and these small shells make up a larger shell coinciding with *abc*. Thus the magnetic field of the current i in *abc* is the same as that of a shell of definite strength whose boundary coincides with *abc*. And the unit current has been so chosen that the strength of the current i is *numerically* the same as the strength of the shell, for it is defined thus (other definitions of unit current are given later):—**The C.G.S. electromagnetic unit of current strength is that current which flowing in any closed circuit has magnetic effects the same as those of a uniform magnetic shell of unit strength whose boundary coincides with the current circuit, the medium being air (strictly a vacuum)**: the **ampere,** the practical unit of current strength, is one-tenth of this unit.

It may be noted that in addition to the *equivalent shell method* there is another method of making calculations on current circuits known as the *current element method* which is, in many cases, easier to apply: and although, in the past, the shell method was considered more satisfactory and the element method as fictitious, the latter is now justified, for we now know that a current in a wire is a movement of electrical particles—electrons. This last statement will be understood later when both methods of making the calculations will be explained.

B—MAGNETIC FIELDS DUE TO MAGNETS

8 The Magnetic Field off the End of a Bar Magnet

We have seen that the intensity of the field at distance d cm. from a pole of strength m webers is m/d^2 gauss (in air), for this is the force on unit pole at the point. To find the field at a point due to a magnet we imagine a unit N. pole put there, calculate the force on it due to both poles of the magnet, and find the resultant by the parallelogram law: this resultant force in dynes gives the field in gauss. We will generally assume the magnet to be small compared with the distance of the point from it, and it will help the student if we consider first a numerical example.

Suppose the pole strength of NS (Fig. 83) is 8 units, its length 4 cm., and let us find the field at P *on its magnetic axis produced* and 100 cm. from the neutral line.

$$\text{Repulsion on unit north pole at P due to N} = \frac{8 \times 1}{98^2} = \frac{8}{9604} \text{ dyne.}$$

$$\text{Attraction on unit north pole at P due to S} = \frac{8 \times 1}{102^2} = \frac{8}{10404} \text{ dyne.}$$

These forces are indicated by the arrows;

$$\therefore \text{ Resultant force} = \frac{8}{9604} - \frac{8}{10404} = \frac{6400}{99{,}920{,}016} \text{ dyne.}$$

This force on unit pole in dynes gives the intensity of the field at P in gauss due to NS. Further, it will be accurate enough to write 100,000,000 for 99,920,016;

$$\therefore \text{ Field at P} = \frac{6400}{100{,}000{,}000} = \frac{2 \times 32}{(100)^3} = \frac{2M}{d^3} \text{ gauss,}$$

for 32 is the moment (M) of the magnet NS, and 100 cm. is the distance (say d) from the centre of the magnet to P. Note that the length of the magnet (4 cm.) is small compared with the distance d (100 cm.).

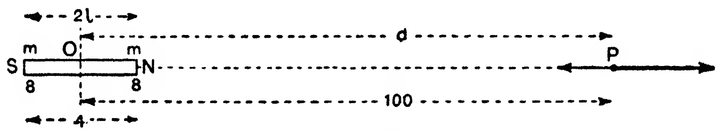


FIG. 83.

Considering now a general case, let m be the pole strength of NS, and $2l$ the distance between the poles: then:—

$$\text{Intensity at P due to N} = \frac{m}{(d-l)^2} \text{ along NP,}$$

$$\text{Intensity at P due to S} = \frac{m}{(d+l)^2} \text{ along PS,}$$

$$i.e. \text{ Field at P} = \frac{m}{(d-l)^2} - \frac{m}{(d+l)^2} = \frac{4mld}{(d^2-l^2)^2} = \frac{2ml \times 2d}{(d^2-l^2)^2};$$

$$\therefore \text{ Field at P} = \frac{2Md}{(d^2-l^2)^2} \text{ gauss (along NP)} \quad \ldots\ldots (9)$$

In this case the magnet is said to be "end on" to the point P. If the magnet be short compared with the distance d we can neglect the l^2 and we get (as above):—

$$\text{Field at P} = \frac{2M}{d^3} \text{ gauss} \quad \ldots\ldots\ldots\ldots\ldots (10)$$

In practice there is generally another field at P, viz. the earth's field, and the actual field is the resultant of the two: we are concerned at present, however, with the magnet's field only.

9. The Magnetic Field off the Middle of a Bar Magnet

The problem here is to find the field at P (Fig. 84), a point on the *equatorial line* of the magnet. Denoting the pole strength by m and the distance between the poles by $2l$ we have:—

$$\text{Intensity at P due to N} = \frac{m}{r^2} \text{ in the direction NP.}$$

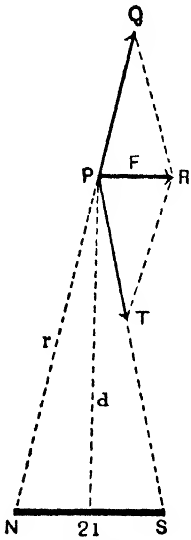


FIG. 84.

$$\text{Intensity at P due to S} = \frac{m}{r^2} \text{ in the direction PS.}$$

Denoting these by PQ and PT, and completing the parallelogram, the diagonal PR represents the resultant field F at P. From the figure, NPS and PQR are similar triangles: hence—

$$\frac{\text{PR}}{\text{PQ}} = \frac{\text{NS}}{\text{NP}}; \quad \therefore \frac{\text{F}}{\frac{m}{r^2}} = \frac{2l}{r}, \ i.e.\ \text{F} = \frac{2ml}{r^3} = \frac{\text{M}}{r^3};$$

$$\therefore \text{ Field at P} = \frac{\text{M}}{(d^2 + l^2)^{\frac{3}{2}}} \text{ gauss (along PR)} \quad (11)$$

In this case the magnet is said to be "broadside-on" to the point P. If again the magnet is short compared with the distance d we can neglect the l^2 and we get:—

$$\text{Field at P} = \frac{\text{M}}{d^3} \text{ gauss} \quad \ldots\ldots\ldots (12)$$

Note that the field at a point due to an end-on magnet ($2\text{M}/d^3$) is twice the field at the point due to the same magnet broadside-on at the same distance (M/d^3). Note also that the resultant field in Fig. 84 is parallel to the magnet.

Example.—*A uniformly magnetised bar magnet,* 10 *cm. long and of moment* 200 *C.G.S. units, is placed in a horizontal position with its axis in the magnetic meridian and its north pole towards the north. A small compass* 10 *cm. due east of the centre of the magnet is in neutral equilibrium. Find the horizontal intensity of the earth's magnetic field.*

Let R (Fig. 85) be the position of the compass. Since the latter is in equilibrium the field F at R due to the magnet must be equal and opposite to

the earth's horizontal field H (see null point, Fig. 43), *i.e.* since the magnet is "broadside on" to R

$$F = H = \frac{M}{(d^2 + l^2)^{\frac{3}{2}}} = \frac{200}{(10^2 + 5^2)^{\frac{3}{2}}};$$

$$\therefore\ H = \cdot 14 \text{ gauss.}$$

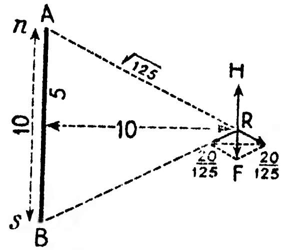


FIG. 85.

83. Magnetic Field of a Bar Magnet at any Point

Let NS (Fig. 86) be the magnet of moment M, and P the point at which the field is required. Let OP $= d$ and let α be the angle which OP makes with the magnetic axis of the magnet. (We shall assume the magnet small compared with the distance d.)

Resolve the magnetic moment M into two components, one, M cos α, in the direction OP, and the other, M sin α, at right angles to OP. The former (M cos α) is "end-on" to the point P and the latter (M sin α) is "broadside-on" to P. Hence:—

$$\text{Field at point P due to M}\cos\alpha = \frac{2M\cos\alpha}{d^3} \text{ along PQ.}$$

$$\text{Field at point P due to M}\sin\alpha = \frac{M\sin\alpha}{d^3} \text{ along PT.}$$

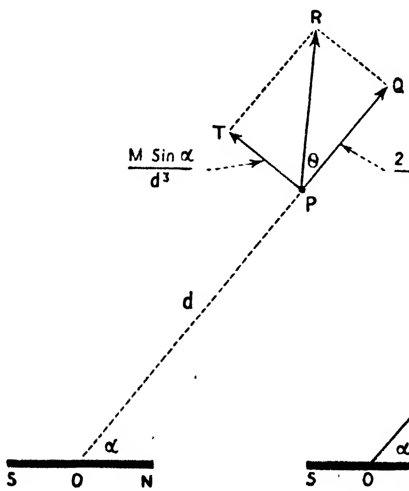


FIG. 86.

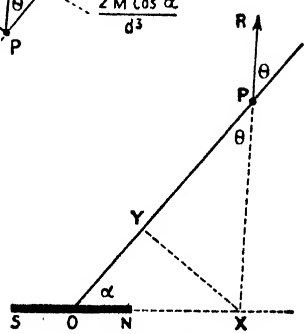


FIG. 86*a*.

Let PQ and PT represent these: then PR represents the resultant

$$PR^2 = PQ^2 + PT^2, \textit{ i.e. } PR = \sqrt{PQ^2 + PT^2};$$

$$\therefore\ PR = \sqrt{\left(\frac{2\ M \cos \alpha}{d^3}\right)^2 + \left(\frac{M \sin \alpha}{d^3}\right)^2}$$

$$\textit{i.e. } PR = \frac{M}{d^3}\sqrt{4 \cos^2 \alpha + \sin^2 \alpha} = \frac{M}{d^3}\sqrt{3 \cos^2 \alpha + (\cos^2 \alpha + \sin^2 \alpha)};$$

$$\therefore \text{Field at P} = \frac{M}{d^3}\sqrt{3 \cos^2 \alpha + 1} \quad \ldots\ldots\ldots\ldots (13)$$

From (13) it follows that if P is on the axial line $\alpha = 0$ (or 180°), $\cos^2 \alpha = 1$, and the field is $2M/d^3$ as in Art. 8 (end-on). If P be on the equatorial line, $\alpha = 90°$ (or 270°), $\cos^2 \alpha = 0$, and the field is M/d^3, as in Art. 9 (broadside-on).

The *direction* of the field makes an angle θ with OP such that $\tan \theta$ is one half of $\tan \alpha$ (which latter is, of course, *known*). This is readily seen:—

$$\tan\ \theta = \frac{QR}{PQ} = \frac{PT}{PQ} = \frac{M \sin \alpha}{d^3} \Big/ \frac{2M \cos \alpha}{d^3};$$

$$\therefore\ \tan\ \theta = \tfrac{1}{2} \tan \alpha \textit{ or } \tan \alpha = 2 \tan \theta \quad \ldots\ldots\ldots (14)$$

If we merely wish to *find the direction graphically* of the field at P we can proceed thus:—Join OP (Fig. 86*a*) and take OY equal to $\frac{1}{3}$ of OP. Draw YX perpendicular to OP and produce to meet the magnet's axis at X. Join PX. Then XP is the direction of the field at P, for:—

$$\tan \alpha = \frac{XY}{OY}, \quad \tan \theta = \frac{YX}{PY}; \quad \therefore \frac{\tan \alpha}{\tan \theta} = \frac{PY}{OY} = \frac{2}{1},$$

$$\textit{i.e. } \tan \alpha = 2 \tan \theta \text{ (as required).}$$

C—TWO IMPORTANT FORMULAE FOR EXPERIMENTAL WORK

Suppose at O (Fig. 87) there is a magnetic field F represented by OP, and another, H, at right angles to it represented by OQ. Then OR represents the resultant field and a compass or suspended magnet placed at O will come to rest with its magnetic axis along OR, *i.e.* at an angle of deflection from H of, say, $\theta°$. From the figure:—$F = H \tan \theta$. In the next two sections we deal with the action of a suspended magnet or compass when it is placed at a point where there are two component fields at right angles.

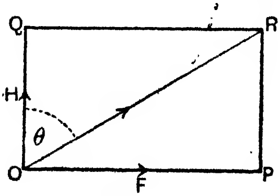


FIG. 87.

11. The Tangent A or End-on Formula

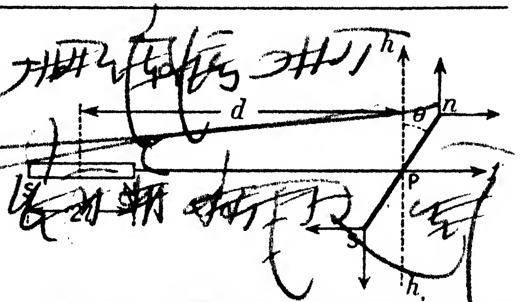


FIG. 88. Magnet *ns* is, of course, small.

Suppose P (Fig. 88) is a point on the table, and the dotted line *h'h* is the magnetic meridian through P. Let H represent the earth's horizontal field (about ·18 gauss) then H is the magnetic field at P acting in the direction P*h*.

On the west side (or the east side) put down a small bar magnet "end-on" to the point P. Its magnetic field (call it F) acts at P in the direction P*f*. We have at P then two magnetic fields at right angles. Now imagine a small suspended magnet or compass placed at P, and we will assume the suspended magnet is so small that the field F is the same where its poles *n* and *s* are as it is at P. The earth exerts a couple on it tending to turn it into the meridian, and the magnet exerts a couple on it tending to turn it along P*f*, *i.e.* at right angles to the meridian. It comes to rest with its magnetic axis along the direction of the resultant field, and

$$F = H \tan \theta,$$

where θ is the "deflection" of the suspended magnet from the meridian, *i.e.* the angle it makes with H.

But NS is "end-on" to the point P, so that if M is the moment of NS and d the distance from its neutral line to P,

$$F = \frac{2Md}{(d^2 - l^2)^2}; \quad \therefore \frac{2Md}{(d^2 - l^2)^2} = H \tan \theta,$$

$$i.e. \ \frac{M}{H} = \frac{(d^2 - l^2)^2}{2d} \tan \theta \ldots\ldots\ldots\ldots (15)$$

This is known as the "Tangent A or End-on Position of Gauss," and it is extensively used in experimental and test-room work. If the deflecting magnet NS be small compared with d, then $2M/d^3$ may be used for F, and we get:—

$$\frac{M}{H} = \frac{d^3 \tan \theta}{2} \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

In the above one of the fields has been assumed to be the earth's field, but the same reasoning applies to any two fields at right angles.

Fig. 89 shows the case in greater detail than Fig. 88, m being the pole strength of the suspended magnet. As stated:—

Deflecting couple = Controlling couple.

$$m\frac{2Md}{(d^2 - l^2)^2} \times \overline{NK} = mH \times \overline{SK}: \text{ and } \frac{SK}{NK} = \tan\theta;$$

$$\therefore \tan\theta = \frac{2Md}{H\,(d^2 - l^2)^2} \quad or \quad \frac{M}{H} = \frac{(d^2 - l^2)^2}{2d}\tan\theta.$$

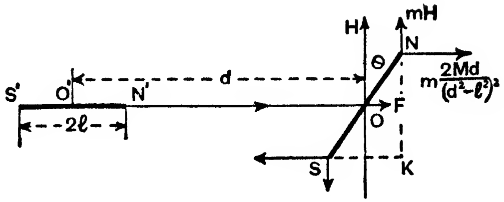


FIG. 89.

12. The Tangent B or Broadside-on Formula

As before, take P (Fig. 90) on the table with the earth's field H acting in the direction PE. Now on the south (or north) put down a small bar magnet "broadside-on" to the point P. Its field (F) acts in the direction Pf at right angles to H. Imagine a small compass at P: it comes to rest along the direction of the resultant field, making an angle of deflection from the direction of H equal to, say, θ, and we have $F = H\tan\theta$. But NS is "broadside-on" to P, so that—

$$F = \frac{M}{(d^2 + l^2)^{\frac{3}{2}}}; \quad \therefore \frac{M}{(d^2 + l^2)^{\frac{3}{2}}} = H\tan\theta,$$

$$i.e.\ \frac{M}{H} = (d^2 + l^2)^{\frac{3}{2}}\tan\theta \quad \ldots\ldots\ldots\ldots\ldots (17)$$

This is known as the "Tangent B or Broadside-on Position of Gauss," and, like the A position it is largely used in experimental work. As before, if the deflecting magnet is small compared with d, we can neglect l^2 and we get:—

$$\frac{M}{H} = d^3\tan\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

Examples.—(1) *A short bar magnet is placed at Gibraltar perpendicular to the magnetic meridian and "end-on" towards a compass needle from which it is distant* 100 *cm. When the experiment is repeated with the same apparatus at*

Portsmouth, the magnet has to be placed at a distance of 110 *cm. from the compass to produce the same deflection. Compare the earth's horizontal magnetic field at Gibraltar and Portsmouth.*

Let H_g and H_p = Horizontal fields at Gibraltar and Portsmouth;

$$\frac{M}{H_g} = \frac{100^3 \tan\theta}{2} \quad \text{and} \quad \frac{M}{H_p} = \frac{110^3 \tan\theta}{2};$$

$$\therefore \frac{\text{H at Gibraltar}}{\text{H at Portsmouth}} = \frac{H_g}{H_p} = \frac{110^3}{100^3} = 1{\cdot}33.$$

(2) *A small magnet* A *is placed "broadside-on" to a compass needle at a distance* 35 *cm. due magnetic south of it, and the deflection of the compass is* 30°. *Another small magnet* B *placed in the same position produces a deflection of* 45°. *Compare the magnetic moments of* A *and* B.

$$\frac{M_a}{H} = 35^3 \tan 30° \quad \text{and} \quad \frac{M_b}{H} = 35^3 \tan 45°;$$

$$\therefore \frac{\text{Moment of A}}{\text{Moment of B}} = \frac{\tan 30°}{\tan 45°} = {\cdot}58.$$

D—THE VIBRATING MAGNET

13. Time of Vibration of a Suspended Magnet

If a suspended magnet be deflected from its position of rest in a magnetic field, it at once experiences a couple urging it back into that position, and when the deflecting influence is removed it oscillates backwards and forwards about its equilibrium position, until finally it becomes stationary again. In such cases a double swing, *i.e.* a to *and* fro movement, is a *complete vibration*, the number of complete vibrations in one second is the *frequency*, the time taken for one vibration is the *period*, and the extent of the motion from the central position to an extreme position is the *amplitude*. Experiment shows that although the amplitude of the vibrations gradually gets less as the magnet gradually comes to rest, the time of performing each vibration is (for small swings) the same, and further that the time of a vibration depends upon the size, shape, mass, and magnetic moment of the magnet, and upon the strength of the field in which it vibrates. So far as the latter two are concerned, the greater the magnetic moment of the magnet and the stronger the field in which it is vibrating, the quicker it

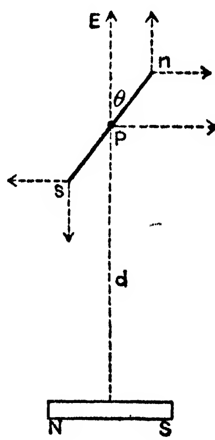


FIG. 90.

will vibrate, *i.e.* the less the period and the greater the frequency, *i.e.* the greater the number of vibrations it will make in a given time.

It is shown in books on Mechanics that if a suspended body, when displaced from its position of rest, experiences a restoring couple whose moment is proportional to its angular displacement, then its motion will be of the kind known as *simple harmonic motion*, and in such cases it is shown that the time of a complete (torsional) vibration t is given by:—

$$t = 2\pi\sqrt{\frac{K}{c}},$$

where K is the *moment of inertia* of the body, a constant which depends on its *mass*, *shape*, *size*, etc., and c is the moment of the restoring couple when the angular displacement is one radian: c, in fact, is the ratio (restoring couple)/(angle of displacement). The student must look up this and its proof in a book of Mechanics.

Now it was shown on page 62 that the restoring couple when a magnet of moment M is displaced through an angle a is MH sin a. If the displacement is small, not more than a few degrees, sin a and a (in radians) do not differ appreciably, and we can write MHa for the restoring couple: hence:—

$$\frac{\text{Restoring couple}}{\text{Angle of displacement}} = \frac{\mathrm{MH}a}{a} = \mathrm{MH};$$

$$\therefore \text{ Time of vibration} = t = 2\pi\sqrt{\frac{K}{\mathrm{MH}}} \quad \ldots\ldots\ldots(19)$$

Squaring we get $t^2 = 4\pi^2\mathrm{K/MH}$: if n be the frequency $n = 1/t$, and we get $n^2 = \mathrm{MH}/4\pi^2\mathrm{K}$. From these it follows that $t^2 \propto 1/\mathrm{MH}$ and $n^2 \propto \mathrm{MH}$: hence:—

(*a*) If the same magnet be caused to vibrate in different fields, the square of the period is *inversely* proportional to the field strengths, and the square of the frequency is *directly* proportional to the field strengths (see Chapter IV.) *if we neglect any change in the moment of the magnet due to the inductive action of the field.*

(*b*) If two magnets have *equal moments of inertia* (K), and if they be caused to vibrate in the same field, the squares of the periods are *inversely* proportional to the magnetic moments, and the squares of the frequencies are *directly* proportional to the magnetic moments (see Chapter IV.).

Below are the formulae for calculating K if required (1) for a rectangular bar magnet length a cm., breadth b cm., and mass m grammes vibrating about an axis through its centre, and perpendicular to the surface bounded

ANDRÉ-MARIE AMPÈRE, 1775-1836.

Born at Lyons. In 1793 the revolutionary army entered Lyons and André's father was executed: this so preyed on his mind that he nearly became insane. Gradually he recovered, became Professor at Paris, and established important relationships between magnetism and electricity. To perpetuate his name the unit of current strength is called the *ampere*.

by a and b, and (2) for a cylindrical magnet length l cm., radius r cm., and mass m grammes vibrating about an axis through its centre and perpendicular to the axis of the cylinder.

$$\text{Bar magnet, } K = m\frac{a^2 + b^2}{12}; \text{ Cylindrical magnet, } K = m\left(\frac{l^2}{12} + \frac{r^2}{4}\right).$$

Examples.—(1) *A suspended magnet makes* 27 *vibrations per minute in London and* 36 *per minute in Sydney. The earth's field at London is* ·18 *unit. What is the strength of the earth's field at Sydney?*

Call the earth's horizontal field at London H and at Sydney H_1. Let M be the moment of the vibrating magnet (which we will assume to be constant).

$$\frac{(\text{Frequency at Sydney})^2}{(\text{Frequency at London})^2} = \frac{MH_1}{MH} = \frac{H_1}{H} = \frac{\text{Earth's field at Sydney}}{\text{Earth's field at London}};$$

$$\therefore \frac{(36)^2}{(27)^2} = \frac{H_1}{\cdot 18}; \quad \therefore H_1 = \frac{(36)^2 \times \cdot 18}{(27)^2} = \cdot 32.$$

(2) *A small suspended magnetic needle makes* 20 *vibrations per minute under the influence of the earth alone. When a bar magnet is placed due magnetic south of the needle, the magnetic axis of the magnet being in the meridian and its north pole pointing towards the needle, the latter makes* 40 *per minute. Compare the field (at the place occupied by the needle) due to the magnet with the earth's field.*

An examination of the case will show that the earth's field and the magnet's field are *in the same direction* at the place occupied by the needle.

$$20^2 \propto \text{earth's field}, \quad 40^2 \propto \text{magnet's field} + \text{earth's field};$$

$$\therefore 40^2 - 20^2 \propto \text{magnet's field};$$

$$\therefore \frac{\text{Field due to magnet}}{\text{Earth's horizontal field}} = \frac{40^2 - 20^2}{20^2} = \frac{1200}{400} = 3.$$

(3) *Two magnets are arranged parallel one above the other. When like poles are together the combination makes* 20 *vibrations per minute, and when unlike poles are together* 10 *vibrations per minute. Compare their magnetic moments.*

Clearly K is the same in both cases. In Case 1 the magnets are in the same direction, so that if M_1 and M_2 denote their moments, t_1 the period, and n_1 the number of vibrations per minute,

$$t_1 = 2\pi\sqrt{\frac{K}{(M_1 + M_2)H}}.$$

If t_2 be the period in the second case, and n_2 the number of vibrations per minute,

$$t_2 = 2\pi\sqrt{\frac{K}{(M_1 - M_2)H}}.$$

$$\therefore \frac{t_1^2}{t_2^2} = \frac{n_2^2}{n_1^2} = \frac{M_1 - M_2}{M_1 + M_2}; \quad \therefore \frac{M_1}{M_2} = \frac{n_1^2 + n_2^2}{n_1^2 - n_2^2} = \frac{20^2 + 10^2}{20^2 - 10^2} = \frac{5}{3}.$$

E—MAGNETIC POTENTIALS DUE TO MAGNETS

14. Magnetic Potential at a Point due to a Pole

The point P at which the magnetic potential is required is d cm. from the north pole N of strength m units: P, Q, R, S, T are points *very close* together, T being at distance d_1 cm. from N (Fig. 91).

$$\text{Intensity of field at P} = \frac{m}{NP^2}; \text{ Intensity at Q} = \frac{m}{NQ^2};$$

$$\therefore \text{ Average intensity between P and Q} = \frac{m}{NP.NQ},$$

and this is therefore the force on a unit north pole between P and

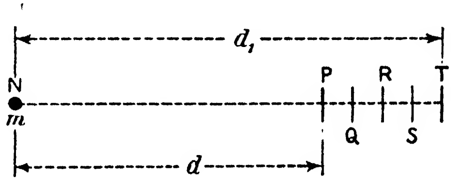


FIG. 91.

Q. Further, as work = force × distance, *the work done in moving a unit north pole* from Q to P will be:

$$\frac{m}{NP.NQ} \times QP = \frac{m}{NP.NQ} \times (NQ - NP) = \frac{m}{NP} - \frac{m}{NQ},$$

and similar expressions will be obtained for each of the other short steps. Hence we can write:—

$$\text{Work in moving unit N. pole from Q to P} = \frac{m}{NP} - \frac{m}{NQ},$$

$$\text{,, ,, ,, ,, ,, ,, ,, R to Q} = \frac{m}{NQ} - \frac{m}{NR},$$

$$\text{,, ,, ,, ,, ,, ,, ,, S to R} = \frac{m}{NR} - \frac{m}{NS},$$

$$\text{,, ,, ,, ,, ,, ,, ,, T to S} = \frac{m}{NS} - \frac{m}{NT},$$

and the total work done in moving a unit north pole from T to P will be the sum of these four expressions, which gives the result:—

$$\text{Work from T to P} = \frac{m}{NP} - \frac{m}{NT} = \frac{m}{d} - \frac{m}{d_1}.$$

Strictly, a large number of *very* small steps should be taken: the result would, of course, be as above. But the work done in moving unit north pole from T to P measures the magnetic potential difference between the two points (Art. 3): hence:—

$$\text{Magnetic potential difference} = \frac{m}{d} - \frac{m}{d_1} \quad \ldots\ldots\ldots (20)$$

If T be at infinity, then m/d_1 is zero, and the work done in moving the unit north pole from infinity to P becomes m/d: but this measures the *potential at P* (Art. 3), so that for the potential at a point at distance d cm. from a magnet pole of strength m webers we have:—

$$\text{Magnetic potential at P} = \frac{m}{d}\ \text{units} \quad \ldots\ldots\ldots\ldots (21)$$

If m be replaced by a south pole of the same strength $(-m)$ the potential at P is $-m/d$. If the medium is not air but one of permeability μ the magnetic potential is $\pm m/\mu d$.

An application of the Calculus provides a much neater solution than the one above: thus for the magnetic potential at P we have:—

$$H = -\frac{dv}{dx} \text{ (page 61) or } dv = -H dx;$$

$$\therefore \text{Potential at P} = \int_{\infty}^{d} - H dx = \int_{\infty}^{d} - \frac{m}{\mu x^2} dx = -\frac{m}{\mu}\left[-\frac{1}{x}\right]_{\infty}^{d}$$

$$\textit{i.e.} \text{ Potential at P} = \frac{m}{\mu d} = \frac{m}{d} \text{ for air.}$$

15. Potentials off the End and off the Middle of a Bar Magnet

(1) MAGNETIC POTENTIAL ALONG THE AXIAL LINE.—Here the point P at which the magnetic potential is required is on the axial line of the bar magnet (Fig. 83). If $2l$ be the length of NS, m its pole strength, and d the distance of P from the neutral line:—

Potential at P = Potential due to N + Potential due to S,

$$\textit{i.e.} \text{ Potential at P} = \frac{m}{(d-l)} + \frac{-m}{(d+l)} = \frac{2ml}{d^2 - l^2};$$

$$\therefore \text{Magnetic potential at P} = \frac{M}{d^2 - l^2} \text{ units or } \frac{M}{d^2} \text{ units} \ldots (22)$$

the latter expression being obtained on the assumption that the magnet is small compared with the distance of P so that l^2 in the denominator can be neglected. If the medium is not air but one of permeability μ the magnetic potential is $M/\mu\,(d^2 - l^2)$ or $M/\mu d^2$.

(2) MAGNETIC POTENTIAL ALONG THE EQUATORIAL LINE.—It is readily seen that the magnetic potential at *any* point on the equatorial line of a bar magnet (*e.g.* at P in Fig. 84) is *zero*, for the point is always equidistant from N and S so that the potential due to S is equal and of opposite sign to the potential due to N: thus the potential at P in Fig. 84 is $m/r - m/r = 0$.

16. Magnetic Potential at any Point due to a Bar Magnet

Consider the magnetic potential at the point P (Fig. 86) due to the bar magnet NS of moment M: let OP $= d$ and let a be the angle OP makes with the axial line of NS. As in Art. 10 resolve the magnetic moment M into two components, one, M cos a, in the direction OP, and the other, M sin a, at right angles to OP.

Now P is on the axial line of M cos a, and (assuming our magnet small compared with d) the potential at P due to M cos a is M cos a/d^2. Again P is on the equatorial line of M sin a so that the potential due to M sin a is zero. The resultant potential due to the two components is therefore M cos a/d^2, and this is the potential due to NS, *i.e.*

$$\text{Magnetic potential at P} = \frac{\text{M}\cos a}{d^2} \quad \ldots\ldots\ldots\ldots(23)$$

If P is on the axial line $a = 0°$, $\cos a = 1$, and the potential is M/d^2 as in Art. 15. If P is on the equatorial line $a = 90°$, $\cos a = 0$, and the potential is zero as in Art. 15. If the medium is not air but one of permeability μ formula (23) becomes M cos $a/\mu d^2$.

F—POTENTIAL AND FIELD DUE TO A MAGNETIC SHELL

17. Magnetic Potential at a Point due to a Shell

Investigations on the magnetic potential and the magnetic field at a point due to a magnetic shell involve the conception and measurement of a *solid angle*. These the student should refer to in some book on Geometry, but the following brief note will answer our present purpose.

Imagine a sphere with centre P and radius r cm., and consider any area a sq. cm. of the surface of the sphere. From numerous points on the boundary of a picture lines drawn to P, the whole forming a kind of cone with the surface a (curved) as base and apex at P. If another sphere be drawn with centre P and radius r_1, the cone will intercept an area a_1 of this sphere, and it is found that the two areas a and a_1 are proportional to the squares of their corresponding radii r and r_1—in other words, the ratio, Area/(Radius)^2 is

constant. By analogy with the method of measuring a plane angle, this ratio is said to measure the solid angle subtended at P by the areas, *i.e.* solid angle at P $= a/r^2 = a_1/r_1^2$.

Now let AB (Fig. 92) be a ***small*** area a at right angles to the plane of the paper and P a point in the plane of the paper, and suppose lines are drawn forming a cone as before. With P as centre and PB as radius describe a sphere cutting the cone in the closed curve BC. The solid angle APB is given by:—Solid angle

$$= \frac{\text{Area of BC}}{(\text{PB})^2} = \omega.$$

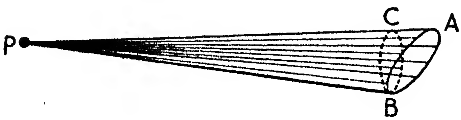


FIG. 92.

If the linear dimensions of BC are small compared with PB, then BC is practically plane and we can write Area BC = Area AB $\times \cos \alpha$, where α = the angle of inclination between the surfaces or the angle between the normals to AB and CB. Hence for the solid angle which the surface AB subtends at P we have:—

$$\text{Solid angle} = \omega = \frac{a \cos \alpha}{r^2} \quad \dots\dots\dots\dots\dots\dots (24)$$

We can now find an expression for the magnetic potential at a point P (Fig. 93) due to a magnetic shell. Consider a small element of the shell at O. The length of the element taken parallel to the direction of magnetisation, that is normal to the shell, is equal to t, the thickness of the shell at O. Let the area of the face of the element be denoted by a, where a is very small. Then the magnetic moment M of the element is equal to Iat, where I denotes the intensity of magnetisation. (Ia corresponds to m and t to l in the usual expression for moment, viz. M = ml.)

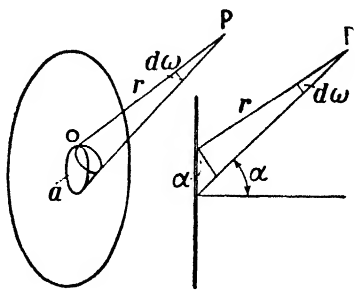


FIG. 93.

Now by Art. 16 the potential at P due to this small element is M cos α/r^2 : that is:— Potential due to element $= \dfrac{\text{I}at \cos \alpha}{r^2}$.

But It is the strength ϕ of the shell (Art. 7), and $a \cos \alpha/r^2$ is the small solid angle (call it dw) subtended at P by the element: thus the potential at P due to the element is $\phi\ (dw)$. Further, if the strength of the shell be the same at all points of it, and w be the solid angle subtended at P by the *whole shell*:—

Magnetic potential at P = Strength of shell $\times$ Solid angle at P,

i.e. Potential at P due to shell $= \phi w$ ....(25)

If the medium is not air but one of permeability μ the magnetic potential at P is $\phi w/\mu$, *i.e.* $1/\mu$ of the potential in air.

If P is indefinitely near the shell, $w = 2\pi$ and the potential is $+2\pi\phi$ at one side and $-2\pi\phi$ at the other, the potential difference between two such points being $4\pi\phi$.

18. Magnetic Field at a Point due to a Shell

In this book we need only take one example of this, viz. the field at a point on the axis of a *plane circular shell of uniform strength,* and even this the student may omit on a first reading. The case is shown in Fig. 94 where AB is the shell of radius r and P the point at distance x along the axis. Now:—

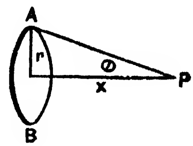


FIG. 94.

Potential at P

$= V =$ Strength $\times$ Solid angle at P $= \phi w$.

It is shown in the Appendix that in this special case the solid angle subtended at P by the shell is equal to $2\pi(1 - \cos\theta)$: hence:—

$$V = \phi w = 2\pi\phi\,(1\text{-}\cos\theta) = 2\pi\phi\left(1 - \frac{x}{(x^2 + r^2)^{\frac{1}{2}}}\right) \quad ..(26)$$

and to find the magnetic field at P from this potential we have:—

$$H = -\frac{dv}{dx} = -2\pi\phi\frac{d}{dx}\left(1 - \frac{x}{(x^2 + r^2)^{\frac{1}{2}}}\right),$$

$$i.e.\ H = 2\pi\phi\left\{\frac{(x^2 + r^2)^{\frac{1}{2}} - \frac{1}{2}(x^2 + r^2)^{-\frac{1}{2}}2x.x}{x^2 + r^2}\right\};$$

$$\therefore\ H = 2\pi\phi\frac{(x^2 + r^2)^{-\frac{1}{2}}(x^2 + r^2 - x^2)}{x^2 + r^2} = \frac{2\pi r^2\phi}{(x^2 + r^2)^{\frac{3}{2}}}$$

$$i.e.\ \text{Magnetic field at P} = \frac{2\pi r^2\phi}{(x^2 + r^2)^{\frac{3}{2}}}\ \text{gauss} \quad ......(27)$$

Now if AB is a coil of wire carrying a current i e.m. units, the magnetic field at P due to the current will be given by:—

$$\text{Magnetic field due to current} = \frac{2\pi r^2 i}{(x^2 + r^2)^{\frac{3}{2}}}\ \text{gauss} \quad ......(28)$$

and putting $x = 0$ we get that the magnetic field at the centre of a circular coil carrying a current i e.m. units is $2\pi i/r$ gauss. That this is so will also be seen in Chapter XI. (See pages 317, 318.)

G—TRANSLATORY FORCE ON A MAGNET IN A NON-UNIFORM FIELD

19. Translatory Force on a Magnet

We have seen that when a magnet is in a uniform field there is no force on it tending to move it bodily from one part of the field to another, but that when it is in a non-uniform field there is a translatory force on it. In this book we need only consider one simple case—two *small* magnets laid under constraint, *e.g.* laid on the table, as shown in Fig. 95—and the problem is to find the force, say, on magnet B *tending to move it bodily*. Let M be the moment of magnet A, and M′ that of magnet B.

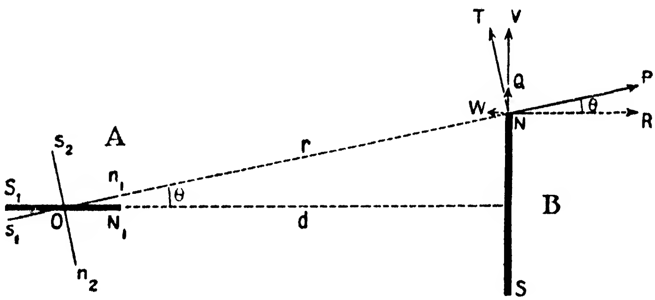


FIG. 95.

To find the force on N adopt the method used in Arts. 10, 16, *i.e.* resolve the moment M of magnet A into n_1s_1 of moment M cos θ along ON, and n_2s_2 of moment M sin θ at right angles to ON. If m be the strength of pole N the force on it due to n_1s_1 is $m2\mathrm{M}\cos\theta/r^3$ along NP, and the resolved part of this force in the direction NQ is $(m2\mathrm{M}\cos\theta/r^3) \times \sin\theta$. We will neglect the other component along NR (there is an equal and opposite one on the other pole S.

Similarly, the force on N due to n_2s_2 is $m\mathrm{M}\sin\theta/r^3$ along NT, and the resolved part of this in the direction NV is $(m\mathrm{M}\sin\theta/r^3) \times \cos\theta$. As before, and for the same reason, we neglect the other component along NW.

Adding the two we get that there is a force on N acting upwards in the figure and given by:—

$$\text{Upward force on N} = \text{NQ} + \text{NV} = \frac{3m\mathrm{M}}{r^3}\sin\theta\cos\theta.$$

If we estimate the force on S in the same way we get that there is a force on it equal to the above and also acting upwards in the figure. Hence for the total force on the magnet B we have:—

$$\text{Force (upwards) on B} = \frac{6mM}{r^3} \sin\theta \cos\theta.$$

Further, since the magnets are small compared with d, we can write $r = d$, $\cos\theta = 1$, and $\sin\theta = l/d$, where $2l$ = length of magnet B (so that its moment $M' = 2ml$), and we get:—

$$\text{Translatory force on B} = \frac{3MM'}{d^4} \text{ upwards} \quad \ldots\ldots (29)$$

and it can be shown, in the same way, that the translatory force on A has also this value, but it acts downwards in the figure.

It is easier to show that if the two magnets be in line the translatory force on each is $6MM'/d^4$, the forces being in opposite directions along the line joining them, but we leave this as a simple exercise for the student.

If the horizontal components on N and S (which were neglected above) be considered, it will be found that they form a couple tending to *rotate* the magnet (see end of Art. 4).

H—MAGNETIC INDUCTION. PERMEABILITY. SUSCEPTIBILITY

20. Magnetic Induction

Up to this point in the present chapter, the magnets have been *permanent* magnets *in air* (strictly *in vacuo*), and therefore the necessity for any differentiation between magnetic force and magnetic induction, tubes of force and tubes of induction, etc., referred to in Chapter II. has not arisen. Should the medium consist of one of the "magnetic" materials, however, further considerations are necessary; these are of the utmost importance in electromagnetic theory and in the study of applied magnetism.

Consider two poles of strengths m and m_1 situated d cm. apart in a medium of *indefinite extent* and of permeability μ; the force between them is $mm_1/\mu d^2$, and if m_1 be a unit pole the force $m/\mu d^2$ on it measures the strength of the field in C.G.S. units at distance d cm. from the pole m. Thus the **field intensity** (H) depends on the strength of the pole, on the distance, and *on the medium*, so that the condition of the field at the point cannot be used to specify the pole m completely, since the medium is also involved.

It is convenient therefore to have a quantity which depends only on the pole and on the distance from the pole, *i.e.* which is

always the same at distance d from a pole m, and this quantity is called the **magnetic induction** (B). Clearly, from the preceding it must be taken as μ times the field intensity (*i.e.* $B = \mu H$) for then the induction at distance d cm. from the pole m becomes $\mu \times m/\mu d^2$ or m/d^2, *i.e.* it depends only upon m and d. To summarise:—for a point at distance d cm. from a pole of strength m in a medium of permeability μ:—

$$H = \text{Field Intensity} = \frac{m}{\mu d^2},$$

$$B = \text{Magnetic Induction} = \mu H = \frac{m}{d^2},$$

and if the medium be a vacuum (or air) $\mu = 1$, and both intensity H and induction B are measured by the same expression, viz. m/d^2.

Just as a line of force is such that the tangent at any point of it gives the direction of the field at that point, so we take a line of induction as such that the tangent at a point gives the direction of the induction. Further, just as, by convention, the number of *unit tubes of force* per sq. cm. (at right angles to the field) measures the field intensity H, so lines are grouped into tubes of induction such that the number of *unit tubes of induction* per sq. cm. measures the induction B. In air (strictly *vacuo*) $B = H$ and lines (and tubes) of induction and lines (and tubes) of force are the same. Note again, however (page 37), that in actual practice *tubes of induction* do not start or stop anywhere but form closed curves: *a tube of induction is never discontinuous but will, if traced far enough, double back on itself to form a closed tube*: if part of it passes through air (strictly *vacuo*) that part is also a tube of force.

21. Gauss's Theorem

Imagine *any* closed surface surrounding an amount of "pole" m in a medium of permeability μ, and consider a small area a containing a given point (Fig. 96). Let H denote the field intensity at this point and let α be the angle between the direction of H and the *outward* drawn normal to the surface at the given point. The component of H along this normal is $H \cos \alpha$, and since magnetic induction $= \mu \times$ field intensity, the induction in this direction is $\mu H \cos \alpha$. The product $\mu H \cos \alpha \times a$ is spoken of as the **normal magnetic induction** or **magnetic flux** over the small area a. The total normal flow of induction or the total normal magnetic induction or magnetic flux over the whole closed surface is obtained by supposing the whole surface divided up into a very large number

of small areas such as *a* and summing up the values of μH cos $\alpha . a$ for all these areas. Denoting this total normal induction by T.N.M.I. we have:—

$$\text{T.N.M I.} = \mu\Sigma\text{H} \cos \alpha . a.$$

Now in Fig. 96 if *d* be the distance of *m* from the point in the area *a* we have:—

$$\text{Normal induction over } a = \mu\text{H} \cos \alpha . a = \mu \frac{m}{\mu d^2} \cos \alpha . a,$$

$$= m \frac{\cos \alpha . a}{d^2} = m\omega,$$

for cos $\alpha . a/d^2$ is the solid angle ω subtended at *m* by the area *a*; hence for the total normal induction or magnetic flux *over the whole closed surface* we have:—

$$\text{T.N.M.I.} = m\Sigma\omega = 4\pi m \quad \ldots\ldots\ldots\ldots\ldots (30)$$

for $\Sigma\omega$ is the solid angle subtended by the whole closed surface and is equal to 4π. This is a simple proof of Gauss's Theorem applied to magnetism which states that "**the total normal magnetic induction or magnetic flux over a closed surface drawn in a magnetic field is 4π times the total 'magnetism' inside.**" A proof of the same theorem as applied to electrostatics is given on page 185.

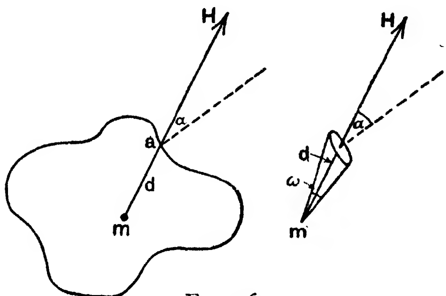


FIG. 96.

If *m* be *outside* the closed surface, the total normal induction over the surface is zero for the flux *inwards* is equal to that *outwards*.

The theorem holds if there are several " poles " inside, and if some are + and some −; in this case the *sum* of the pole strengths is involved. It follows that if the closed surface be drawn round a magnet the total normal induction over the surface is zero: the lines of induction passing *outwards* also pass *inwards* to complete their closed circuits.

In the case of an air medium μ is taken as unity, induction and intensity coincide, and the reader will come across the statement " **the total normal magnetic intensity over a closed surface drawn in a magnetic field is 4π times the sum of the strengths of all the magnetic poles inside.**" It will now be seen that the fact stated on page 59 was really a special case of the general Gauss theorem: the medium was air, and as *intensity is measured by the number of*

unit tubes of force per unit area, the *total unit tubes of force* passing outwards from the pole m was $4\pi m$.

Distinguish between the *total induction* or *total flux* over an area and the *induction B* of Art. 20 or *flux density* which refers to unit area.

SIMPLE APPLICATION OF GAUSS'S THEOREM.—Let us take first an imaginary case of a plane sheet of north magnetism of *infinite* extent, the amount of magnetism per unit area being denoted by ρ and *let it be required to find the intensity of the field at P due to the sheet* (Fig. 97). Picture a unit area at P parallel to the sheet. From the boundary of this area draw lines perpendicular to the sheet thus forming a prism, the other end of the prism being a unit area at P_1. In this closed surface formed by the prism the total amount of magnetism is that on the unit area S, viz. ρ. By Gauss's theorem the total induction over the closed surface is $4\pi\rho$, and as the induction is everywhere perpendicular to the sheet, the induction over the sides of the prism is zero whilst that over the two ends together is $4\pi\rho$. Half of this is at P and half at P_1 so that the induction at P is $2\pi\rho$. As the medium is air this also measures the intensity at P; hence

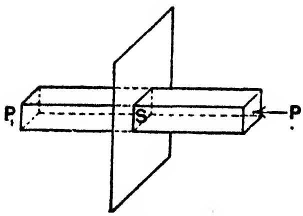


FIG. 97.

$$\text{Intensity at P} = 2\pi\rho,$$

and is independent of the distance of P from the infinite sheet.

From the above it is easy to find **the intensity of the field at a point between two poles** (assumed close together) as indicated in Fig. 98. If I be the intensity of magnetisation, *i.e.* the amount of pole per unit area, then if the poles are very close together the intensity at P due to N may be taken as $2\pi I$ and the intensity due to S as also $2\pi I$ (poles assumed equal). These are in the same direction, viz. right to left, so that the total intensity at P is given by

FIG. 98.

$$\text{Intensity at P} = 4\pi I \ldots\ldots\ldots\ldots\ldots\ldots (31)$$

22. Iron in a Magnetic Field

Consider first the three simple experiments briefly indicated below:—

(*a*) Note the magnetic lines between the unlike poles of two magnets in line (Fig. 35). Repeat with a bar of soft iron between the poles. The iron

is magnetised and the diagram (Fig. 99) seems to indicate that the magnetic lines are crowding into the iron as if they found it easier to go through iron than through air and were "taking the path of least resistance." The full explanation of this is, however, given below.

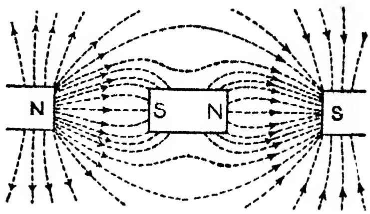


FIG. 99.

(*b*) Place a soft iron ring between the unlike poles of the two magnets in line (Fig. 100), and map the lines by filings. The filings do not set themselves in definite directions *inside* the ring. There are *practically* no lines in the central air space: they seem to pass along the iron, preferring to keep to this rather than to cross the central space. This space is therefore more or less *screened* from the influence of the magnets. This is used in certain galvanometers, ammeters, and other measuring instruments: they are placed inside a thick cylinder of soft iron which protects the needle, etc., of the instrument from magnetic fields outside.

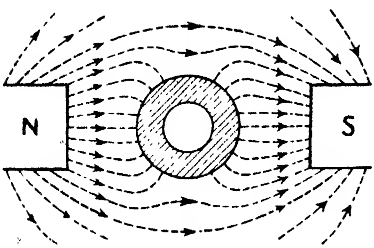


FIG. 100.

(*c*) Similarly, Fig. 101 shows how a compass at C is more or less screened from the effect of the magnet N by interposing a thick sheet of soft iron as shown. If the iron were replaced by a sheet of glass, wood, copper, etc., the magnetic lines would pass through to the other side almost the same as with air, and the compass would be affected.

From these and similar experiments we may, for practical purposes, regard every substance as possessing a certain power of conducting magnetic lines and of offering a certain resistance to the passage of the lines. Thus from the experiments we might deduce that iron is a better conductor of magnetic lines than air, or, as we say, is more "permeable" or has greater **permeability** than air. Put another way, we say that the magnetic resistance or **reluctance** of air is greater than that of iron. From this point of view we might say in general terms, and

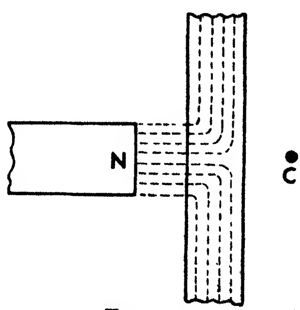


FIG. 101.

certainly rather vaguely, that *the permeability of a material is its conducting power for magnetic lines as compared with air,* or that *the permeability of a material is the degree to which the magnetic field can penetrate or permeate the material,* or that *the permeability of a material is the power it possesses of acquiring a magnetic flux when subjected to magnetic influence.*

Now let us consider the first experiment above (Fig. 99) in greater detail, but this time we will imagine the iron to be put in a *uniform field.* Let the dotted lines of Fig. 102 represent the lines of force in a uniform field (in air), the direction of the field being from left to right. Now let the bar of soft iron be placed in the field (Fig. 103). The iron is magnetised, a south pole on the left and a north on the right, and we now have, as it were, two sets of

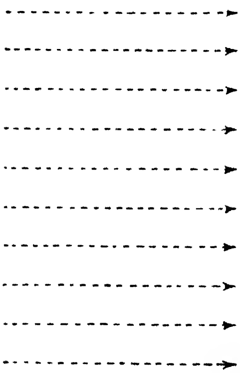


FIG. 102.

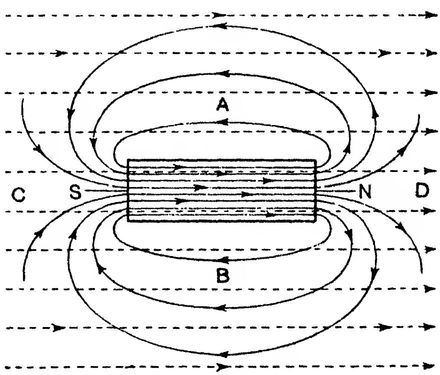


FIG. 103.

magnetic lines, those of the original field and those of the magnetised iron. Of course, in practice we get a "resultant" effect, but for the moment we will imagine them existing together.

First consider the iron itself. The magnetisation of the iron results in magnetic lines passing along it from left to right, *i.e.* from S to N: in addition we have, also passing along the iron from left to right, the lines of the original field: and we may have lines from right to left, *i.e.* N to S, owing to the end poles (demagnetising effect of end poles—page 31). The total *lines of induction,* as we call them, in the iron is therefore the resultant of these.

Now consider the air space outside the iron. At A and B we have the magnetic lines of the magnet which have come out of N going from *right to left* to enter S, and we have the magnetic lines

of the original field going from *left to right.* The two sets are in opposite directions here and partly cancel each other, the final result being that we have, as it were, a weakening effect in the space at A and B. At C and D the two sets of magnetic lines are more or less in the same direction and help each other, the final result being that the field is *strengthened* at C and D by the presence of the iron. Put another way, there will be a *concentration* of lines at C and D and a *separation* of lines at A and B. The actual "resultant" is shown in Fig. 104. The crowding of the lines at C and D and the separating at A and B are therefore due to the fact that the iron is magnetised by the field, that we have therefore two agents at work, that these help each other at some places and oppose each other at other places: and the "resultant" effect of the two gives the arrangement shown in Fig. 104.

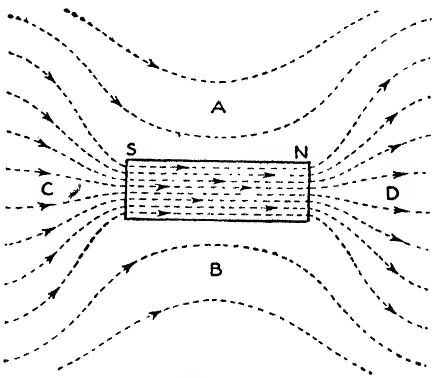


FIG. 104.

If a bar of diamagnetic material instead of iron had been placed between the poles of Fig. 99 or in the uniform field of Figs. 102-104 it would have been magnetised (very weakly) *the other way about,* its N. end being on the left and its S. end on the right. Reasoning like the above will show that there would be a slight opposition or weakening effect at C and D and a helping effect at A and B—a separation of lines at C and D and a concentration at A and B, but both only slight. Fig. 105 shows the resultant effect for a diamagnetic bar corresponding to Fig. 104 for the iron bar.

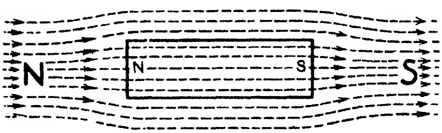


FIG. 105.

23. Permeability and Susceptibility

We have seen that the intensity (H) of a magnetic field at a point in air is measured by the force it exerts on a unit north pole

put there. If, then, we wish to consider in this way the magnetic intensity, or magnetising field, or magnetising force, say H_1 *actually effective in the iron* of Fig. 104 we must imagine a hole or cavity in the material in which to place the unit pole. A cavity of any shape will not do, however, for the walls of the cavity will exhibit magnetism and exert an influence on the unit pole, so that the force on the latter will not be H_1, the effective magnetising force in the iron. Consider, however, a cavity such as is shown at RS (Fig. 106), viz. a long indefinitely thin tunnel in the direction of magnetisation. The sides of the tunnel will exhibit no magnetisation, the " poles " at the ends of the tunnel will be weak and far away from the centre, and will not appreciably affect a unit pole put there; thus the field intensity at the centre of the tunnel will give the actual value of the magnetising force H_1 in the iron. Hence **the magnetic or magnetising force or magnetising field H_1 inside the iron is measured by the force in dynes on a unit north pole placed at the centre of a long and indefinitely narrow tunnel in the direction of magnetisation.**

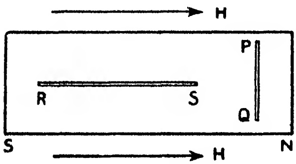


FIG. 106.

Reference has been made to the demagnetising effect of the end poles of a magnet (page 31). It should be noted, then, that the force H_1 on the unit pole in the above tunnel, although not affected by any magnetisation of the walls of the tunnel, is really the resultant of two forces, viz. H, due to the original field acting from left to right in Fig. 103 (this is known as the *apparent* magnetic force and is the force on a unit pole before the iron is put there), and h due to the *end poles* of the iron acting from right to left along the iron in Fig. 103 (this is known as the *self-demagnetising* force). Usually H and h are directly opposed, and the *effective* magnetic force H_1 on the unit pole in the tunnel is given by $H_1 = H - h$ (Fig. 107). This is referred to again later.

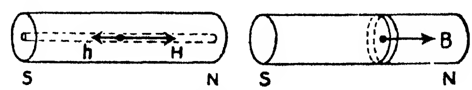


FIG. 107.

Now consider an indefinitely thin crevasse PQ cut across the specimen (Fig. 106), its plane faces being perpendicular to the direction of magnetisation, and of course infinitely large compared with the distance between them, and imagine a unit north pole in the crevasse. Here the magnetisation on the walls *does* effect the

unit pole: there is a N. wall at one side (left) and a S. wall at the other: if I be the intensity of magnetisation the force on the unit pole due to the walls of the cavity is $4\pi I$ (Art. 21). In addition, there is of course the force H_1 on the pole defined above. Thus the total force on the unit pole is $H_1 + 4\pi I$ and this measures the induction B: that is:—

$$B = H_1 + 4\pi I \quad \ldots\ldots\ldots\ldots\ldots\ldots (32)$$

Hence the magnetic induction B in the iron is measured by the force in dynes on a unit north pole placed in an indefinitely narrow crevasse with its parallel faces at right angles to the magnetisation.

It is interesting to look at the crevasse PQ from the point of view of the magnetic lines. It is clear that if a be the area of the walls of the crevasse PQ all the lines passing through area a of the iron—those due to the effective magnetising field and those due to the magnetisation—will go across the *very narrow* air gap instead of round the edges. Thus the number of unit tubes per square centimetre in the air gap equals the number of unit tubes per square centimetre in the iron. The field in the air gap PQ tends therefore to the value of the induction in the iron as the gap is made infinitesimally thin, and we therefore define B as being measured by the force on unit pole in the crevasse. On the other hand, the field (force on unit pole) in the tunnel RS is entirely due to the effective applied field.

The ratio of the magnetic induction (B) to the *effective* magnetic or magnetising force (H_1) both as defined above measures the **permeability** (μ) of the iron, *i.e.*

$$\mu = \text{Permeability} = \frac{\text{Magnetic Induction}}{\text{Magnetising Force}} = \frac{B}{H_1}; \quad \therefore B = \mu H_1.$$

The iron is magnetised and if I be its intensity of magnetisation the ratio of the intensity of magnetisation to the effective magnetising force H_1 measures the **susceptibility** (κ) of the iron, *i.e.*

$$\kappa = \text{Susceptibility} = \frac{\text{Intensity of Magnetisation}}{\text{Magnetising Force}} = \frac{I}{H_1}; \therefore I = \kappa H_1.$$

The relationship between these two quantities is readily established. We have seen above that $B = H_1 + 4\pi I$: hence:—

$$\mu = \frac{B}{H_1} = \frac{H_1 + 4\pi I}{H_1} = 1 + 4\pi\kappa \text{ and } \kappa = \frac{\mu - 1}{4\pi} \quad ..(33)$$

In the above both the magnetic induction and the magnetising force in the iron have been accurately defined from the point of view of the force on unit pole—B with the pole in the crevasse and H_1 with the pole in the tunnel—and the *exact* relation

$B = H_1 + 4\pi I$ has been established. This is the method of treatment the student should learn, but the following is an alternative method of treatment (less accurate) which is often used for practical purposes.

Referring again to Fig. 103, let H be the number of unit tubes of force per square centimetre in the space before the iron is put there. Then *if we assume that there is no opposing force due to end poles when the iron is there* we can say that H, which really measures the original field, also measures the *magnetising force* when the iron is there. When the iron is put there it becomes magnetised and the number of magnetic lines (called lines of induction) now passing along from left to right is much greater, for there are those due to the magnetisation as well as those due to the field. If B be the number of unit tubes of induction per square centimetre in the iron, then B measures the *magnetic induction* or *flux density*.

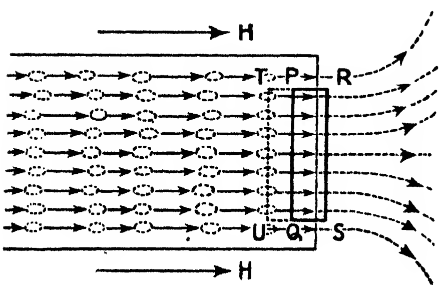


FIG. 108.

For the permeability μ and the susceptibility κ of the iron we have:—

$$\mu = \frac{\text{Magnetic Induction or Flux Density}}{\text{Magnetising Force}} = \frac{B}{H}; \quad \therefore\ B = \mu H$$

$$\kappa = \frac{\text{Intensity of Magnetisation}}{\text{Magnetising Force}} = \frac{I}{H}; \quad \therefore\ I = \kappa H.$$

The relationships between the two can also be established thus. At the end of the iron (Fig. 108) consider a closed surface PQRS with faces PQ and RS each of area a parallel to the end. The "amount of pole" inside is Ia, and by Gauss the *total magnetic induction* or *magnetic flux* is $4\pi Ia$. This all crosses RS, and when added to that of the original field, viz. Ha, we get for the flux across RS the expression $Ha + 4\pi Ia = (H + 4\pi I)\,a$. Next suppose PQ moved back to TU so that our closed surface includes the end vertical batch of complete magnetic particles. The total "amount of pole" inside the new closed surface is equally N. and S. (+ and −), so that the total magnetic flux over the closed surface is zero. Thus the flux (inwards) over TU is equal to that over RS, viz. $(H + 4\pi I)\,a$. Reasoning in this way we see that the

magnetic flux passing along a section a of the iron and leaving at the end is $(H + 4\pi I)\,a$, and the flux per unit area is $(H + 4\pi I)$, which, of course, is the magnetic induction or flux density B, *i.e.* $B = H + 4\pi I$: hence:—

$$\mu = \frac{B}{H} = \frac{H + 4\pi I}{H} = 1 + 4\pi\kappa \text{ and } \kappa = \frac{\mu - 1}{4\pi}.$$

Note again that the magnetising force to be used should really be that *in the iron* (H_1) and not the field value (H) before the iron is put there, and H_1 is less than H owing to the self-demagnetising force (h) of the *end poles* ($H_1 = H - h$ usually). If the length of the iron is 500 or more times its diameter, h is very small and negligible, so that in any experimental test on the permeability, etc., of such a rod of material, H the original field value can be used. Again, if the iron or other material is in the form of a ring magnetised by a current as in Fig. 71 there are no poles and no demagnetising effect, so that the magnetising force in the iron is the same as that (H) in the empty coil. In general, $h = NI$, where I is the intensity of magnetisation and N is a factor depending on the shape and dimensions of the specimen. If the specimen be a wire whose length is 500 times its diameter, N is only about ·0003. Long, thin wires and rings are used for testing purposes.

24. Ferromagnetics, Paramagnetics, and Diamagnetics

The student should read again pages 52-55 before proceeding with this section.

If, say, the north pole of a magnet be brought near one end of a bar of magnetic material (ferromagnetic or paramagnetic) the bar is magnetised by inductive influence, the near-end being a south pole, and there is attraction between them: the bar is, in fact, magnetised in the direction of the field, and tends to move from weaker to stronger parts of the field. If the bar be diamagnetic material it is magnetised the other way about, the near-end being a north (page 91), and is repelled, *i.e.* tends to move from stronger to weaker parts of the field. If a bar of magnetic material, say iron, be suspended in a uniform field it will rotate until its longest axis is along the field so that the magnetic lines which prefer to go through iron (Fig. 104) have a long path in the material: the dispersion of the magnetic lines in the diamagnetic bar (Fig. 105) would cause it to rotate until its longest axis was at right angles to the field.

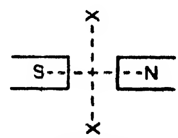


FIG. 109.

In the early experiments with solids, small bars of various substances were suspended between the poles of a powerful

electro-magnet. If the substance was paramagnetic (or ferromagnetic) it set itself *axially, i.e.* along the line NS; if diamagnetic it set itself *equatorially, i.e.* along the line XX (Fig. 109).

Liquids were enclosed in thin glass tubes and suspended between the poles, with the result that they set themselves either along NS or along XX and were classified accordingly. Plücker placed the liquids, in turn, in a watch-glass resting on the pole pieces as shown in Fig. 110. If the pole pieces are not more than about $\frac{1}{10}$ inch apart a paramagnetic liquid sets as at (*a*), *i.e.* it congregates where the field is strongest, and if diamagnetic it sets as at (*b*), *i.e.* it moves away from this strongest part of the field. If the pole pieces are not close together the field is strongest *round about the poles* and the liquids set accordingly, so that the results are practically opposite to those indicated above.

In dealing with gases the latter, rendered evident by traces of some other substance (*e.g.* ammonia and hydrochloric acid), were allowed to ascend between the poles, and it was noted whether they spread out between the poles or across them.

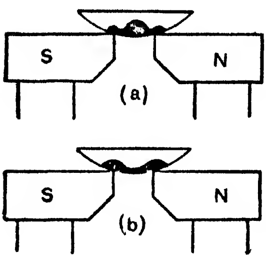


FIG. 110.

Experiments also indicated that the medium affected the results; thus a paramagnetic will act like a diamagnetic if it is surrounded by a medium more paramagnetic than itself.

For air (strictly a vacuum) B is taken equal to H, therefore $\mu = B/H$ is unity, and κ is zero (since $\mu = 1 + 4\pi\kappa$). For a paramagnetic and ferromagnetic B is greater than H, therefore μ is greater than unity, and κ is positive and greater than zero. For a diamagnetic B is less than H, μ is less than unity, and κ is negative; κ in this case is *very small*, so that μ, although less than unity, never becomes negative. For reference purposes a few values are given in the Appendix.

In ferromagnetics the magnetisation is not proportional to the magnetising force, and neither μ nor κ are constant. The magnetisation also depends very much on the temperature (page 45): this variation depends on the value of the magnetising force at the time, but with medium fields κ decreases as the temperature is raised, and when the critical temperature is reached drops fairly suddenly—the ferromagnetic properties disappear and the body becomes merely paramagnetic. With paramagnetics the magnetisation is proportional to the magnetising force. The effect of temperature is somewhat similar to the above, *i.e.* κ decreases with rise of temperature (it sometimes

becomes negative, *i.e.* the substance becomes diamagnetic): over a wide range the variation of κ with temperature follows Curie's law, viz. "with a given magnetising force κ is inversely proportional to the absolute temperature." With diamagnetics κ is constant—does not vary with the field or the temperature (bismuth is an exception).

The fact that temperature changes have no effect on the property of *diamagnetism* is what would be expected from the explanation of diamagnetism referred to on page 55, for whilst in ferromagnetism and paramagnetism we are concerned with *orientation of atoms or groups of atoms,* diamagnetism is the result of *changes occurring inside the atoms.*

CHAPTER IV

MAGNETIC MEASUREMENTS

FOR the comparison and measurement of magnetic fields and magnetic moments some type of what is known as a *magnetometer* is usually employed. There are three main forms, viz. *deflection magnetometers* (tangent and sine), *oscillation magnetometers*, and *torsion magnetometers*, and as the first named has the widest application in practice we will deal first with magnetic measurements by deflection methods.

1. The Deflection Magnetometer

One form consists of a magnetic needle pivoted at the centre of a circular scale, the deflection being read by means of a light pointer generally fixed at right angles to the needle at its centre. The deflecting magnet is placed on the graduated arms, A, B, so that its distance from the needle is known. In this form, when the pointer is at zero, the arms A, B are at right angles to the needle, so that the deflecting magnet is placed "end-on" (Fig. 111).

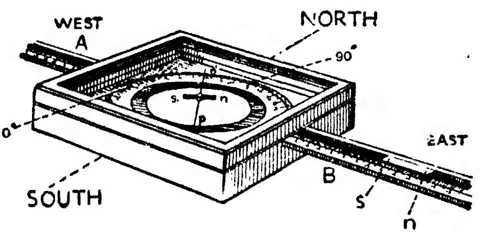


FIG. 111.

To correct for errors due to the "centering" of the needle and for the position of the poles of NS, it is usual to read *both ends* of the pointer, and to arrange the magnet in the four positions shown (Fig. 112): the mean of the eight deflections is taken.

In a "mirror magnetometer", the magnet is fixed horizontally to the back of a small plane or concave mirror, or fixed to a light frame carrying the mirror (Fig. 113), and the whole is suspended by a silk fibre in a suitable wooden case. Light from a lamp passes through a slit in a stand set opposite the mirror, falls on the mirror, and is reflected to a scale fixed to the stand, producing an image of the slit on the scale. If the mirror be plane, a lens must be used to obtain a *real* image on the scale. If the magnet be deflected

through an angle θ, the reflected rays move through an angle 2θ, so that, if s be the deflection of the image along the scale (in cm., say), and x cm. be the distance between the scale and mirror, we have (Fig. 113) $\tan 2\theta = s/x$; $\therefore \tan\theta = s/2x$ (approx.), so that θ is known from tables if required.

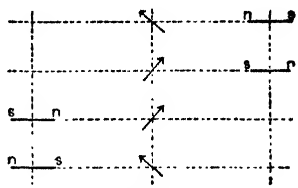


FIG. 112.

Sometimes the scale carries an electric lamp and lens fitted with a vertical thin thread, the image of which is formed on the scale (Fig. 114). In another form a telescope just above the scale is used to view the image of the scale in the mirror (plane); when the mirror moves, another mark on the scale comes into view and the deflection is therefore read. Fig. 115 shows a type of mirror magnetometer arranged for the " broadside-on " position.

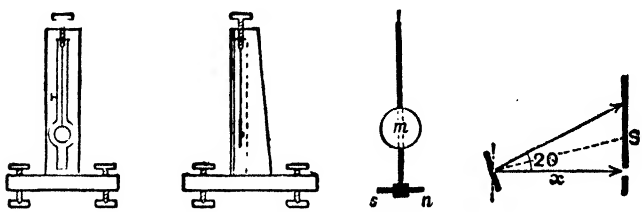


FIG. 113.

2. Simple Experiments with the Deflection Magnetometer

In the following experiments it will be assumed that the magnets are short compared with their distance from the magnetometer needle. Further experiments with the magnetometer are dealt with in subsequent pages.

(1) **To compare the moments of two small bar magnets.**—Set up the magnetometer for (say) the A position of Gauss. Place the first magnet A " end-on," its neutral line at distance d (great compared with the length of the magnet), and read the deflections with the magnet in the positions shown in Fig. 112. Let θ_1 be the mean of the eight readings. Repeat with the

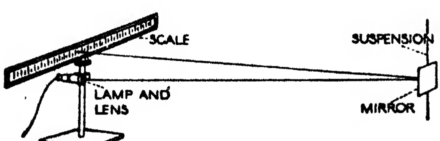


FIG. 114.

second magnet B at the same distance d, and let θ_2 be the mean of the eight readings. Clearly:—

$$\frac{M_1}{H} = \frac{d^3 \tan\theta_1}{2} \text{ and } \frac{M_2}{H} = \frac{d^3 \tan\theta_2}{2}; \quad \therefore \frac{M_1}{M_2} = \frac{\tan\theta_1}{\tan\theta_2},$$

where M_1 and M_2 are the moments of the magnets. Of course, if H be known the actual value of the moment, say M_1, can be calculated from the experiment, viz. $M_1 = (Hd^3 \tan\theta_1)/2$.

If the magnetometer be of the mirror type four readings of the deflection of the image on the scale are taken for each magnet (*i.e.* for the four positions shown in Fig. 112). Then, using the lettering of Fig. 113:

$$\frac{\text{Moment of A}}{\text{Moment of B}} = \frac{\tan\theta_1}{\tan\theta_2} = \frac{s_1/2x}{s_2/2x} = \frac{s_1}{s_2} = \frac{\text{Deflection by A}}{\text{Deflection by B}}.$$

Another method consists in placing the magnets on opposite sides of the needle so that the field due to one acts opposite to the field due to the other, and *adjusting the distances until the needle is not deflected.* If d_1 = distance of M_1, d_2 = distance of M_2, and $2l_1$ and $2l_2$ = lengths of M_1 and M_2, then:—

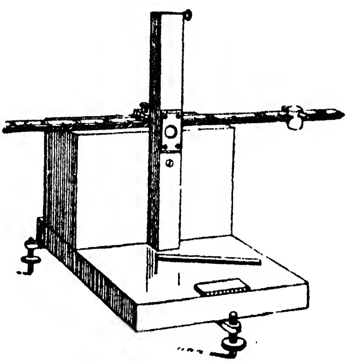


FIG. 115.

$$\frac{2M_1d_1}{(d_1^2 - l_1^2)^2} = \frac{2M_2d_2}{(d_2^2 - l_2^2)^2};$$

$$\therefore \frac{M_1}{M_2} = \frac{d_2\,(d_1^2 - l_1^2)^2}{d_1\,(d_2^2 - l_2^2)^2} = \frac{d_1^3}{d_2^3},$$

if the magnet lengths are small compared with the distances.

If the magnetometer is not of a type constructed for the purpose so that the distances d_1 and d_2 from the needle have actually to be *measured,* it is better to proceed thus:—Balance as above at distances d_1 and d_2: then balance at two other distances D_1 and D_2. Then:—

$$\frac{M_1}{M_2} = \frac{d_1^3}{d_2^3} = \frac{D_1^3}{D_2^3}; \quad \therefore \sqrt[3]{\frac{M_1}{M_2}} = \frac{d_1}{d_2} = \frac{D_1}{D_2} = \frac{D_1 - d_1}{D_2 - d_2};$$

$$\therefore \frac{M_1}{M_2} = \frac{(D_1 - d_1)^3}{(D_2 - d_2)^3},$$

and the *differences* in the distances, viz. $(D_1 - d_1)$ and $(D_2 - d_2)$, are more easily and accurately measured than the actual distances from the needle.

(2) **To compare the earth's horizontal field at two places.**—At the first place let θ_1 be the mean of the eight readings when the neutral line of the magnet is d cm. from the needle. The same magnet and magnetometer are taken to the second place and the experiment repeated with the magnet at the same distance d cm. Let θ_2 be the mean of the eight readings, H_a the

earth's field at the first place, and H_b the earth's field at the second place; then, if the "end-on" position has been used:—

$$\frac{M}{H_a} = \frac{d^3 \tan \theta_1}{2} \text{ and } \frac{M}{H_b} = \frac{d^3 \tan \theta_2}{2}; \quad \therefore \frac{H_a}{H_b} = \frac{\tan \theta_2}{\tan \theta_1},$$

$$i.e. \quad \frac{\text{H at the first place, A}}{\text{H at the second place, B}} = \frac{\text{Tan deflection at B}}{\text{Tan deflection at A}}$$

For another method see worked example, page 100.

(3) **To verify the inverse cube law for a small magnet.**—We have seen that the field at a point due to a *small* magnet *varies inversely as the cube of the distance* (see again Chapter III., Arts. 8, 9, 10). For this experiment a good mirror magnetometer should be employed and "distances" should be as great as possible consistent with readable deflections. The small magnet is placed (say) "end-on" and the deflection s and distance d noted. This is repeated at various distances. Taking any pair of results, it will be found that approximately

$$\frac{\text{Deflection } s_1 \text{ with magnet at distance } d_1}{\text{Deflection } s_2 \text{ with magnet at distance } d_2} = \frac{d_2^3}{d_1^3}.$$

Now $F = H \tan \theta$, *i.e.* F is proportional to $\tan \theta$, and therefore proportional to s; hence

$$\frac{\text{Field due to magnet at distance } d_1}{\text{Field due to magnet at distance } d_2} = \frac{s_1}{s_2} = \frac{d_2^3}{d_1^3},$$

which verifies the inverse cube law.

In connexion with several experiments in this chapter the fact should be borne in mind that the field at a point a *relatively short* distance off the end of a *long* magnet is proportional to m and inversely proportional to the *square* of the distance *from the pole* ($F = m/d^2$), but the field a *relatively great* distance off the end of a *short* magnet is proportional to M and inversely proportional to the *cube* of the distance *from the centre* ($F = 2M/d^3$).

3. Magnetometer Method of Determining Magnetic Properties

The extensive use of magnetic material, particularly iron and its alloys, in the construction of dynamos, motors, transformers, etc., has made the determination of the magnetic properties of materials one of great importance—a daily necessity in practice—and an early elementary treatment is desirable, for nowadays the student will constantly come across references to the various points in his reading.

In Fig. 52 the magnetic field in the case of a solenoid carrying a current was depicted, and it will be noted that inside it the lines of force run *more or less* parallel, *i.e.* the magnetic field at the centre

is more or less uniform. Further, it can be shown that in the case of a closely wound solenoid, if its length is great compared with its diameter, the *whole* magnetic field inside *is* uniform (at any rate to within a short distance of each end). Now it will be seen later that the strength of this field inside or the number of unit tubes of force per unit area (H) is given by:—

$$H = \frac{1{\cdot}257 \times \text{Current in amperes} \times \text{Number of turns on solenoid}}{\text{Length of solenoid (cm.)}},$$

and since the length and number of turns of wire are known, we can easily calculate the magnetising force H when any particular current (amperes) flows in the solenoid. The ampere is the practical unit of current strength and is one-tenth of the electromagnetic unit referred to on page 68: it is given directly by an ammeter placed in the circuit. Amperes and ammeters are fully dealt with later, but they are well-known and even household names to-day. Incidentally, the number of turns divided by the length is the "turns per centimetre," and number of amperes multiplied by number of turns is called the "ampere-turns"; hence H = 1·257 times the ampere-turns per cm.

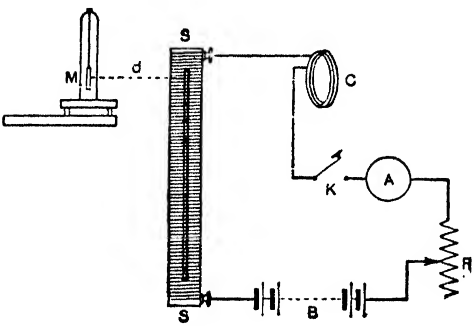


FIG. 116.

We can now consider the main principles of one method, the *deflection magnetometer method,* of finding the magnetic properties of, say, iron. In Fig. 116, M is a magnetometer of the mirror type, whilst SS is a long, thin, closely wound solenoid standing vertically, say due magnetic east of the magnetometer needle. B is a battery to supply the current, the strength of which can be varied by varying the resistance or rheostat R, and measured in amperes by the ammeter A. The current circuit also includes a small coil C, which will be dealt with presently. The specimen of iron under test is in the form of a wire, its length being at least 500 times its diameter: it is placed vertically in SS, with its upper end on a level with the magnetometer needle.

Assume a certain current passing: this magnetises the iron, and the needle of M is deflected, say, $\theta°$. If F be the ***horizontal*** field at M due to the iron and h the earth's horizontal field, then $F = h \tan \theta$, for the two fields F and h are at right angles. If the effect of the bottom pole of the iron be neglected (*long* thin specimen), then F, being due to the top pole only, is m/d^2, where m is the pole strength of the vertical wire magnet, and d (cm.) is the distance from the pole to M. Hence:—

$$\frac{m}{d^2} = h \tan \theta, \quad i.e. \; m = d^2 h \tan \theta,$$

and as d and θ are observed, and h, the earth's field, is known (·18, say), the strength of the pole induced in the iron (*i.e.* m) is found.

Now the intensity of magnetisation (I) of the iron is obtained by dividing the pole strength m by the cross-sectional area a ($I = m/a$), so that on working out the cross-section (a) of the specimen in sq. cm. the intensity of magnetisation (I) is determined from this relation. The magnetising force H is next calculated from the relation previously given (page 102), the amperes being given by A and the length and number of turns being known. Finally the magnetic induction or flux density B is calculated from the relation $B = H + 4\pi I$, the permeability (μ) from $\mu = B/H$, and the susceptibility (κ) from $\kappa = I/H$. We have, therefore, found for this specimen of iron when subject to this one particular current and magnetising force, the values of B, I, μ, and κ, and these are the quantities usually required in considering the magnetic properties of a material and its suitability for various practical purposes. By altering the rheostat R we alter the current and repeat, thus finding the values of B, I, μ, and κ with various magnetising forces.

Now a word about the coil C. When a current is passed through the solenoid, the latter will deflect the needle even with no specimen inside. To correct for this, the "compensating coil" C is used. Before the iron is inserted, a current is passed, and the position of C is adjusted until it acts "equal and opposite" to the empty solenoid, and the needle is not deflected: C then remains in that position throughout the experiment, and whatever current is passing it will still cancel the effect of the solenoid on M, so that only the iron will be deflecting the needle.

As the specimen is fixed vertically it will be magnetised (by induction) by the vertical component of the earth's magnetic field (the strength of this vertical field is about ·43). To correct for this, a *separate* coil is wound on

SS and joined to a *separate* battery, and a current is kept flowing in this second coil of such a strength and in such a direction that it creates a field equal to the earth's vertical field, but in the opposite direction, thus cancelling the effect of the earth. This separate coil and battery are not shown in the figure.

A "reversing switch" is generally included in the circuit of Fig. 116 instead of the simple "on and off" switch K, so that the current can be made to flow in the opposite direction in the solenoid if desired.

For a more accurate determination the bottom pole of the magnet must be taken into account. Instead of m/d^2 being used for the *horizontal* field F at the magnetometer due to the specimen the expression to be used becomes:—

$$F = m\left\{\frac{1}{d^2} - \frac{d}{(d^2 + l^2)^{\frac{3}{2}}}\right\},$$

where l is the *magnetic* length of the specimen (which may be taken as three quarters the actual length), and d, as before, is the distance from the upper pole to M. This is readily seen from Fig. 116 (a), for the field F at O (the point where M is situated) in the direction NO due to the magnet is:—

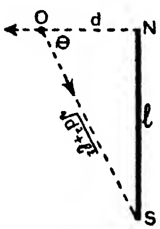


FIG. 116 (a).

$$F = \frac{m}{d^2} - \frac{m}{d^2 + l^2}\cos\theta = \frac{m}{d^2} - \frac{m}{d^2 + l^2}\cdot\frac{d}{\sqrt{d^2 + l^2}},$$

which reduces to the expression given above.

It will be noted that we have assumed the magnetising force H to be the same as that of the empty solenoid, *i.e.* we have ignored the demagnetising effect of the end poles produced in the iron. As explained, this can be done in the case of such a specimen as is used in the experiment (page 95): for shorter specimens, however, it may be necessary to estimate the *effective force* in the iron. Now Ewing showed that:—

Effective magnetising force = H − NI (see page 95),

where I is intensity of magnetisation and N is a factor depending on the size of the wire. Thus I could be estimated as above, H found from the current and solenoid dimensions as above, and N taken from Ewing's tables (see below): hence the effective magnetising force would be found and used in working out the other quantities.

VALUES OF N FOR WIRE SPECIMENS

$\frac{\text{Length}}{\text{Diameter}}$	N	$\frac{\text{Length}}{\text{Diameter}}$	N
50	0·01817	300	0·00075
100	0·00540	400	0·00045
200	0·00157	500	0·00030

Further methods of determining the magnetic properties of materials are dealt with in Chapter XVII. It will be seen there that by employing a different method of testing the specimen can be in the form of a ring similar to Fig. 71. This latter has the advantage of absence of any free poles, and therefore any demagnetising effect, so that the *effective* magnetising force may actually be taken as H, *i.e.* that calculated from the dimensions of the coil, number of turns, and the current. The ring must be of small (radial) breadth compared with its diameter to minimize the crowding of the magnetic lines towards the inner face of the ring, and the consequent error in assuming the flux density (B) to be uniform, and it should not be forged, as the welded joint would introduce an error. On the whole rods are more suited to elementary laboratory testing, and if the ratio length/diameter be large there will be no serious inaccuracy in the calculated value of H from the coil dimensions and the magnetising current.

The necessity for the almost daily determination of the magnetic properties of iron and its alloys in practical electrical engineering work has resulted in the introduction of several commercial instruments to facilitate measurements on a practical scale: one or two of these are referred to later.

4. Magnetisation Curves

The results of these experiments are generally shown graphically by *magnetisation curves*. It is instructive for the beginner, however, to first get a general idea of the shape of a magnetisation curve. Suppose in an experiment the current has been increased by $\frac{1}{4}$ ampere at a time from zero up to a maximum of 3 amperes, and that at each step we have noted the current strength and the deflection. Now the magnetising force H at each step is proportional to the current: the scale deflection depends on the pole strength, and therefore on the intensity of magnetisation I, so that we can say that I at each step is proportional to the deflection. We take, then, corresponding values of *current* and *deflection* and plot a curve.

Fig. 117 shows roughly the kind of curve which would be obtained for a soft iron wire and for a similar one of hard steel. With the iron the magnetism increases rapidly at first, and then slowly as it approaches "saturation": the steel creeps up much more slowly. An iron electromagnet gives a more powerful magnet with a certain current than steel does, and is therefore cheaper to run in any practical appliance. Thus with a current OQ the strength of

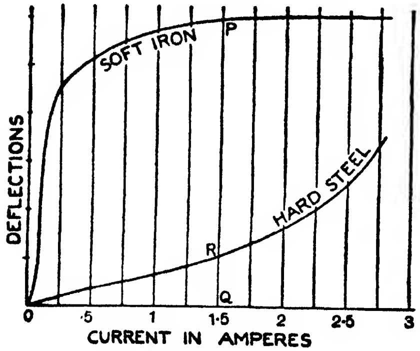


FIG. 117.

the iron magnet is represented by QP, and it is practically "saturated," but the strength of the steel is only QR.

If, however, we work out the actual values of the magnetic induction B and the magnetising force H at each step, and plot the corresponding values, we get a proper magnetisation curve--a *BH curve*, as it is called, and the ratio of B to H at any point of the curve gives the *permeability* of the specimen at that stage, *i.e.* the permeability of that specimen when it is subjected to that particular magnetising force. Similarly we can plot corresponding values of I and H giving an *IH curve*, and the ratio of I to H at any point gives the *susceptibility* at that stage. Similarly too, μ can be plotted against H to show graphically how permeability changes with the magnetising force, and so on.

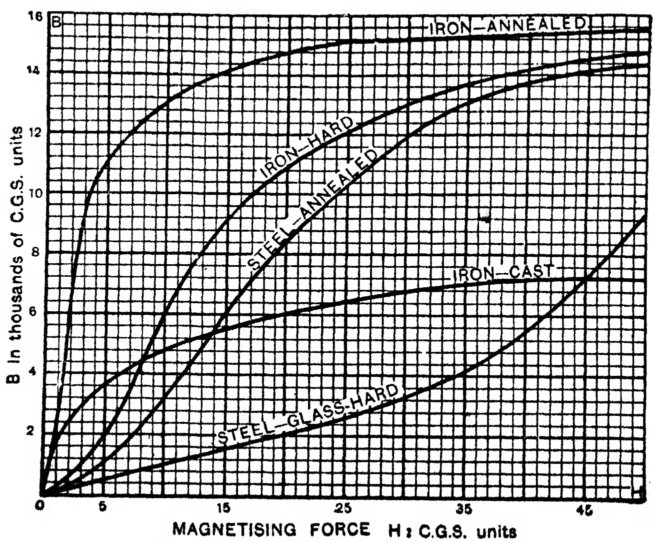


FIG. 118.

Fig. 118 gives the BH curves for various samples of iron and steel, H being plotted horizontally and B vertically. With a magnetising force of only 2·5 the soft iron (annealed) gives a value of B of about 8000, the cast iron about 3000, the hard iron about 1000, and the glass-hardened steel about 400. When H has the value 10 the soft iron curve is becoming almost horizontal, *i.e.* the iron is approaching saturation, but this does not take place with the steel and the hard iron until H is from 40 to 45. When H is 50 the soft iron, hard iron, and steel are all practically saturated, but the glass-hard steel is still rising, and its value of B is only equal to that in soft iron under a magnetising force of about 2·6. The IH curves would be somewhat similar. Note again how the curves confirm the ease with which soft iron can be magnetised compared with hard steel.

If Fig. 118 had been drawn to a larger scale it would have been clear that there are really three different stages in a complete magnetisation curve:—

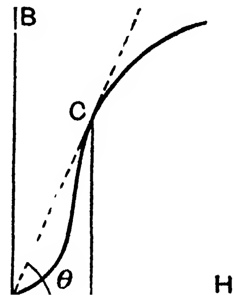


FIG. 119.

(*a*) A short initial stage when the magnetising force is small, in which a change in H produces only a very small change in B, and where the permeability is small. This part of the curve is better seen in Fig. 119. For *very small values of H* the curve is a straight line inclined to the horizontal so that $\mu = B/H$ is constant.

(*b*) A stage where the curve is rising rapidly, *i.e.* where a small change in H produces a large change in B, and where the permeability is increasing rapidly to a maximum value. Since $\mu = B/H = \tan\theta$ (Fig. 119), the permeability has its maximum value at C in that figure.

(*c*) The saturation stage or nearly horizontal part of the curve. Here a large increase in H produces *comparatively* little effect on B, and the permeability drops to quite low values.

The above changes in the permeability μ as the magnetising force H changes are roughly indicated in Fig. 120. Note, as stated, that when the magnetising field is weak μ is practically constant: it then rises rapidly to a maximum value (at *d*) as H increases, after which it falls as H continues to increase. For comparison, an IH curve with the corresponding changes in the susceptibility κ ($= I/H$) as H increases is roughly indicated in Fig. 121. Fuller details of the changes in μ and κ with changes in magnetising force (and with temperature changes) are given in Chapter XVII.

In Chapter II. reference was made to certain magnetic alloys, and magnetisation curves can be plotted for them from which their suitability for various purposes can be seen at a glance. As an example take the case of alloys of the *permalloy* and *mumetal* types. It was stated that these were easily strongly magnetised even by

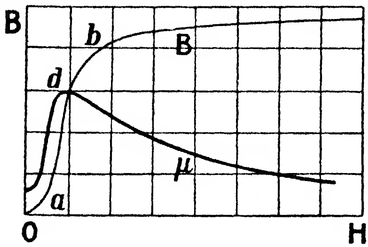


FIG. 120.

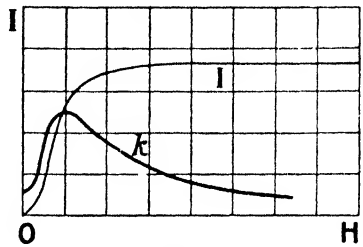


FIG. 121.

weak magnetic fields. Fig. 122 gives the BH curve for permalloy using small magnetising forces, and the curve for iron *in these weak fields* is plotted for comparison. Note how quickly the permalloy rises in magnetisation compared with the iron: with a small magnetising force of ·5 the flux density or induction in permalloy is about 9000, but it has only reached about 500 in the iron. Remember, however, that as the magnetising field increases the induction in iron exceeds the induction in permalloy and mumetal, *i.e.* iron can be made into a stronger magnet.

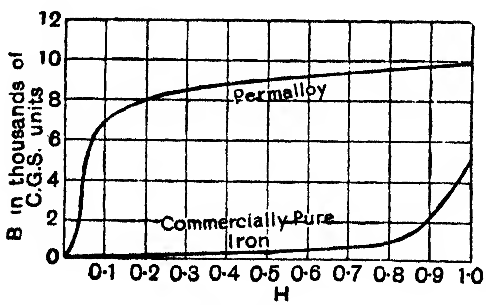


FIG. 122.

Referring again to the BH curves of Fig. 118 the scale used for B is much smaller than that used for H since the changes in H are much smaller than the changes in B. If the same scale were used for both, the saturation part of the *BH curve* would be a straight line, not horizontal, but inclined at 45° to the axes, showing that the change in B is merely equal to the change in H. This is also clear from the relation $B = H + 4\pi I$, since I has become constant on saturation.

Further details of magnetisation curves and the variations in the magnetic properties of materials are given in Chapter XVII.

5. Hysteresis

If the current flowing round the iron in Art. 3 be gradually increased, thus producing an increase of H, we have seen that B rises quickly at first, then more slowly, and finally, when the condition of magnetic saturation has been reached, no further increase in current will cause *appreciable* increase in B. (See note at end of Art. 4.)

If the current be now reduced step by step, the flux produced will lessen too, but when the current is reduced to zero, as at the beginning of the experiment, the flux will not have fallen to zero; some will remain. This remaining flux is termed the **residual magnetism.** If, further, we then commence to send the current round the wire in the reverse direction, it will have to reach some particular value before the previous flux will be quite removed; this particular value of H in the reverse direction which is necessary to wipe out the flux measures the **coercive force** of the specimen. Increase of the reverse current beyond this amount will cause flux in the reverse direction; but, as before, after the current reaches a certain amount the flux will be practically stationary, indicating saturation in the reverse direction.

If, now, we lessen the reverse current by steady steps, the reverse flux will lessen too; but, as before, when the current has reached zero value again, the flux will not be zero, but some will remain. If, again, the current be reversed into the old direction, and then steadily increased by steps, the reverse flux will lessen, become zero, and it too will then go back into the old direction, and so on.

Such increase of current and flux from zero to a maximum in one direction, and then back through zero to a maximum in the other direction, and finally back again through zero to the first maximum, is spoken of as a **magnetic cycle.** The effect which is brought out in the experiment, viz. that although the current causing flux is brought to zero, the flux itself does not return to zero, is termed **hysteresis.** That is, *hysteresis is the lagging of the magnetic flux behind the magnetic force (or magnetising force) producing it.*

Fig. 123 shows such a cycle plotted out to scale. H measured to the right is due to current flowing in one direction, while H measured to the left is due to the reversed current. So, too, the magnetic flux B measured upwards is that due to the direct current, while that measured downwards is that due to the reversed current.

Residual Magnetism.—The intercept OA in the diagram shows the magnetic flux remaining when the current is reduced to zero; that is, OA is the residual magnetism to scale.

Coercive Force.—So, too, the intercept OC shows the amount of reversed magnetic force needed to just remove all the residual magnetism, and this measures the *coercive force.*

Hysteresis Loop.—The whole loop is termed the **hysteresis loop,** and it can be shown that *the area of the figure is proportional to the energy wasted in the iron due to the rapidly changing magnetic condition.* In fact it will be proved in Chapter XVII. that in the BH case of Fig. 123 if the area is estimated in terms of B and H *and then divided by* 4π, the result is the waste of energy (in ergs) per cubic centimetre per cycle, and if IH curves be plotted, the area, estimated in terms of I and H, is the waste of energy (in ergs) per cubic centimetre per cycle.

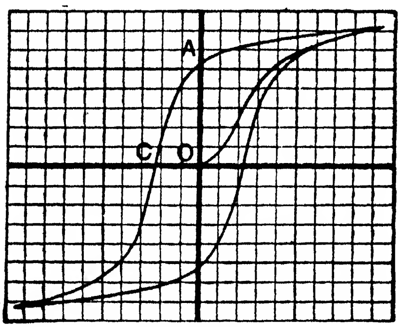


FIG. 123.

Clearly, the narrower the loop the less energy wasted in hysteresis. Naturally all such wasted energy means heat developed in the iron undergoing magnetisation, and it is obviously to the advantage of the engineer to choose such iron that the energy wasted (and, of course, heat produced in the appliance) shall be as small as possible, at any rate in all cases where iron is subjected to fluctuating magnetic forces, as it is, for example, in transformers, etc., used in alternating current work. It will be remembered that an alternating current is always going through a complete cycle—say 50 times per second—for it rises to a maximum then dies down to zero, then rises to a maximum in the opposite direction, then dies down to zero, and so on. What the engineer desires, then, is that his iron (or iron alloy) should give a narrow curve or loop, which means less energy lost and less heating, and it is to this end that magnetic testing of iron supplied for electrical purposes is a daily necessity.

Fig. 124 shows the *hysteresis loops* for equal sized specimens of iron and steel, and they bring out several points mentioned in preceding pages. Thus the residual magnetism OX of the iron is

greater than OY, that of the steel, but the coercive force OP of the iron is less than that, OQ, of the steel. It was stated on page 48 that iron would show more residual magnetism, and therefore have greater retentivity, than steel if protected from the least " disturbing influence," but that a small amount of " disturbance " would wipe it out, *i.e.* its coercivity was less than that of steel. Again, the loop for iron is narrower and has a less area than the loop for steel, which means that if subjected to reversals of magnetisation as occurs in A.C. work, more energy would be wasted in steel than in iron, and the steel would be heated to a greater extent. All these points are of importance to the electrical engineer.

Some general idea of the magnitude of these effects will be gathered from these figures:—For stalloy the hysteresis loss is from 1600 to 3000 ergs per c.cm. per cycle, for Swedish iron it is of the order 7300, for cast iron (annealed) 10,000 to 14,500, for hardened steel 100,000 to 150,000. We have seen that for *permanent* magnets tungsten steels and cobalt steels are used: but if these had to be subjected to magnetic cycles the hysteresis loss would be of the order 300,000 ergs per c.cm. per cycle. An idea of the heating effect is readily obtained. Suppose the hysteresis loss for a certain iron is 50,000, that the density of the iron is 7·7 grm. per c.cm., and its specific heat ·11: if $t°$ = temperature rise:—

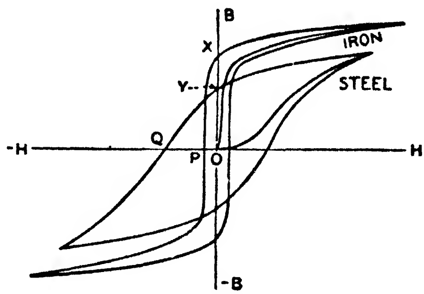


FIG. 124.

$$(7{\cdot}7 \times {\cdot}11 \times t°) \times (4{\cdot}2 \times 10^7) = 50{,}000,$$

which gives $t = {\cdot}0014°$ C. If the A.C. is 50 cycles per second, this means a temperature rise of ·07° per second, or 4·2° per minute.

It is often necessary to demagnetise a piece of steel. This is best done by placing it in a solenoid, passing an A.C., and gradually reducing the current to zero. This subjects the steel to a succession of magnetic cycles which get less and less until they get so small that the magnetism is practically zero. Putting the magnet into the solenoid and then gradually withdrawing it is a plan often adopted in the laboratory.

The above is an introductory elementary treatment only of the subject of hysteresis for the purpose of giving the reader some general idea of the subject and its importance: further details appear in subsequent chapters.

6. The Oscillation Magnetometer and Experiments with it

This consists essentially of a small magnet hanging by a silk thread in a suitable enclosure, and Fig. 125 shows three quite usual types. In working with it the amplitude of the oscillations must be small—not more than 3° or 4° on either side of the rest position—and there must be no twist on the suspension to begin with.

(1) **To compare the earth's horizontal field at two places.**—At the first place A start the needle vibrating by bringing slowly towards it for a moment a bar magnet presented "end-on." Note the time to execute, say, 30 complete vibrations. From this find the time of one vibration t_1 and the number of vibrations per minute n_1. Repeat the experiment at the second place B: let t_2 be the time of one vibration and n_2 the number of vibrations per minute.

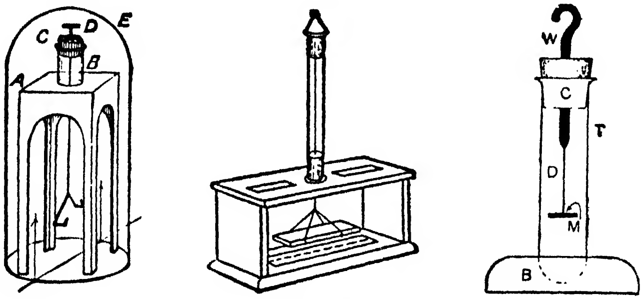


FIG. 125.

Then, assuming that the moment of the magnet does not alter when taken from one field to the other, we have (page 76):—

$$\frac{\text{Earth's field at A}}{\text{Earth's field at B}} = \frac{(\text{Time of a vibration at B})^2}{(\text{Time of a vibration at A})^2} = \frac{t_2^2}{t_1^2}$$

$$\text{and } \frac{\text{Earth's field at A}}{\text{Earth's field at B}} = \frac{(\text{No. of vibrations per min. at A})^2}{(\text{No. of vibrations per min. at B})^2} = \frac{n_1^2}{n_2^2}.$$

(2) **To compare the pole strengths of two long magnets.**—Place the first magnet A on the table with its axis in the meridian and its north pole pointing northwards, and select a point P on the axial line due magnetic north of it and, say, 3 in. from its north pole. At P the earth's field and the magnet's field are in the *same* direction. Hence place the magnetometer at P and find the number of vibrations per minute (n_1) under the influence of the earth plus the magnet. Remove the magnet, and find the vibrations per minute (n_2) under the influence of the earth alone. Then:—

$$n_2^2 \propto \text{earth's field at P};$$

$$n_1^2 \propto \text{earth's field at P} + \text{magnet's field at P};$$

$$\therefore\ n_1^2 - n_2^2 \propto \text{the A magnet's field at P}.$$

Repeat with the second long magnet B, its north pole being in the same position, and let n_3 be the number of vibrations per minute. Then as before:—

$$n_3^2 - n_2^2 \propto \text{the B magnet's field at P.}$$

But as the magnets are long we can neglect their south poles and say that the fields at P are due to their north poles only, and are therefore proportional to their pole strengths. Hence:—

$$\frac{\text{Pole strength of A}}{\text{Pole strength of B}} = \frac{\text{Field due to A}}{\text{Field due to B}} = \frac{n_1^2 - n_2^2}{n_3^2 - n_2^2}.$$

NOTE.—The *moments* of two *small* magnets may be compared in the same way, the distance from the magnets to the needle being, however, *great* compared with the lengths of the magnets: the field at the needle due to a magnet is, in this case, $2M/d^3$, and is therefore proportional to M.

(3) **To compare the moments of two magnets.**—Suspend the magnets A and B in turn in the oscillation box and find for each the time (t) of one vibration and the number of vibrations (n) per minute. Let t_1 and n_1 be the values for A, and t_2 and n_2 the values for B. Let M_1 = magnetic moment of A, and K_1 its moment of inertia: let M_2 and K_2 denote the values for B. Then:—

$$t_1 = 2\pi\sqrt{\frac{K_1}{M_1H}};\ t_2 = 2\pi\sqrt{\frac{K_2}{M_2H}};\quad \therefore t_1^2 \propto \frac{K_1}{M_1} \text{ and } t_2^2 \propto \frac{K_2}{M_2};$$

$$\therefore \frac{\text{Moment of A}}{\text{Moment of B}} = \frac{M_1}{M_2} = \frac{K_1}{K_2}\left(\frac{t_2}{t_1}\right)^2 = \frac{K_1}{K_2}\left(\frac{n_1}{n_2}\right)^2,$$

and if the two magnets have equal moments of inertia this becomes:—

$$\frac{\text{Moment of A}}{\text{Moment of B}} = \frac{M_1}{M_2} = \frac{n_1^2}{n_2^2} = \frac{t_2^2}{t_1^2}.$$

The drawback to the method above is that it invariably necessitates the determination of the moments of inertia K_1 and K_2. This can be avoided by strapping the magnets together with their axes parallel and allowing them to oscillate (1) with like, (2) with unlike poles adjacent. With the usual notation:—

$$t_1 = 2\pi\sqrt{\frac{K_1 + K_2}{(M_1 + M_2)H}}.\qquad t_2 = 2\pi\sqrt{\frac{K_1 + K_2}{(M_1 - M_2)H}};$$

$$\therefore \frac{M_1 + M_2}{M_1 - M_2} = \frac{t_2^2}{t_1^2} = \frac{n_1^2}{n_2^2},\quad i.e.\ \frac{M_1}{M_2} = \frac{t_2^2 + t_1^2}{t_2^2 - t_1^2} = \frac{n_1^2 + n_2^2}{n_1^2 - n_2^2}.$$

See worked example, page 77.

7. The Torsion Magnetometer

(1) LABORATORY TYPE.—This consists of a small magnet generally suspended by two quartz threads, one of which is attached to a torsion head (which can be rotated over a scale) at the top, and the other to a spring at the bottom (Fig. 126). Sometimes a small bar electromagnet is used instead of a permanent magnet: in such cases the bar is generally of mumetal and the suspensions

are phosphor bronze strips which serve to lead the current to and from the magnetising coil on the mumetal rod. One advantage of this over the permanent magnet is that the sensitivity can be altered by altering the current. The suspended magnet generally carries a mirror so that deflections of it can be noted, if necessary, by the lamp and scale method.

Such an instrument has various uses in experimental work: as an illustration we may consider its application to the comparison of two magnetic fields. Set up the magnetometer so that the magnet is at rest in its zero position. Then apply the first field A of strength H_a *so that its direction is at right angles to the magnet.* Thus if the field in question is that due to a current in a coil, the latter would be placed on one side of the magnet as in Fig. 79: if the field is that off the end of a magnet, this magnet would be similarly placed with its axis perpendicular to the suspended magnet. In either case the suspended magnet is deflected.

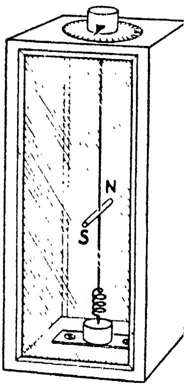


FIG. 126.

Now turn the torsion head until the suspended magnet is brought back to its zero position: let θ_1 be the angle the torsion head is turned. As the suspended magnet is *at right angles* to the applied field A, the couple due to the field is MH_a where M is the moment of the suspended magnet, and this is balanced by $\theta_1°$ of torsion: hence $MH_a \propto \theta_1° = k\theta_1°$, where k is a constant for the suspension.

Repeat with the second field B of strength H_b (due to a coil or magnet), and let $\theta_2°$ be the angle of rotation of the torsion head: then $MH_b = k\theta_2°$;

$$\therefore \frac{\text{Strength of field A}}{\text{Strength of field B}} = \frac{H_a}{H_b} = \frac{\theta_1°}{\theta_2°}.$$

(2) THE TORSION BALANCE.—This instrument, due to Coulomb (see also page 233), is mainly of historical interest and finds no place in the equipment of a modern laboratory: it is only introduced here because of its connexion with the history of the subject and to illustrate certain principles.

Essentially it consists of a fine vertical silver wire carrying at its lower end a stirrup to hold a horizontal magnet and attached at its upper end to a torsion head which can be rotated (Fig. 127), the whole being contained in a suitable glass case (see Fig. 128). A scale of degrees is etched on the glass case on a level with the

suspended magnet so that the deflection of the latter in any experiment can be noted. Through an aperture in the top of the case a vertical magnet can be inserted so that its lower end just comes against, and level with, a pole of the suspended magnet and opposite the zero of the scale, and, of course, the suspended magnet is deflected (say θ°—(Fig. 128). Before commencing an experiment the suspension must be free from any twist when the suspended magnet lies in the meridian.

To compare the moments of two magnets.—Suspend the first magnet A so that it hangs in the meridian when the wire is without twist, and then turn the torsion head through an angle β_1°, say to the right, to deflect the magnet through an angle α° to the right: the twist on the wire is $(\beta_1 - \alpha)^\circ$. Repeat with the second magnet B and let β_2° be the angle the torsion head must be turned through to deflect the magnet through the same angle α°.

The couple due to the torsion in the first case is $c\ (\beta_1 - \alpha)$ where c is a constant for the wire (couple due to unit torsion), and this balances the couple due to the earth, viz. $M_a H \sin \alpha$, where M_a is the moment of the first magnet; hence:—

$$M_a H \sin \alpha = c\ (\beta_1 - \alpha).$$

Similarly, in the second case,

$$M_b H \sin \alpha = c\ (\beta_2 - \alpha);$$

$$\therefore \frac{\text{Moment of A}}{\text{Moment of B}} = \frac{M_a}{M_b} = \frac{\beta_1 - \alpha}{\beta_2 - \alpha}.$$

(See worked example, page 63).

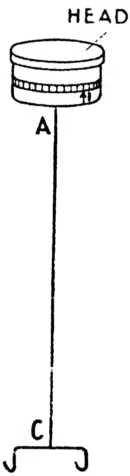


FIG. 127.

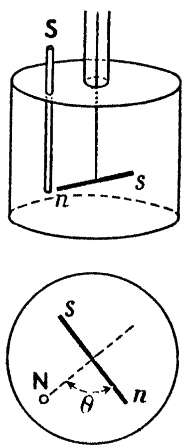


FIG. 128.

Another type of experiment with the torsion balance will be best understood from the following example:—

Example.—*The torsion head of a torsion balance is turned through* 50° *and the suspended magnet is deflected through* 10°. *If the instrument be brought back to its initial position and the vertical magnet inserted, the suspended magnet is deflected* 30°. *How much must the torsion head be turned to reduce this deflection to* 15°? (*Inter. B.Sc.*)

(*a*) The twist on the wire in the first case is $(50 - 10)^\circ = 40^\circ$. Hence 40° of torsion balances the earth's action for 10° deflection. Thus we may assume that, approximately, the earth's action when the deflection is 1° is equivalent to 40/10, *i.e.* 4° of torsion on the wire.

(*b*) In the second case the repulsion between the pole of the vertical magnet and the pole of the suspended magnet causes a deflection of 30°. Acting against and balancing this force of repulsion we have the twist on the wire and the action of the earth both of which are tending to bring the magnet back. These are represented by $30° + (30 \times 4)° = 150°$ of torsion.

(*c*) In the third case the torsion head is turned, say, $x°$ in the *opposite* direction to the deflection to bring the deflection down to 15°. The repulsion between the poles is balanced by the twist on the wire and the action of the earth. The former is $(x + 15)°$, for the top of the wire has been turned $x°$ in one direction and the bottom is still 15° in the other direction: the earth's action is represented by $(15 \times 4)° = 60°$ of torsion. Thus the total balancing torsion now is $(x + 15°) + 60° = (x + 75)°$: hence:—

$$\frac{\text{Repulsion between poles in Case }(b)}{\text{Repulsion between poles in Case }(c)} = \frac{150}{x + 75}.$$

Now if we assume—which is not quite true—that the distance between the poles has been reduced to one-half (30° to 15°), then by the law of inverse squares the force of repulsion is four times. Hence—

$$\frac{1}{4} = \frac{150}{x + 75}; \quad \therefore x = 525°.$$

Coulomb used this method to prove the law of inverse squares, *i.e.* $x°$ was observed and it was shown that the balancing torsion in (*c*) *was about* four times that in (*b*), when deflections were *small*, so that at half the distance the force was four times. For a more exact treatment of, and general formula for, the torsion balance, see *Advanced Textbook of Electricity and Magnetism.*

In torsion balance experiments such as the above the instrument is often mounted on a turn-table device so that it can be rotated as a whole, and before a reading is taken the whole is turned so that the magnet is lying in the meridian. Thus the earth's effect vanishes (*i.e.* the MH sin α effect or its equivalent torsion does not enter into the calculation).

8. Absolute Determination of a Field and a Magnetic Moment

So far our experiments in this connexion have been concerned mainly with *comparisons* of field strengths and of magnetic moments, and of course if one was known the other was determined. We can, however, find an absolute value for a magnetic field or a magnetic moment by combining a deflection and an oscillation experiment. The determination is an important one, although so far as field strength is concerned, methods which depend upon the magnetic effects of currents flowing in conducting circuits are much more accurate. To explain the method we will take the case of the

determination of the earth's horizontal field H and the moment M of the magnet used in the experiment.

(*a*) *Deflection Experiment.*—A good mirror magnetometer is arranged (say) for the A position of Gauss. A small deflecting magnet is then placed "end on" with its neutral line at a convenient distance *d* cm. from the needle, and the mean of the four deflections with the magnet in the four positions shown in Fig. 112 is determined; let it be *s* cm. and let *x* cm. be the distance between the magnetometer mirror and the scale. If $\theta°$ be the actual deflection of the needle $s/x = \tan 2\theta$; thus $2\theta°$ may be found from tables and therefore $\theta°$ is known. Now

$$\frac{M}{H} = \frac{(d^2 - l^2)^2}{2d} \tan \theta,$$

where l is half the distance between the magnet poles (say half its length approximately). Thus all the terms on the right-hand side are known: that is:—

$$\frac{M}{H} = \text{some number} = A.$$

(*b*) *Oscillation Experiment.*—The deflecting magnet is suspended in the oscillation box (Fig. 125) and the time taken to execute (say) 50 complete vibrations is observed and from this the time (*t* seconds) of one vibration is calculated. This is repeated once or twice and the mean value of *t* taken. The moment of inertia K of the magnet is next found by weighing and measuring and then applying one or other of the formulae given on page 77. Now

$$MH = \frac{4\pi^2 K}{t^2},$$

and all the terms on the right-hand side are known: that is:—

$$MH = \text{some number} = B.$$

$$\text{Now } MH \div \frac{M}{H} = H^2; \quad \therefore H^2 = \frac{B}{A}, \ \textit{i.e.} \ H = \sqrt{\frac{B}{A}},$$

and as both B and A are known numbers, H, the horizontal component of the earth's field, is determined. The actual form of the expression is, of course, easily shown to be:—

$$H = \frac{2\pi}{t\,(d^2 - l^2)} \sqrt{\frac{2Kd}{\tan \theta}},$$

but it is better to work out the values of M/H (*i.e.* the number A) and MH (*i.e.* B) separately.

In an actual experiment the following were the results:—(1) *Deflection*: $d = 40$ cm., $\theta = 10{\cdot}5°$, $l = 7{\cdot}64$ cm.; $\therefore$ M/H $= 6371{\cdot}4$. (2) *Oscillation*: $t = 19{\cdot}5$ sec., length of magnet $= 15{\cdot}28$ cm., breadth of magnet $= {\cdot}65$ cm., weight of magnet $= 95{\cdot}34$ grm.; $\therefore$ K $= 1858$ and MH $= 192{\cdot}93$. Hence:—

$$\mathrm{H} = \sqrt{\frac{192{\cdot}93}{6371{\cdot}4}} = {\cdot}174.$$

The same experiment enables us to determine the moment of the magnet used in the experiment. Thus:—

$$\mathrm{MH} \times \mathrm{M/H} = \mathrm{M}^2; \quad \therefore \ \mathrm{M}^2 = \mathrm{A} \times \mathrm{B},\ i.e.\ \mathrm{M} = \sqrt{\mathrm{A} \times \mathrm{B}}.$$

9. Eliminating the Magnetic Length of the Deflecting Magnet

The length l, which should be used in the deflection experiment, is really half the distance between the poles of the magnet, and is approximately equal to half the length of the magnet, only for a very thin magnet. Where l cannot be determined directly with sufficient accuracy it may be eliminated from the result by the following method. We have for the field F due to the deflecting magnet:—

$$\mathrm{F} = 2\mathrm{M}\frac{d}{(d^2 - l^2)^2},$$

which may be expanded and written as follows:—

$$\mathrm{F} = \frac{2\mathrm{M}}{d^3}\left\{1 + 2\left(\frac{l}{d}\right)^2 + 3\left(\frac{l}{d}\right)^4 + \ldots\right\}.$$

Now as l/d is a small quantity, the higher powers of it may be neglected, and we get as a sufficiently accurate result:—

$$\mathrm{F} = \frac{2\mathrm{M}}{d^3}\left(1 + 2\frac{l^2}{d^2}\right).$$

Again, since l^2 is unknown, we may write x^2 for $2l^2$ and we get the following expression for F:—

$$\mathrm{F} = \frac{2\mathrm{M}}{d^3}\left(1 + \frac{x^2}{d^2}\right).$$

$$\text{But } \mathrm{F} = \mathrm{H}\tan\theta; \quad \therefore \ \mathrm{H}\tan\theta = \frac{2\mathrm{M}}{d^3}\left(1 + \frac{x^2}{d^2}\right);$$

$$\therefore \ \tan\theta = \frac{2\mathrm{M}}{\mathrm{H}}\left(\frac{1}{d^3} + \frac{x^2}{d^5}\right).$$

Now if two observations of θ are made for distances d_1 and d_2, and if θ_1 be the mean of the four deflections at distance d_1 and θ_2 the mean deflection at distance d_2,

$$\tan \theta_1 = \frac{2M}{H}\left(\frac{1}{d_1^3} + \frac{x^2}{d_1^5}\right),$$

$$\tan \theta_2 = \frac{2M}{H}\left(\frac{1}{d_2^3} + \frac{x^2}{d_2^5}\right),$$

and eliminating x from these results we get an expression for M/H which does not include l the half *magnetic* length of the magnet, viz.—

$$\frac{M}{H} = \frac{d_1^5 \tan \theta_1 - d_2^5 \tan \theta_2}{2(d_1^2 - d_2^2)}.$$

Hence to eliminate the error due to l, the deflection experiment is done for *two* distances d_1 and d_2 instead of one, and this formula is used in calculating M/H instead of the formula in Art. 8.

Another method is to find the actual value of l^2 for the deflecting magnet, and then to use this value in the formulae of the preceding article. Thus, with θ_1, θ_2, d_1, and d_2 having the same meaning as above,

$$\frac{M}{H} = \frac{(d_1^2 - l^2)^2}{2d_1} \tan \theta_1 \text{ and } \frac{M}{H} = \frac{(d_2^2 - l^2)^2}{2d_2} \tan \theta_2;$$

$$\therefore \frac{(d_1^2 - l^2)^2}{2d_1} \tan \theta_1 = \frac{(d_2^2 - l^2)^2}{2d_2} \tan \theta_2.$$

Solving for l^2 and neglecting higher powers of l, we get—

$$l^2 = \frac{d_1^3 \tan \theta_1 - d_2^3 \tan \theta_2}{2(d_1 \tan \theta_1 - d_2 \tan \theta_2)}$$

from which the value of l^2 is once for all determined, and this value is then used for l^2 in the formulae for M/H of Art. 8.

10. Eliminating the Torsion Error by the Sine Method

When the needle in the deflection experiment is deflected there is, of course, a twist on the suspension making the deflection *less* than it would otherwise be. In practice the error is not great, and it is usual to neglect it. It can, however, be eliminated by using what is known as the "sine" method instead of the "tangent" method previously given.

Imagine the magnetometer so constructed that the whole can be rotated, and let this be done until the magnet is always at right angles to the needle (Fig. 129), in which case the magnet and needle

are in the same relative position as at the "starting" position and *there is no twist on the suspension.* Clearly:—

$$m\frac{2Md}{(d^2 - l^2)^2} \times SN = mH \times ST; \quad \therefore \frac{M}{H} = \frac{(d^2 - l^2)^2}{2d}\sin\theta.$$

Thus the tangent of the angle must be replaced by the sine in the deflection formula. Similarly for the formula eliminating l we would have

$$\frac{M}{H} = \frac{d_1^5 \sin\theta_1 - d_2^5 \sin\theta_2}{2(d_1^2 - d_2^2)}.$$

For *very* accurate determinations of H (and M) further corrections are necessary. Thus in the oscillation experiment the "twist" on the suspension reduces the period of vibration: again in the deflection experiment the magnet is at *right angles* to the meridian, whereas in the oscillation experiment it is more or less in the meridian and subject to the earth's inductive action, so that its moment M is not quite the same in the two experiments. For details of these corrections, and also of the Kew magnetometers for very accurate work, see *Advanced Textbook of Magnetism and Electricity.*

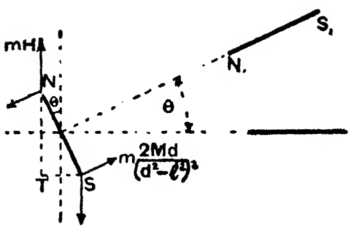


FIG. 129.

11. Null Point Measurements

(*a*) Place a *long* magnet on the table with its length in the meridian and *its S pole towards the north,* and find the null point (Y in Fig. 44) by a small compass. The field at Y due to the *long* magnet is m/d^2 (d = distance *from the S pole to* Y). The field H at Y due to the earth is ·18, say. But the two fields are equal since Y is a null point. Hence $m/d^2 = \cdot 18$ or $m = \cdot 18d^2$. Thus the *pole strength* of the magnet is determined.

(*b*) Again repeat the experiment with a *short* magnet, and let d = distance of its *neutral line from* Y. In this case the field at Y due to the magnet is $2M/d^3$ if d is great compared with the length of the magnet. Hence $2M/d^3 = \cdot 18$ or $M = \cdot 09d^3$. Thus the *moment* of the magnet is determined. Incidentally if M be known, the experiment gives the earth's field H.

(*c*) More accurate results will be obtained if the more exact formulae for the field at the null point due to the magnet be employed. Thus in (*b*) we have at the point Y:—

$$H = \frac{2Md}{(d^2 - l^2)^2} \text{ and } M = \frac{H(d^2 - l^2)^2}{2d},$$

from which either H or M is found if the other be known. Here d is as before the distance from the neutral line to Y, and l is half the magnetic length of the magnet. Similarly, if the magnet be placed as in Figs. 43 or 45 and the null points X or Z be found by experiment, the strength of the earth's field H or the moment M of the magnet may be found by applying the appropriate formula for the field due to the magnet. (See Chapter III.)

12. Proving the Law of Inverse Squares

(1) DEFLECTION AND OSCILLATION METHODS.—We have seen that the force between isolated point poles varies inversely as the square of the distance between them. It is not a simple matter to prove the law, for magnets are used in the experiments and the poles are neither isolated nor concentrated at points. Below, however, are a few laboratory experiments to approximately verify the law—they will show that the errors consequent upon assuming the law must be negligibly small.

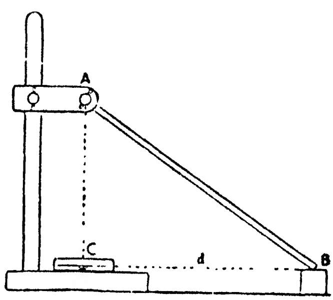


FIG. 130.

(*a*) First Method — Deflection.—In Fig. 130 C is a pivoted compass which moves over a scale. AB is a long magnet pivoted at A *straight above* the centre of C so that the pole A has practically *no turning effect* on the compass: it cannot deflect it from the meridian. The apparatus is arranged that the other pole B is at a certain distance due east of C, and the deflection is noted (α_1). Then A is raised until B is half the distance from the compass and the deflection is again noted (α_2). Now the field F at C due to AB may be taken as due *only to one pole* B, and at half the distance it ought to be four times. In an experiment, when B was 20 cm. away the deflection was 28°, and when 10 cm. away it was 65°.

$$\frac{F_2}{F_1} = \frac{H \tan \alpha_2}{H \tan \alpha_1} = \frac{\tan 65^\circ}{\tan 28^\circ} = \frac{2{\cdot}14}{{\cdot}53} = \frac{4}{1}, \text{ nearly.}$$

(*b*) Second Method—Deflection.—On pages 68 and 70 it is proved that the field F_1 due to a small "end-on" magnet is twice the field F_2 due to the same magnet "broadside-on" at the same distance, and *in the proof it was assumed that the law of inverse squares for a magnetic pole was true.* Now let a magnet be placed "end-on" to the magnetometer needle and let θ_1 be the mean of the eight deflections. Let the same magnet be then placed "broadside-on" at the same distance and let θ_2 be the mean of the eight deflections. Since $F = H \tan \theta$ we have

$$F_1 : F_2 = \tan \theta_1 : \tan \theta_2.$$

But *if the law of inverse squares be true* $F_1 = 2F_2$, and therefore *if the law be true* $\tan \theta_1$ should be twice $\tan \theta_2$; this will be found to be so within the range of experimental error.

(*c*) **Third Method—Oscillation.**—Consider a long magnet lying in the meridian, its N. pole northwards. The field *due to the magnet* at a point P on the axial line and near the north pole of the magnet may be regarded as *due to this pole only* and equal to m/d^2 if the inverse law be true, where d is the distance between the pole and the point P, and the law may be verified by an oscillation magnetometer at P.

Let n_1 = the number of vibrations in a given time under the influence of the earth (H) alone, and n_2 the number in the same time when the long magnet is placed as above, its north pole being (say) 2 inches from the centre of the oscillating needle; then $n_2^2 - n_1^2$ is proportional to the force due to the pole 2 inches distant. This is repeated with the bar magnet at various distances. Taking any pair of results it will be found that approximately

$$\frac{\text{Force due to pole at distance } d_1}{\text{Force due to pole at distance } d_2} = \frac{d_2^2}{d_1^2},$$

which means that the force is inversely as the square of the distance.

(*d*) **Fourth Method—Oscillation and Graphical.**—The student can carry the above a step further. Perform the experiment with the magnet pole at various distances from the needle and obtain for each position the number proportional to the force due to the pole (*i.e.* the numbers $n_2^2 - n_1^2$). Now assume the law is that the force varies inversely as the pth power of the distance where p is a number to be found. If this is so we can write (if F denotes force and d distance):—

$$F \propto \frac{1}{d^p}; \quad \therefore \; Fd^p = \text{a constant, } i.e. \log F + p \log d = \text{a constant.}$$

Thus from the various numbers obtained proportional to F at various distances find the various values for log F and log d, and plot log F against log d; the tangent of the angle of slope of the curve will give p, and this will be found to be approximately 2, which verifies that the law is an inverse *square* law.

(2) MAGNETIC BALANCE METHODS.—Several pieces of apparatus have been specially devised for proving the law both as applied to magnets and to currents in solenoids, and Fig. 131 will indicate the general principle of many. In the figure A is a long, thin solenoid fixed horizontally to a graduated stand. B is another long solenoid suspended horizontally at its centre of gravity, the suspension frequently consisting of the wires which serve to lead the current to and from the coil. When currents are passed in the same direction in A and B the poles *a* and *b* are unlike poles and *b* is attracted, and the force of attraction is balanced, *i.e.* B is brought back to its horizontal position, by means of a sliding brass weight or

rider W which is hung on B. By raising or lowering A the experiment is repeated with the poles at various distances apart and the inverse square law is verified. Further, by keeping the distance fixed and varying the currents it can be shown that the force is proportional to the product of the current strengths.

Hibbert's magnetic balance works on the above principle.

(3) GAUSS'S METHOD.—By far the most valuable work on the verification of the law was done by Gauss. He assumed that *the force between two poles varies inversely as the pth power of the distance*, in which case a simple extension of the mathematics of pages 68 and 70 leads to the results that (*a*) the field (F_1) due to an "end on"

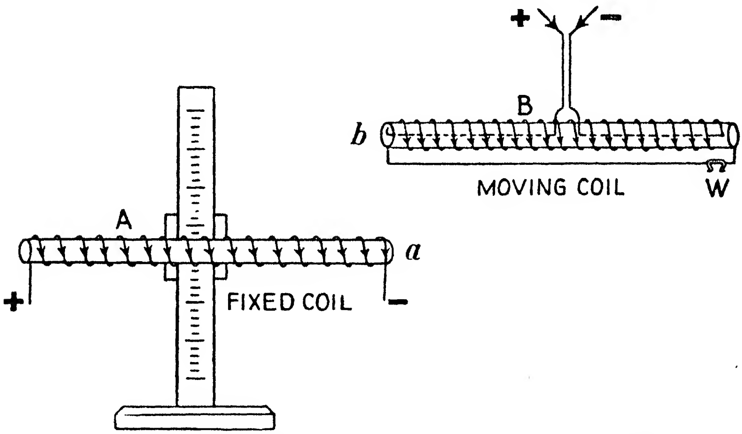


FIG. 131. The coils are shown wound on cardboard tubes.

magnet is equal to $\frac{pM}{d^{p+1}}$, and (*b*) the field (F_2) due to the same magnet "broadside on" at the same distance is equal to $\frac{M}{d^{p+1}}$ so that $F_1 = pF_2$. Now $F_1 = H \tan \theta_1$ and $F_2 = H \tan \theta_2$, where θ_1 and θ_2 are the deflections produced by a magnet at equal distances in the "end on" and "broadside on" positions respectively. Thus $F_1 : F_2 = \tan \theta_1 : \tan \theta_2$, and, since $F_1 = pF_2$, we have $\tan \theta_1/\tan \theta_2 = p$. Experiment shows that $\tan \theta_1/\tan \theta_2 = 2$; hence $p = 2$, *i.e.* the force between magnetic poles varies inversely as the *square* of the distance. (Compare with Experiment (*b*) above.) For a full treatment of the mathematics involved, see *Advanced Textbook of Electricity and Magnetism.*

13. Worked Examples

(1) *The period of vibration of a uniformly magnetised magnetic needle is 3 seconds. The needle is then broken into exact halves. What is the period of vibration of each half?*

$$t_1 = 2\pi\sqrt{\frac{K_1}{M_1H}}, \quad t_2 = 2\pi\sqrt{\frac{K_2}{M_2H}}; \quad \therefore \frac{t_1}{t_2} = \frac{\sqrt{K_1 \times M_2}}{\sqrt{K_2 \times M_1}}$$

$$K_1 = w_1\left(\frac{l_1^2}{12} + \frac{r_1^2}{4}\right) = \frac{w_1 l_1^2}{12} \text{ approx. and } K_2 = \frac{w_2 l_2^2}{12};$$

$$\therefore \frac{K_1}{K_2} = \frac{w_1 l_1^2}{w_2 l_2^2}; \text{ further, } \frac{M_2}{M_1} = \frac{ml_2}{ml_1} = \frac{l_2}{l_1};$$

$$\therefore \frac{t_1}{t_2} = \frac{\sqrt{w_1 l_1^2 \times l_2}}{\sqrt{w_2 l_2^2 \times l_1}} = \frac{\sqrt{w_1 \times l_1}}{\sqrt{w_2 \times l_2}}. \quad \text{But } w_1 = 2w_2 \text{ and } l_1 = 2l_2;$$

$$\therefore \frac{3}{t_2} = \frac{\sqrt{2w_2 \times 2l_2}}{\sqrt{w_2 \times l_2}} = \frac{2}{1}, \text{ i.e. } t_2 = 1\tfrac{1}{2} \text{ seconds.}$$

(2) *A small compass needle makes* 10 *oscillations per minute under the influence of the earth's magnetism. When an iron rod* 80 *cm. long is placed vertically with its lower end on the same level with and* 60 *cm. from the needle and due (magnetic) south of it, the number of oscillations is* 12 *per minute. Calculate the strength of pole of the iron rod* (i) *neglecting,* (ii) *taking account of, the influence of the upper end.*

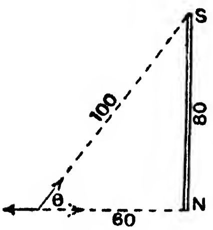


FIG. 132.

(*a*) *Neglecting upper pole.* The bar is magnetised, the lower end being a north pole, and the horizontal field due to this pole only is $\frac{m}{60^2}$. Now (Fig. 132)—

$$\frac{F+H}{H} = \frac{12^2}{10^2} = 1{\cdot}44, \text{ i.e. } \frac{F}{H} = {\cdot}44; \quad \therefore F = {\cdot}44 \times H;$$

$$\frac{m}{60^2} = {\cdot}44 \times {\cdot}18; \quad \therefore m = {\cdot}44 \times {\cdot}18 \times 60^2 = \mathbf{285{\cdot}12 \text{ units.}}$$

(*b*) *Taking upper pole into account.* The field due to the upper pole is $m/100^2$ in the direction shown and the horizontal component of this is $\frac{m}{100^2}\cos\theta$, *i.e.* $\left(\frac{m}{100^2} \times \frac{60}{100}\right)$. The total horizontal field F due to the magnet is:—

$$F = \frac{m}{60^2} - \left(\frac{m}{100^2}\cdot\frac{60}{100}\right) = m\left(\frac{1}{60^2} - \frac{60}{100^3}\right)$$

$$\frac{F}{H} = {\cdot}44, \text{ i.e. } F = {\cdot}44 \times {\cdot}18; \quad \therefore m\left(\frac{1}{60^2} - \frac{60}{100^3}\right) = {\cdot}44 \times {\cdot}18;$$

$$\therefore m = \frac{{\cdot}44 \times {\cdot}18 \times 36000000}{7840} = \mathbf{363{\cdot}6 \text{ units.}}$$

The values are quite different: the rod is too short (80) compared with the distance (60) to neglect S.

CHAPTER V

GENERAL PRINCIPLES OF ELECTROSTATICS

WE have seen (Chapter I.) that a positively charged body is one in which some atoms are deficient in electrons, whilst a negatively charged body is one in which some atoms have gained electrons. We have also seen that protons repel each other and electrons repel each other, but protons and electrons attract. Thus in electrification by the rubbing experiments (the object of which was to secure close and extended *contact*) of Chapter I., electrons were transferred from some of the atoms of one body where they were less firmly held by their attracting nuclei to the atoms of the other body where they were more firmly held, so that the first body was positively charged, the second negatively, and the two attracted each other. As already mentioned, an atom (or group of atoms) which has lost electrons is called a *positive ion*, and one which has gained electrons a *negative ion*.

Only a very small fraction of the total number of electrons present in the bodies is actually moved: when vulcanite is rubbed with fur it is only about one in every million million electrons present which is transferred, but the resulting force of attraction between the bodies is quite a distinct measurable quantity—we can measure forces considerably less. The force of attraction between the protons and electrons inside the atom must therefore be very great: in fact if we could drag *one electron only* out of *every* atom of a *single* drop of water and take them half a mile away from the drop, it would require a force of several thousand million tons weight to stop the electrons moving back to the drop.

1. Electrical Potential

On pages 19-21 reference was made to *electrical potential*. The exact meaning of electrical potential and its relation to work and energy is dealt with in the next chapter (the conception is somewhat similar to that of magnetic potential—see page 59), so that for the present we can continue to look upon it, as we did in Chapter I., merely in a general sense. As stated, an isolated positively charged brass ball, say, was said to be at a *positive potential* (higher than earth potential which, in practice, is taken as zero potential): if joined to earth by a conducting wire it becomes neutral and we now know that this is because electrons flow in the direction earth to ball, *i.e.* lower to higher potential. If the ball be negatively

charged it was said to be at a *negative potential* (lower than earth potential), and when earthed we know that electrons flow in the direction ball to earth. This flow of electrons in the wire in the direction lower to higher potential is the true electronic current: the early scientists assumed the "flow" of electricity to be, of course, in the opposite direction in the wire, *i.e.* higher to lower potential, and this, as we have seen, is referred to as the conventional current direction.

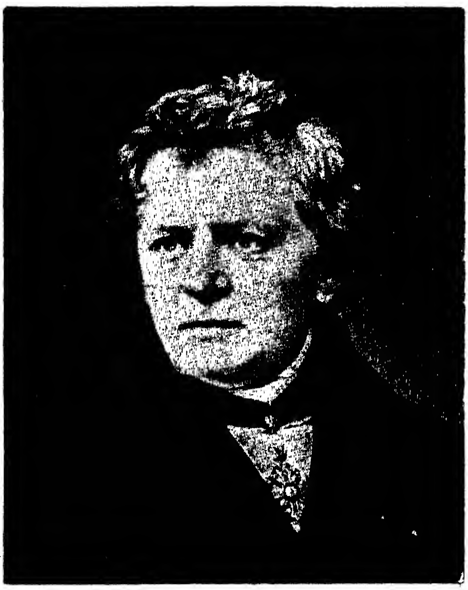
It will be seen later that the electrical potential of a body depends not only on the "kind" of its electrification, but also on the amount of electrification and on the size of the body; it is also affected by neighbouring bodies and charges and by the medium in which the body is placed. Thus, to quote one case only, there may be *two positively charged* bodies, both at positive potentials, but the positive potential of one may be greater than that of the other, so that when they are joined by a wire, *electrons* will flow from the one of lower to the one of higher positive potential. Similarly, two negatively charged bodies may be at different negative potentials, so that electrons will flow along a connecting wire from the one which has the *greater negative potential* to the other, for the former is really at the lower potential—further below zero potential.

For the present, then, we can summarise in a general way as follows:—**The electrical potential of a body is its electrical condition which settles whether the body will give electricity to the earth or another body, or get electricity from the earth or another body when they are joined.** *Taking the modern idea*, a body is at a positive potential if electrons flow in the direction earth to it when they are joined by a conducting wire; it is at a negative potential if electrons flow in the direction from it to the earth, and it is at zero potential if there is no flow: similarly A is at a higher potential than B if electrons flow in the direction from B to A along a wire joining them. *Taking the old idea*, a body is at a positive potential if a (conventional) current goes from it to the earth, and at a negative potential if it goes from the earth to it: A is at a higher potential than B if a (conventional) current goes from A to B.

Clearly:—(*a*) A potential difference (P.D.) is necessary in order to transfer "electricity" from one point to another. (*b*) All parts of a *conductor* in electrical equilibrium are at the same potential. (*c*) When a conductor is earthed and everything is "steady" it is at zero potential. It is clear, too, that positively charged bodies will tend to move from a place where the electrical potential is high to a place where the potential is lower, whilst negatively

ALESSANDRO VOLTA,
1745-1827.

Italian physician, born at Como. Professor at Pavia and later Director of Philosophy at Padua. Came to England in 1782 and met Priestley, then at Birmingham. Constructed the voltaic pile, the forerunner of modern batteries. To perpetuate his name the unit of electric pressure is called the *volt*.



GEORG SIMON OHM,
1787-1854.

Born at Erlangen, Germany. When lecturer at Cologne he discovered his great law, and published *The Galvanic Chain Mathematically Worked Out.* Disappointed at its unfavourable reception, he resigned and for six years did odd jobs. London, however, honoured him with the Copley Medal, and he resumed his work, becoming, at 62, Professor at Munich. To perpetuate his name the unit of resistance is called the *ohm*.

charged bodies will tend to move in the opposite direction. In a metallic substance (*e.g.* the connecting wire above) only electrons are free to move, but it follows from the preceding that if in any substance there were positive and negative ions *which were free to move*, the positive ions would move from high to low potential and the negative ions from low to high potential (see Art. 2, below).

One further point should be noted. If an insulated and positively charged brass ball A be made to touch an insulated neutral brass ball B, electrons will flow from B to A until the two come to the same (positive) potential: B will now be positively charged, for it has lost some electrons, whilst A will be less positively charged than before for it has got back some electrons. For brevity it is sometimes said that " A has shared its positive charge with B." If A be negatively charged, electrons will flow from A to B until the two come to the same (negative) potential: B is now negatively charged, for it has excess electrons, whilst A is less negatively charged than before for it has parted with some of its surplus electrons: we say A has shared its negative charge with B.

2. Conductors and Insulators

Every material offers a certain amount of opposition to the flow of electricity through it. At this stage *resistance* may be defined in a general way as that property of a body which opposes the flow of electricity. *An insulator is a substance possessing such enormous resistance that practically it does not permit electricity to flow through it: a conductor is a substance of such small resistance that it readily allows electricity to flow through it.* The best conductors are the metals, notably silver, copper, and aluminium: amongst the good insulators are *dry* air, *dry* glass, *dry* paper, paraffin wax, mica, vulcanite, shellac, india-rubber, sulphur, sealing-wax, silk, and certain oils of the paraffin family.

In practical electrical work *copper* and *aluminium* are largely used as conductors: for overhead transmission lines hard-drawn copper is largely employed, but aluminium (fitted with a steel core to give it strength) is coming into more extended use (*e.g.* in the transmission lines of the " Grid "), and where great strength (tensile) is required, *bronzes* composed of copper with tin, silicon, etc., are often used. Alloys such as *manganin* (copper, manganese, nickel), *german silver* (copper, zinc, nickel), *platinoid* (copper, zinc, nickel, tungsten), and *resista* (iron, nickel, manganese) are used for special purposes.

Amongst the practical insulators are *glazed porcelain, slate and marble, asbestos, celluloid, bakelite, ebonite, india-rubber, gutta-percha, mica* and *micanite, oil, presspahn, paraffin wax, shellac,* and *vitreous enamel.* As a protection for electric wires and cables *vulcanised india-rubber*, which is rubber cured with sulphur at a high temperature, is largely used, but to avoid the bad effect of sulphur on the copper a layer of pure rubber is always put between them.

Mica has the advantage, for certain purposes, that it can be obtained in very thin sheets. Micanite (thin sheets of mica held together by an insulating cement), under the application of heat, can be moulded into almost any convenient form. Paper impregnated with insulating oil is largely used in cable insulation. Certain mineral oils are good insulators and many electrical appliances, *e.g.* switches (or cut-outs), condensers, etc., used for high P.D. or high pressure (voltage) work are oil-immersed to obtain high insulation. Distilled water is an insulator, but the slightest impurity makes it a conductor; hence, in practice, moisture ruins the insulating properties of any material which absorbs it.

Conductors become worse conductors when they are heated, i.e. *the resistance they offer to the flow of electricity becomes greater.* Most alloys have a fairly big resistance and they also increase in resistance when heated, but they do not vary so much as pure metals and are therefore largely used for testing and other purposes where the conductivity is required to remain constant. Insulators become worse insulators when they are heated, i.e. *the resistance they offer to the flow of electricity becomes less,* and some, if strongly heated, may even become fairly good conductors. Further, a body may be an insulator when the applied P.D. is low, but a conductor when the P.D. is sufficiently high: thus normally air (dry) is an insulator, but the lightning flash passes through miles of it.

The modern theory of conduction and insulation has been briefly referred to (Chapter I.), and further details appear in later chapters. Our picture of conduction in, say, a copper wire is, as already indicated, a drift of electrons through a more or less fixed framework of positive ions. The *outer* electrons of the atoms of a good conductor such as copper are not rigidly held by their attracting nuclei (and they are not far from nuclei of adjacent atoms and are under some attraction from them), so that there is, at any instant, a number of outer electrons detached from their parent atoms and existing as " free " electrons moving about in the spaces between the atoms, passing from the " orbital " system of one atom to that of another, and so on. The positive ions in the copper form a kind of framework (or " space lattice " as it is termed) which is fixed—at any rate they occupy a definite mean position about which they merely oscillate with a motion of thermal agitation. When a P.D. is applied the " free " electrons drift through the spaces or avenues between the ions in the direction lower to higher potential, this " movement " constituting the current in the copper: the positive ions, apart from a minute elastic displacement are unaffected by the applied P.D. In a good insulator there are no " free " electrons, each nucleus holding on firmly to its own electrons. When a P.D.

is applied the "charges" inside the atoms certainly experience forces tending to move them in their appropriate directions, but they only undergo small elastic displacements: in simple language the internal electrical make-up of the atoms is "distorted" only. If the P.D. is made sufficiently great, however, electrons are at last "torn" out of their atoms, and a movement of electrons takes place through the body of the insulating material: but this represents the breakdown or rupture of the insulation, the molecular constitution of which has now been destroyed.

In the case considered above, viz. that of a P.D. being applied to a metallic conductor (copper), it is, as explained, electrons only which move from lower to higher potential, any ions in the conductor being more or less fixed. In a class of liquids known as electrolytes there always exists a number of *both positive and negative ions which are free to move:* thus in a solution of sodium chloride (common salt) there are +ve sodium ions and −ve chlorine ions moving about in a haphazard way (page 12). Hence if a P.D. be applied, *e.g.* if two metal plates connected to a battery be put a certain distance apart in the electrolyte, the negative ions will move through the liquid in the direction lower to higher potential, and the positive ions will move in the opposite direction, higher to lower potential. This is dealt with in Chapter XIII.

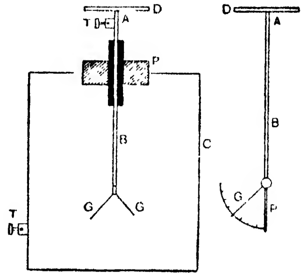
Again, gases normally are insulators, but, by various means, they can be rendered conducting by forcing electrons out of some of the atoms, in which case electrons and positive ions are produced in the gas, and *both are free to move* (this is spoken of as the **ionisation** of the gas). If, then, a suitable P.D. be applied to two plates put in the gas, electrons will move in the direction lower to higher potential and positive ions from higher to lower potential. In their movement through the gas the detached electrons make collisions with other atoms and they may detach further electrons producing further positive ions, and these also will move to the respective plates as indicated.* Further, in the course of its motion in the gas a positive ion may encounter an electron (or electrons) moving with sufficiently low velocity to be drawn into the positive ion's electron system so that the positive ion again becomes neutral (referred to as **recombination**). Finally, in its movement

* Most gases exist as *molecules* made up of two or more atoms, and when ionised by detaching electrons the positive ions produced, consist of *molecules* which have lost an electron: this is a detail however, and, for the present, we shall regard gases as if they were made up of single atoms (which *is* the case with gases at *very high* temperatures and with an inert gas—page 12).

through the gas an electron may become "loaded" by attracting and attaching to itself a neutral molecule of the gas, and the same applies to the moving positive ion. But all these points are dealt with in detail in Chapter XIX.

3. The Gold-Leaf Electroscope

In the early study of electricity this instrument is used to detect a charge, to decide whether it is positive or negative, and to compare the amounts of the charges. Strictly, however, it measures, and its action depends on, "potential."



(a) (b)
FIG. 133.

A simple form is shown in Fig. 133 (*a*). It consists of a tin box C of which two opposite sides are replaced by glass. P is an india-rubber plug with a central vulcanite rod. A brass rod AB passes through the vulcanite, and carries at its upper end a brass disc D, and at its lower end a horizontal brass piece, to each side of which a gold-leaf G is attached; in the normal position these two gold-leaves hang vertically side by side, but if they become charged they repel each other. Binding screws T′ and T are fitted to the "case" and to the rod. Sometimes a scale is fitted behind the gold-leaves to show the amount of their movement. Sometimes, too, there is a fixed metal plate P and only one leaf (Fig. 133 (*b*)).

A more sensitive type is the *condensing electroscope*: it is a gold-leaf electroscope with a *condenser* (page 199) attached, and is dealt with on page 225.

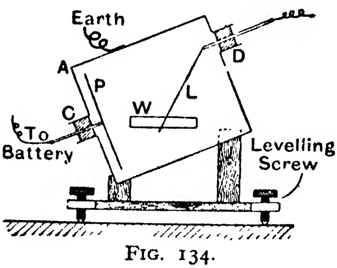


FIG. 134.

A much more sensitive type is the *tilted gold-leaf electroscope*, designed by C. T. R. Wilson and modified by Dr. Kaye, but it is mainly used for experiments on radio-activity and the conduction of electricity through gases. It consists of a small, nearly cubical brass box with holes at C and D (Fig. 134). Through C there projects an insulated rod carrying a brass plate P, while through D projects a well-insulated metal rod carrying a very narrow gold-leaf L. The leaf is about a millimetre wide, and should be just

long enough not to reach across to P. W is a window through which its motion is observed. The insulation at C may be ebonite, that at D must be sulphur or quartz. When in use P is kept charged to a steady fairly high potential (about 200 volts—see later), and the instrument is placed in an oblique position (30°, about) to increase the sensitiveness. The case A is earthed, and L is joined to the "potential" which has to be measured, this measurement being effected by noting the movement of L relative to P. The deflection is observed by a microscope directed to look through the window.

SIMPLE EXPERIMENTS WITH THE ELECTROSCOPE.—In these, the simple electroscope of Fig. 133 is used.

(1) To charge an electroscope positively, rub a brass ball with india-rubber, and touch the cap of the electroscope with it. The ball "shares its positive charge" with the cap, brass rod, and gold-leaves, and the latter, being both positively charged, repel each other (Fig. 133). (The leaves really begin to diverge *as the ball comes near the cap*: this is explained later.) On removing the ball the leaves remain diverging, and the electroscope is "charged" positively. To "discharge" the electroscope touch the cap with the finger: the leaves collapse. To charge the electroscope negatively use a brass ball rubbed with fur. (A better method of charging is given in Art. 5.)

(2) To test the electrification of a body, take three electroscopes, one positively charged, one negatively, the third neutral. Bring the body near the cap of the neutral electroscope: if the leaves diverge the body is electrified, but if they do not it is not electrified. If the body *is* electrified, bring it near the *positive* electroscope. If the leaves *diverge more* the body is positive, for it is attracting electrons from the leaves, thus *making them more positive* and they repel more. If the leaves *diverge less* the body is negative, for it is repelling electrons into the leaves and *lessening the "positive" charge*; verify by bringing it near the negative electroscope, in which case the leaves will *diverge more*. (See below.)

THE ELECTROSCOPE AND POTENTIAL.—It was stated above that the electroscope really measured, and its action depended on, *potential*, and it is easy to prove the important facts that *the leaves will only diverge if there is a difference between the potential of the leaves and the potential of the case*, and that the divergence is greater the greater this P.D.

In Experiment (1) above a positive charge was given to the leaves, rod, and cap, and the leaves diverged. Here the leaves are at a positive potential: the tin case is on the table, *i.e.* is earthed, and at zero potential: *there is a difference, therefore, between the potential of the leaves and case*, and the leaves are diverging. A similar remark applies when the electroscope is negative.

Now insulate the electroscope by placing C on paraffin wax. Earth the leaves, etc., by connecting T by a wire to the gas-pipes; the leaves will be at zero potential throughout the experiment. Charge a metal ball positively, and let it touch the case C; the ball "shares its charge" with the case until they acquire the same positive potential, and the leaves will be found to diverge. Here, again, *there is a difference in potential between leaves and case*, the leaves being at zero potential and the case at a positive potential. If the ball be negative the case will be at a negative potential, the leaves at zero, and the latter will be diverging.

Discharge the case, disconnect T and the gas-pipe, and join by a wire T′ and T. Since C and the leaves are now connected, whatever potential is given to one will be given also to the other. Touch the cap or case with a positive ball so that they acquire a positive potential: the leaves do not diverge—*leaves and case are at the same positive potential.* If a negative ball be used leaves and case will be at the same negative potential and there will be no divergence.

We see, therefore, that the leaves only diverge if they are at a different potential to that of the case, and the greater that P.D. the greater is the divergence. Thus a positive body brought near the cap of a positive electroscope really makes the leaves at a bigger positive potential: there is therefore a bigger P.D. between leaves and case, and the leaves diverge more. In simple experiments C is earthed so that the potential of the leaves will be the P.D. between leaves and case; thus, *the amount of the divergence will be a measure of the common potential of the leaves and any body joined to them.*

4. Inductive Displacement or Influence

We come now to some experiments which illustrate facts of the greatest importance.

Bring an insulated *neutral* conductor AB (Fig. 135) near an insulated positively charged conductor C (or near a positive glass rod). Touch the end B with a proof plane (which usually consists of a small, flat, circular piece of brass mounted on an insulating handle), and then bring the proof plane near the cap of a positive electroscope. The leaves *diverge more*; hence the proof plane and the end B of the conductor AB are positively charged.

Discharge the proof plane. Touch the end A, and then bring the proof plane near the cap of a negative electroscope. The leaves *diverge more*; hence the end A of AB is negatively charged.

Remove C and test AB: it has no charge. We conclude, therefore, that when C is taken away, the negative at A and positive at B have neutralised and therefore must be equal in amount.

Repeat the above with C negative. In this case A is positive, B negative, and, as before, the two are equal (Fig. 136).

These actions are referred to as **electrostatic induction** or **inductive displacement** or **influence**, the charges at A and B are

spoken of as *induced charges*, and the insulating medium (air) between C and AB which transmits these induction effects is called the **dielectric.** All dielectrics are insulators: when we call them insulators we are referring to the fact that *they will not allow electricity to flow through them*: when we call them dielectrics we are referring to the fact that *they allow these electrical influences to be transmitted through them*, and, in fact, themselves play a part in the action.

By putting a solid dielectric such as glass, wax, mica, etc. (or a

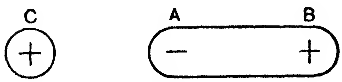


FIG. 135.

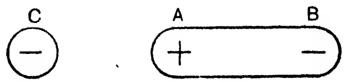


FIG. 136.

liquid dielectric), between C and the conductor AB, the *inductive* effects will be still more marked, for they allow the influence to take place through them better than air, and are said to have a higher *specific inductive capacity* or **dielectric constant** or **permittivity.**

It will be clear that *inductive displacement always precedes the attraction between a charged body and a neutral conductor.* Thus when a negatively electrified rod is held near a suspended pith ball, the near side of the ball acquires a positive charge, and the far side a negative charge; the former charge is nearer the rod than the latter, so that, on the whole, the force is one of attraction.

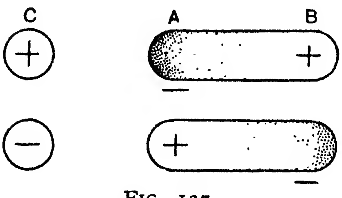


FIG. 137.

An explanation of inductive displacement is simple. If C is positive it has lost some electrons. When AB is brought near, the "predominating positive centres" of the C atoms try to get hold of some of AB's electrons to make up the deficits. Some electrons of AB are therefore attracted towards the end A nearest to C. Thus A has an excess of electrons and is negative, whilst B is deficient and is positive (Fig. 137). When C is removed the atoms at B get back the electrons and AB becomes neutral. Further, the negative charge at A must be equal to the positive at B: all that C does is to "displace" the electrons of AB. If C be negative, some of AB's electrons are repelled towards the far end B, so that A shows a deficit of electrons and is positive, whilst B has a surplus and is negative (Fig. 137).

Considerations of potential also provide an explanation. A positively charged body is at a positive potential and produces a positive potential throughout the whole of the insulating medium (air) round about it, this potential decreasing (at first rapidly and then more slowly) from its positive value at the charged body to zero at the earth, *i.e.* walls, etc., of the room. This is roughly indicated in Fig. 138, where vertical distances denote potentials. (The case is somewhat similar to the magnetic potentials at the various distances from the N. pole in Figs. 75, 91.)

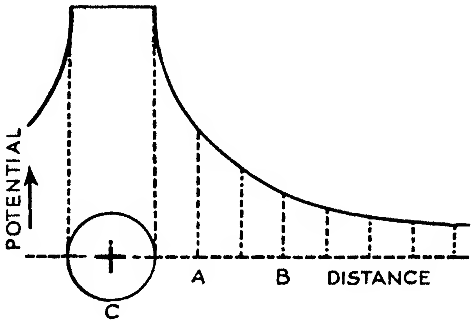


FIG. 138.

In Figs. 138, 139, then, the point A in the air is at a higher potential than B, for it is nearer to C, which is positively charged. This P.D. tends to cause electrons to move in the direction B to A (low to high potential), but the air is an insulator and does not permit this flow: the charges constituting the atoms of the dielectric merely undergo small elastic displacements, thus setting up an opposing influence which balances the effect of the P.D.—there is what we might call an *electric strain* but no " flow " of electrons. When the conductor AB is put there, however, its electrons flow from B to A, causing B to have a deficiency or to be positive, and A to have an equal surplus or to be negative.

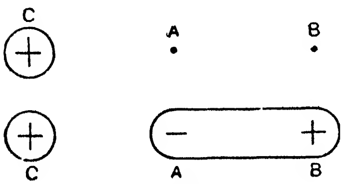


FIG. 139.

So much for the " charges " on AB. With regard to the *potential*, although AB shows a negative charge at A and a positive charge at B, *all parts of* AB *must be at the same potential after the flow stops*, for it is a *conductor*. This uniform potential of AB is, in fact, a positive potential intermediate between that of C and the earth (zero). We might say that the negative at A lowers the potential of this end which is nearest to C, and the positive at B raises the potential of this far end, so that when all is steady AB is

at a uniform potential. An experimental proof that it is a *uniform positive* potential is given below. This positive potential of AB is spoken of as an *induced positive potential*, since it is due to the inductive influence of C.

If C be negative it is at a negative potential and produces a negative potential right throughout the medium, the actual potential of the medium *rising* from its negative value at C to zero at the walls, etc. (earth). In this case the point B is at a higher potential than A, and when the conductor is placed there, electrons flow from A to B (low to high potential) so that A has the deficit of electrons and is positive, and B has the (equal) surplus and is negative. When all is steady AB is at a uniform potential which is negative and intermediate between that of C and the earth: it is an *induced negative potential* due to C.

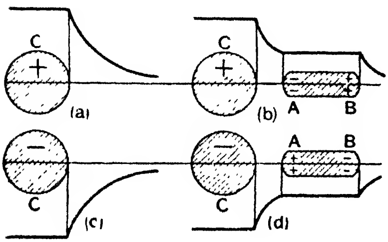


FIG. 140.

Fig. 140 shows these points graphically, the vertical distances denoting potentials: distances above the horizontal line denote positive potentials, and distances below negative potentials. (*a*) shows the positive body before AB is brought near: it is at a uniform positive potential itself, shown by the horizontal line above it, and the curve again shows the potential falling in value as we proceed outwards from it to the earth (zero). (*b*) shows the case when AB is brought near: the positive potential falls as we go from C to AB, then we have a uniform positive potential over AB, and then the potential falls as we pass on from AB to earth. The uniform positive potential of AB is less than that of C. The case where C is negative is shown at (*c*), (*d*).

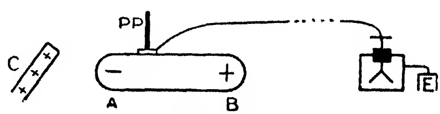


FIG. 141.

It will be noted that when the conductor is in position the potential of C is slightly affected by the induced charges on AB. Thus in Fig. 140 (*b*), the negative at A tends to lower the potential of C, the positive at B tends to raise it; the negative is nearer and has the advantage, so that on the whole the potential of C is slightly lowered. (Note the height of the horizontal line above it.) Similarly, in (*d*), the *negative* potential of C is slightly weakened, *i.e.* the potential of C is slightly *raised* or brought nearer zero.

The fact that the induced potential of AB in Fig. 135 is a *uniform positive potential* is proved by the following experiment:—

Place an electroscope some distance away, *join its cap by a thin wire to the disc of the proof plane*, and place the proof plane on AB (Fig. 141). The conductor AB, proof plane, wire, disc, rod, and leaves all form one conductor. The leaves diverge. Move the proof plane all over AB. The leaves remain diverging by the same amount all the time, showing that the potential of the leaves is the same all the time, and therefore that *the potential of* AB *is the same all over*. That it is a uniform *positive* potential can be proved by bringing a positive body near the electroscope: the leaves diverge *more*. Similar results happen and similar explanations apply if C is negative.

Notice the difference between this last experiment and the one previously used to test the *charges* at A and B. In that A was touched with a proof plane: it took a negative charge certainly, but while it was touching AB it was at the same positive potential as AB. When *it was taken away* its negative charge, being away from the influence of C, immediately made it at a negative potential, so that when it was brought near a negative electroscope it caused a bigger divergence.

That the uniform potential of AB in Fig. 141 is a positive potential will also be seen in the next section.

The following is a further instructive experiment illustrating this matter of *induced potential* due to a neighbouring charge.

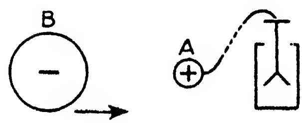


FIG. 142.

Take an insulated brass ball A joined to an electroscope and charge it positively: the leaves will diverge, the amount of divergence being a measure of the positive potential of the leaves, etc., and A. Now take another ball B, give it a strong negative charge and gradually bring it towards A (Fig. 142): the leaves will gradually fall together, showing that the induced negative potential due to B is weakening the positive potential of A. When B reaches a certain position the leaves will collapse, so that their potential is zero, the same as the case of the electroscope; in this position the induced negative potential due to B is exactly cancelling the positive potential of A, due to its own charge. If B be brought still nearer A the leaves will begin to diverge again, being now at a negative potential; the induced negative potential due to B is predominating.

It should be noted that, to begin with, A in Fig. 142 *has a positive charge and is at a positive potential.* When B is in such a position that the leaves collapse A *still has its positive charge, but is at zero potential.* When B is brought nearer, so that the leaves diverge again, A *still has its positive charge, but is at a negative potential.* This effect of neighbouring charges on the *potential* of a charged (or uncharged) body should be carefully noted.

5. Charging a Conductor by Inductive Displacement

Take again AB acted on inductively by the positive body C. AB is joined to an electroscope: the leaves will be diverging (Fig. 143).

Now touch *any part* of AB with the finger. The leaves collapse, showing that now there is no difference between the potential of the leaves and case and, as the latter is zero, showing that AB *is now at zero potential.* Touch A with a proof plane and then *take it away* and bring it near a negative electroscope: the leaves diverge more, showing that *at the end* A *we still have a negative charge.* Similarly test the end B, and it will be found to have no charge. The *positive charge which was at* B *has disappeared on earthing* AB. But for this to happen, electrons must have come

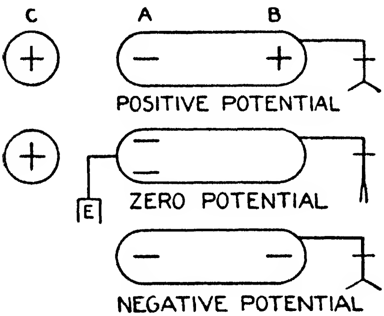


FIG. 143.

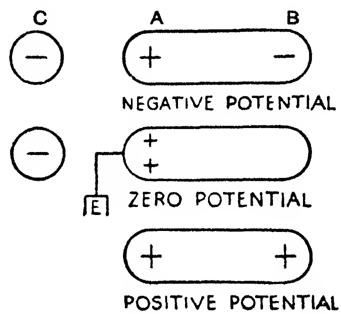


FIG. 144.

up from the earth, and therefore, *before earthing,* the uniform potential of AB must have been a positive potential.

When AB is earthed it is at zero potential although it still has a negative charge at A: the negative potential which the negative charge at A produces is being exactly cancelled by the induced positive potential due to C.

Now take C away (finger having first been removed). The leaves diverge again. Test AB. It shows a negative charge all over and will be at a negative potential due to this charge for C is now out of the way. All this is indicated in Fig. 143.

If C be negative and these operations be carried out the results shown in Fig. 144 will be obtained. AB in this case has (1) a positive charge at one end and a negative charge at the other, and the whole of it is at a uniform induced negative potential (due to C); (2) a positive charge at A and the whole conductor at zero potential; (3) a positive charge all over and at a positive potential.

Note that in the first case AB has been negatively charged (opposite to C), and in the second case positively charged (again opposite to C), by inductive displacement, and in neither case has C's charge been interfered with. The variations in charge and potential in the two cases are represented in Figs. 145 and 146.

A Warning.—When AB (Fig. 143) is earthed, electrons flow into AB and make up the deficit at B, but as a matter of fact, in this case, more electrons flow in in order to bring AB down to zero potential than is necessary to make up the deficit at B, so that at A we have a bigger negative charge than we had before. Now, the charge at A is sometimes called a " bound " charge, and that at B a " free " charge. When earthed, the free charge, it is said, disappears, leaving the bound charge. This is wrong. Do not use the words " bound " and " free " for these *charges*.

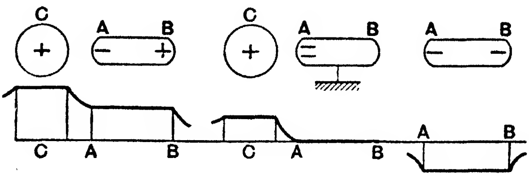


FIG. 145.

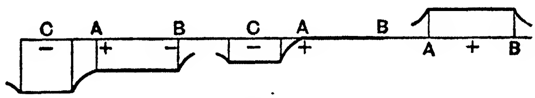


FIG. 146.

Charging an Electroscope by Induction or Influence.—We can use the method of inductive displacement to charge an electroscope positively or negatively. To charge it *positively* proceed thus:—Bring a *negative* rod near the cap, the case of which is earth-connected. Inductive displacement occurs, the cap having a positive charge, the leaves a negative charge, and the cap,

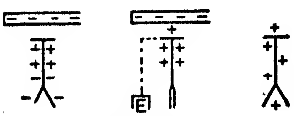


FIG. 147.

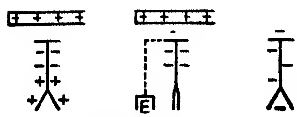


FIG. 148.

rod, and leaves an induced negative potential; the leaves diverge. Touch the cap with the finger; the cap, rod, and leaves are brought to zero potential and the leaves collapse. Remove, *first*, the finger, *then* the rod; the cap, brass rod, and leaves now acquire a positive potential due to the positive charge, and the leaves diverge. The electroscope has been charged positively (Fig. 147). To charge it negatively use a positive rod (Fig. 148). This method of charging an electroscope is much better than that adopted on page 132 and it is the method usually employed: it is, of course, only the previous experiment repeated.

6. The Charge is on the Outer Surface of a Conductor

When a conductor is charged the " charge " is entirely on the outside surface. This applies to both solid and hollow conductors (provided in the latter case there are no charged bodies *inside*), and the fact can be proved by numerous experiments.

(1) Fig. 149 depicts *Biot's experiment.* If the brass ball be positively charged, then the close-fitting brass caps (with insulating handles) be placed on it, and finally the caps be removed, it will be found that the ball and the insides of the caps have no charge but the outsides of the caps are positive.

(2) Fig. 150 shows *Faraday's butterfly net experiment.* The conical net is of linen gauze and, by the silk thread, it can be pulled inside out. The net is charged and the charge is shown to be on the outside, the inside being uncharged. The net is then pulled inside out and the charge is again found on the new outer surface.

(3) If a metal pot be placed on an insulating stand and then a positive charge be given to the outer surface it will be found, on testing, that the inside shows no sign of electrification—the outside only shows the positive

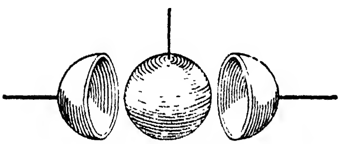


FIG. 149.

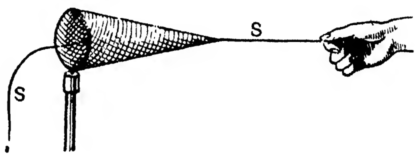


FIG. 150.

charge. Now discharge the pot. Charge an insulated brass ball, positively, and then lower it into the pot, allowing it to touch the inside. Remove the ball and test it: it will be found to be perfectly discharged. On testing the pot it will be found that the inside is uncharged and that the outside is positively charged. When the charged ball touched the inside of the pot it really became part of the latter, and the " charge " immediately disappeared from it, the outside showing the charge.

In the second part of Experiment (3) it is often said, for brevity, that " all the +ve charge of the ball passes to the outside of the can." Of course, *electrons* really pass from the can to the ball making up the latter's deficit and leaving the outside of the can with an equal deficit, *i.e.* an equal positive charge (but see Art. 8 for full explanation). Note also that if a charged conductor touches the outside of a metal pot (or any other conductor) it merely " *shares* its charge with the pot," but if it touches the inside of the pot it " gives up *all* its charge to the pot." Further details of the " charges " and " potentials " of metal pots, *i.e.* hollow conductors, are given in Art. 8: but it might be noted in passing that although

the inside of the pot has no charge it *must* be at the same *potential* as the outside, for the pot is a conductor.

The fact that the charge resides on the outer surface of a conductor may be briefly explained at this stage as follows. The charge always distributes itself in such a way as to possess *minimum potential energy.* In fact, just as objects under the earth's attraction always tend to fall towards the earth and to rest at the lowest attainable level nearest the attracting earth and therefore in the position of least potential energy (gravitational), so electricity passes to its position of lowest attainable potential energy—on the outer surface of the conductor as near as possible to the opposite (attracting) charges it induces on external conductors. Again, as will be seen presently, just as a magnetic pole is surrounded by a magnetic field permeated with magnetic lines, so a charge is surrounded by an electric field with its electric lines which start at a positive charge and end at an equal negative charge, and which tend to contract in the direction of their length and to repel each

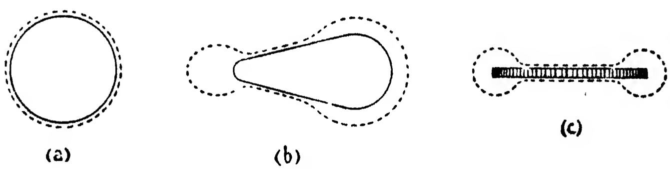


FIG. 151.

other laterally. In, then, say Biot's experiment, electric lines start at the surface of the ball and end at the negative charge induced on the walls, etc., of the room, and when the caps are put on, the "charge" disappears from the ball and "charge" appears on the outside of the caps in accordance with the tendency to longitudinal contraction of the lines (but see Arts. 8, 10).

7. Distribution of the Charge on Conductors. Sharp Points

We have seen that the potential of a charged conductor is the same at all points of it. The *distribution of the charge,* however, depends on its shape (and on its position with regard to other bodies). Thus on an isolated sphere the distribution is uniform (we might impress this on the mind by the method of Fig. 151 (*a*)); on a pear-shaped conductor the charge accumulates most at the pointed end, next at the rounded end, and least on the flat sides (Fig. 151 (*b*)); on a flat circular disc it is greatest round the edge (Fig. 151 (*c*)); on a rectangular metal box it is greatest at the corners, next at the

edges, least on the flat sides. This is usually expressed as follows: *the density of the charge*, i.e. *the charge on unit area, is greatest where the curvature is greatest.*

Give the pear-shaped conductor of Fig. 152, say, a negative charge. Place a small metal can on the cap of a neutral electroscope some distance away. Take two proof planes exactly alike and put one against the pointed end, the other against the rounded end. They form part of the conductor, and take the same potential as the conductor (*whilst they are there*), each taking a charge depending on the charge at the part touched. Now take the proof planes away and lower one into the can on the electroscope, allowing it to touch the inside so that *all its charge* " passes " to the can and electroscope, and note the divergence. Discharge the electroscope and repeat with the other proof plane. The bigger divergence is obtained from the proof plane which touched the pointed end, showing it had taken a bigger charge from the conductor. Begin again, this time touching the rounded end and the flat side: a bigger divergence is obtained from the one which touched the rounded end.

If the conductor contains a re-entrant hollow (*e.g.* Q, Fig. 153) there is

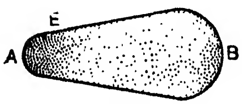


FIG. 152.

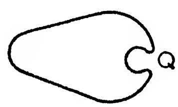


FIG. 153.

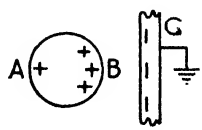


FIG. 154.

practically no divergence when the proof plane is put in the hollow and then taken to the electroscope. The *potential* inside the hollow is, however, the same as the uniform potential of the whole conductor.

Some idea as to why the distribution is as indicated may be gathered thus:—Consider a single electron at E (Fig. 152). This is of course under the repulsive force of all other electrons constituting the charge. But as the whole charge has reached equilibrium distribution, E is stationary: thus the force exerted on E by all the electrons on the *large* surface EB (tending to repel E in one direction) must be equal to the force exerted on it by the electrons on the *much smaller* surface EA (tending to repel E in the other direction). Hence the *concentration* of electrons on the part EA must be greater than on the rest of the surface, and reasoning in this way for other points nearer A it is readily seen that the concentration at the actual pointed end must be the greatest.

Viewing the matter as in Art. 6, if all points on the outer surface of a conductor are positions of equal potential energy for the charge, then the distribution of the charge will be uniform. If, however, there are portions of the surface where the potential energy of the

charge would be less than on other portions, then the charge will accumulate on those portions, until the distribution of the opposite induced charge is such that all points on the surface are positions of equal potential energy for the charge. In this case the distribution of the charge will not be uniform, but the charge per unit area (the *density* of the charge) will vary with the form of the surface, as indicated by the above experiment.

The distribution of the charge is, of course, affected by earth connected and other conductors in the vicinity. Thus, if C (Fig. 154) be an earthed plate close to one side of the positively charged sphere AB, the density of the charge at B will be much greater than the density at A (instead of being uniform as on an *isolated* sphere), owing to the attraction of the induced negative on C: this can also be seen by considering the potentials at A and B as C is brought near.

Action of Points.—It follows that if a sharp point projects from a charged conductor the " density " of the charge at the point will be very great, and the electric force in the air just outside the point will therefore be great. Particles of air and dust in the vicinity may therefore be attracted, become charged by contact with the point, and then repelled, carrying the charge with them, so that there is a continual discharge (called convection discharge) from the point, and the conductor gradually loses its charge. Moreover, in the case of a heavily charged body the density at the point and the electric force may be so great that the air becomes " ionised " (Art. 2), *i.e.* electrons may be torn out of air atoms leaving positive ions, and in colliding with other atoms may drive out other electrons producing other positive ions, and so on. The whole business is that electrons and positive ions (and some negative) are formed in the air: those " particles " with a like charge stream away from the point: those with opposite charge are drawn up towards it, neutralising the charge at the point and discharging the conductor.

Many experiments illustrate the above. If a pin be soldered to the cap of an electroscope and the latter be charged, the leaves will fairly quickly collapse owing to the discharge from the pin point. Again, if a positively charged body be held over the cap of this electroscope it will of course act inductively so that the cap will be negative, the leaves positive, and the latter will be diverging: if the charged body be held over the cap for a few seconds, some of the negative at the cap will be discharged from the point so that when the body is taken away there will be more positive at the leaves than is necessary to neutralise the negative at the cap, and the electroscope will be charged positively, *i.e.* the same as the inducing charge.

The last experiment shows the method of "using points to collect the charge from a charged conductor," as it is often worded. Suppose C (Fig. 155) is a positively charged conductor and A is another conductor with several sharp points facing C. The conductor C acts inductively on A, causing the points to be negative and the other parts of A positive. This negative is "taken away from the points by convection discharge" on to the surface of C, neutralising the positive there, and leaving A positively charged. The final effect is thus the same as if the points on A directly "collected" C's positive charge.

Note that if a sharp point projects from the inside of a charged pot, nothing particular happens, for there is no charge there. But if a charged body inside is acting inductively on the pot, then the previous effects come on.

Practical applications of the action of points are encountered in certain electrical appliances, *e.g.* in electrical machines (Art. 12) and in lightning conductors (page 260). Again, if a large needle, connected to the cap of a very delicate electroscope, be carefully insulated in the open air (some distance above the ground) it will be found that the leaves will, in general, gradually diverge. The needle point "collects" the charge from the air around it, and the divergence of the leaves indicates the kind and amount of the electrification of the air at the point where the needle is placed. When this electrification is very great, a convection discharge often gives rise to a luminous glow diverging from the discharging point. This effect is often seen from the points of ships' masts, flagstaffs, etc., and has been called St. Elmo's fire. *Why* the discharge produces a *luminous* effect is explained later—it is really due to that "recombination" of positive ions and electrons in the ionised air casually referred to in Art. 2.

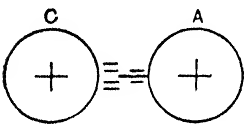


FIG. 155.

Note in passing that *convection discharge* depends on *density of charge*, and it begins when the density is about eight electrostatic units of charge per sq. cm. (see Chapter VI.): "sparks" from a charged body depend on the *potential*. But this is dealt with later.

8. Further Charge and Potential Facts about Hollow Conductors

(1) POTENTIAL OF AIR INSIDE A CHARGED METAL POT.—We have seen that when a hollow conductor (metal pot, say) is charged the charge is on the outside, but the whole pot, inside and outside,

are at the same potential. Further, *the air inside the pot (at any rate to within a short distance of the open top) is at the same uniform potential as the pot itself.* This can be proved by joining a proof plane by a wire to an electroscope some distance away, placing the proof plane on the pot, and moving it all over the surface inside and outside and moving it about in the air inside: the leaves diverge by the same amount all the time. Again, if an electroscope be put *completely* inside a charged metal pot there is no divergence. The case of the electroscope has the potential of the pot: the cap and leaves are in contact with the air inside and have the potential of the air. As there is no divergence the air potential must be the same as that of the pot. (If the cap projects outside the pot, the cap and leaves take the potential of the air *outside* and the leaves diverge.)

The fact that the air inside is at a *uniform* potential shows that there is no "electric force" inside a hollow charged conductor (if there are no charged bodies *inside*), for in order to have an electric

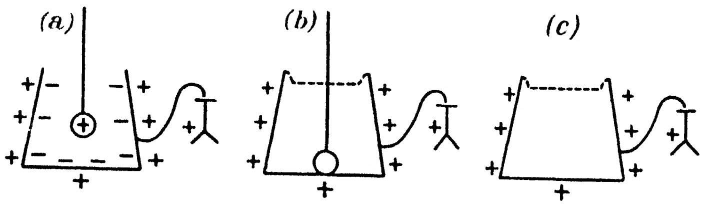


FIG. 156.

force between two points there must be a *potential difference* between them. (See electric screens, Art. 11.)

(2) FARADAY'S ICE-PAIL EXPERIMENT.—In this Faraday used an ice-pail, but any metal pot—preferably of depth at least 2½ times the diameter—will answer the purpose. In the experiment we start off with a positively charged brass ball and a neutral metal pot, the latter standing on wax and joined to an electroscope.

(*a*) Lower the positive ball into the pot: the leaves begin to diverge. When the ball is well inside the leaves have reached their greatest divergence and any movement of the ball about inside does not alter the divergence. On testing with a proof plane, the inside of the pot shows a negative charge and the outside a positive charge (Fig. 156 (*a*)). The ball is, of course, acting inductively on the neutral pot in the usual way.

(*b*) If the ball is taken out without touching the pot the leaves collapse and the pot becomes neutral: the induced negative on the inside and positive on the outside were equal and neutralised when the ball was taken away.

(*c*) Lower the ball again into the pot until the leaves are at their greatest divergence. Now let the ball touch the inside of the pot. *The leaves remain at their greatest divergence.* Take the ball out and test it: it has lost its positive charge. Test the inside of the pot: there is no negative charge there now. The outside of the pot still has its positive (induced) charge (Fig. 156 (*c*)). We conclude therefore that the positive on the ball and the negative induced on the inside of the pot have exactly neutralised each other and must therefore have been equal in amount. And we know that the induced negative on the inside and positive on the outside are equal. Hence in this experiment the induced negative charge on the inside is equal to the induced positive on the outside, and both are equal to the inducing positive charge on the ball. Strictly, this is only absolutely true if the enclosure, *i.e.* the pot, *completely surrounds* the ball.

Note that in Fig. 135 the induced charges on AB were certainly equal, but both were *less* than the positive on C (the balance of the induced negative due to C was on the walls, etc., of the room).

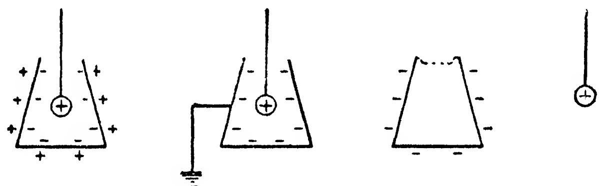


FIG. 157.

Again, in Fig. 143 when AB was earthed *more* electrons flowed into it than was necessary to neutralise the induced positive at B: in the present experiment, however, if the pot be earthed while the ball is hanging inside, just sufficient electrons to neutralise the outside induced positive flow to the pot.

(3) FURTHER EXPERIMENTS.—Three additional experiments on hollow conductors are also instructive:—

(*a*) Lower a positive ball into a neutral pot: it acts inductively on the pot as shown (Fig. 157 (*a*)). Here the air inside is not, of course, at a uniform potential: there is a down gradient of potential from the positive potential of the ball to the slightly lower induced positive potential of the pot. Earth the pot: the outside positive disappears (Fig. 157 (*b*)). Take the ball out without letting it touch the pot: the ball still has its positive charge, and the negative is now on the outside, the inside having no charge (Fig. 157 (*c*)).

(*b*) Now consider Fig. 158 where an *uncharged* ball has been hung in a positive pot. At (*a*) the positive is on the outside of the pot and the whole

pot, air inside, and ball are at the same positive potential: the ball shows no charge and no inductive displacement for all parts of it are at the same potential. At (*b*) the ball has been earthed and brought to zero potential by electrons coming to it from earth, so that it has a negative charge: most of the pot's positive now appears on the inside (as near as possible to the ball's negative), and the pot's positive potential is considerably lowered (nearly to zero) by the induced negative potential due to the ball. At (*c*) the ball is taken out (earth connexion being first removed), and it has a negative charge and negative potential: the positive is again on the outside of the pot and its potential is positive as at first.

(*c*) The metal pot may be used to show that equal and opposite charges are produced in electrification by rubbing. The body rubbed (say, vulcanite) and the rubber (fur on an insulating handle) are enclosed in the pot during the rubbing: an electroscope joined to the pot gives no divergence, but if either the body or the rubber be removed the leaves diverge and collapse again when they are replaced.

One further point may be noted. In Art. 6 we saw that when a charged ball was lowered into a neutral pot and touched the inside its charge disappeared and an equal like charge appeared on the

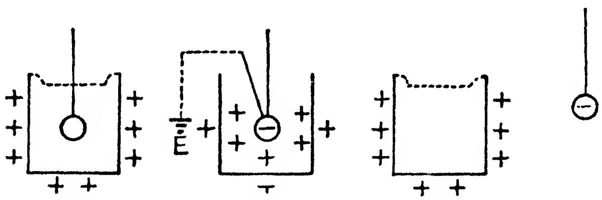


FIG. 158.

outside of the pot. The student will now see exactly what happens from Fig. 156: the positive ball acts inductively on the pot, and when the ball touches, the induced negative on the inside and the positive on the ball neutralise, which means as stated in Art. 6 that electrons have passed from pot to ball.

9. The Electrophorus

This is an early type of instrument, still used to a certain extent in simple experiments, which enables a large charge to be obtained from the one rubbing of a sheet of, say, vulcanite with fur: it is an application of the principles explained in Art. 5.

Typically the electrophorus consists of an ebonite *disc* E (Fig. 159) mounted on a metal base S called the *sole*, and a circular *plate*

of brass B fitted with an insulating handle: *the sole* S *must be earthed when the instrument is in use.* The method of working is as follows:—

Electrify the top surface of E by rubbing with fur. The negative on E creates a negative potential in the space about it, and acts inductively on the walls of the room and other conductors, the greatest induced positive being, however, on the metal sole which is of good conducing material and nearest to the inducing negative with a good dielectric (vulcanite) in between.

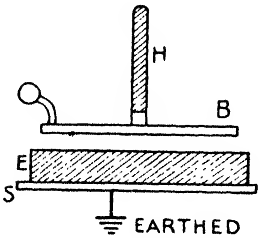


FIG. 159.

Now lower B on the disc (Fig. 160). As it approaches its under side gets a *small* induced positive charge and its upper side a *small* induced negative charge. When B finally rests on E there is true contact at very few points so that, in effect, a narrow air space lies between the two: hence B has an induced positive charge on the under side and negative on the upper side, these charges being greater than they were as B was approaching but *still small*, for the bulk of the inductive action of E is still on the *earth-connected* metal sole S.

Touch B with the finger, thus making it at zero potential, the same as the sole. The negative on B disappears as electrons flow out to bring its potential to zero, but more flow out than is represented by the induced negative, so that the *positive on B is much greater than before earthing.* In fact E's inductive action is now practically *entirely* on B and S and of these B has the greater induced positive, for it is nearer the charge on E.

Remove the finger and then take away the plate B with its positive charge which can be given to other conductors as required. E's charge has not been interfered with (neglecting any small leakage), S returns to its initial state,

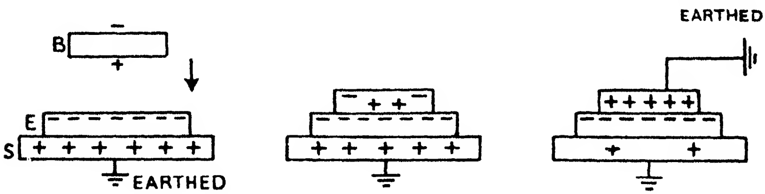


FIG. 160.

and the operation can be repeated thus obtaining a large positive charge from one rubbing of E.

Notice the importance of the sole and that it must be earthed: another point about it is that the attraction between its induced positive and E's negative reduces the tendency of E to lose its charge. Notice also, *in general terms*, the relative magnitudes of the induced

charges at various stages. Thus, suppose the charge on the disc be − 20, and let + 18 denote the induced charge on the sole, and therefore + 2 the induced charge on the walls, etc., of the room. On placing the plate on the disc and earthing the plate, the whole inductive action of the disc is concentrated on the sole and plate, but the latter will contain the greater portion of the induced charge. Hence the distribution may now be, say, + 17 on the plate and + 3 on the sole. On removing the plate the charge + 17 is removed with it, and the original distribution is again attained, + 18 on the sole and + 2 on the walls, etc., as before.

It looks at first as if this instrument was an example of " getting something for nothing," but, of course, it is not. The electrical *energy* possessed by the plate is equivalent to the work done in overcoming the mutual attraction between the negative charge on the disc and the positive charge on the plate when the latter is removed from the former.

10. Electric Fields. Lines and Tubes of Force and Induction

The ideas dealt with in this section are somewhat similar to those dealt with in the corresponding section on magnetism. *An Electric Field is the space surrounding an electrified body (or system of electrified bodies) within which the influence of the charge (or charges) extends.* When considering the forces in the field it is often spoken of as a *field of electric force*; when concentrating on the phenomena associated with induction it is often spoken of as a *field of induction.*

Take first the forces in the field. A free, small, positive charge placed at any point in the field will be urged by a definite force in a definite direction, which latter is indicated by the line of force passing through the point in question. **A line of electric force is a line or curve indicating the direction in which a free small positive charge would travel and it is such that the tangent at any point gives the direction of the electric force at that point.** The positive direction of a line is the direction in which the free *positive* charge tends to move, and as such a charge is always urged from a higher to a lower potential, *the positive direction of a line is the direction in which the potential is falling along it.* In the diagrams which follow, arrows indicate the positive direction of the lines.

Fig. 161 shows the lines of force in the case of a positively charged sphere in the centre of a large room. A negatively charged isolated sphere would give the same diagram, but any arrows would be pointing in the opposite direction. Fig. 162 represents the case of two equal conducting spheres having equal and opposite charges, whilst Fig. 163 depicts the case of two equal

positive point charges. Fig. 164 shows the lines in the case of two unequal positive charges, A of 20 units, and B of 5 units: here N is a **null or neutral point** (cf. pages 33-39), and it will be seen later that AN = 2BN. Fig. 165 represents the lines in the case of two unequal and opposite charges, A = + 20, B = − 5: here, again, N is a null point and AN = 2BN. Fig. 166 is an instructive diagram showing the modifications in the electric lines when the two equal, opposite, charged spheres (a short distance apart) are lowered in between the two charged plates: for simplicity some lines are omitted from the middle diagram.

Note particularly the following points:—(1) *Electric lines start from a positive charge and end on a negative charge somewhere* (*but* see Art. 17, page 197): thus, in Figs. 161, 163 they start from the positive charges and end on the induced negative charges on the walls, etc., of the room, whilst in Fig. 165 some of the lines from A end on B, the remainder on the walls, etc., of the room. (2) Electric lines **do not pass through a conductor** *but end on the surface*; the potential is uniform inside a solid conductor (or inside a hollow conductor if there are no charges inside) and, therefore, the *force* inside is zero: the field, in fact, is annihilated inside. (3) Lines of force cannot cross each other—see page 33. (4) *Lines of force leave* (*and end at*) *a charged conducting surface in a direction normal to the surface* (whatever path they may take in between): if not, there would be a component of the electric force along the surface and the charge would move across it.

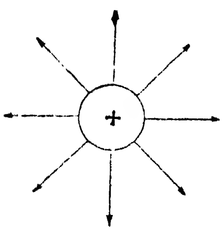


FIG. 161.

If we imagine that each line tends to contract longitudinally, whilst lines proceeding in the same direction tend to repel each other laterally, we have the Faraday explanation of the fundamental facts of attraction and repulsion (see again page 34): this is dealt with in Chapter VI.

One method of mapping the electric lines in an electric field is to fix, say, a circle of metal foil to the underside of a horizontal sheet of glass, join the circle to one pole of an electrical machine (page 155), and sprinkle oxalic acid crystals on the upper surface of the glass. The crystals are acted on inductively and set themselves on the lines just as the filings did in the case of the magnetic field. Other arrangements of conductor can be dealt with similarly.

As in the case of the magnetic field, so here it is usual to conceive the lines of force from a charge to be gathered together into bundles or tubes which touch each other laterally and fill the entire field.

These are called *tubes of electric force*, and when conceived on a definite plan (to be given later) so that a definite number are assumed to start at a given charge, they are called **unit tubes of**

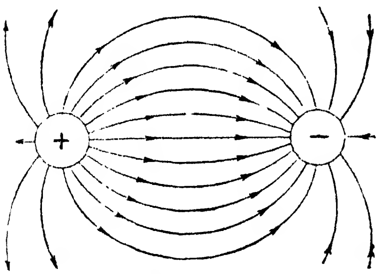


FIG. 162.

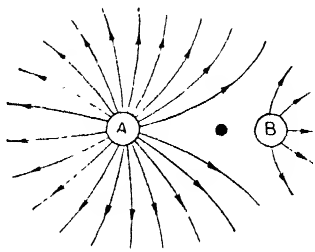


FIG. 164.

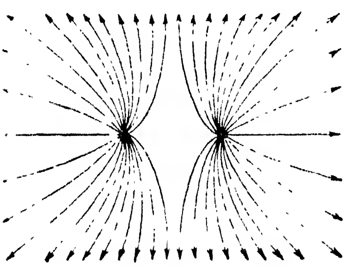


FIG. 163.

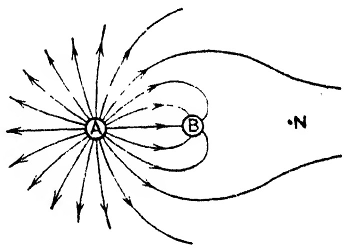


FIG. 165.

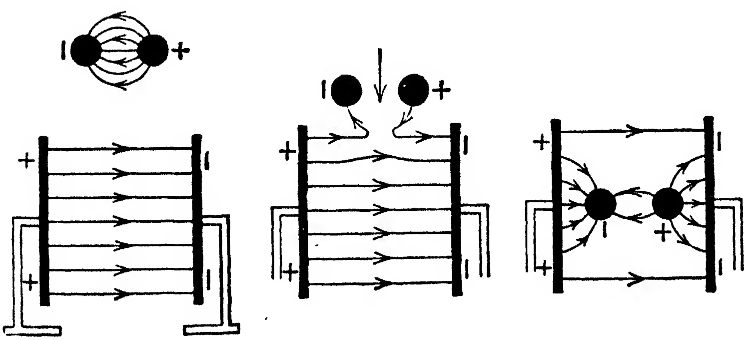


FIG. 166.

electric force. The greater the charge the more unit tubes are conceived to start from it, and the stronger the electric field the more unit tubes are assumed to be passing through a given area: in fact, as will be seen in Chapter VI., the convention adopted

leads to the result that the *intensity* (E) of the electric field is *represented numerically by the number of unit tubes of force passing through unit area.*

Electric lines and tubes of force, then, have reference to the electric forces and their directions in an electric field, but the early physicists went further than that (see again page 37). Just as magnets were regarded as producing magnetic strains in the aether, the magnetic lines indicating the direction of the strain and being in fact *magnetic strain lines in the aether,* so charges were regarded as producing electric strains in the aether of the insulating medium round about, the electric lines indicating the direction of the strain and being in fact *electric strain lines in the aether.* We refer to this again later.

We have seen that modern theory indicates that there are no free electrons in an insulator as there are in a conductor (Art. 2), so that when an electric force is applied there is no " flow " of electrons through the insulator as there is through the conductor. It is assumed, however, that the positive and negative charges (nuclei and electrons) in the atoms of the insulating material will be attracted in opposite directions and become slightly displaced relative to one another, negative displaced one way, positive the other: there is a " polarisation," a displacement of the electrical make-up of the atoms of the insulating medium surrounding a charged body, which soon reaches a limiting value (for the nucleus will not allow its electrons to leave the atom altogether), depending on the electric intensity at the point considered. There might be said to be a certain resemblance between the case of a dielectric and the case of an *elastically strained* solid, and electrical phenomena are sometimes said to be due to what is termed an *electric strain* in the dielectric medium.

Now when these " medium " effects and inductive influence in general are under consideration the field is spoken of as a *field of induction,* and the lines and tubes associated therewith as lines and tubes of induction, and when the tubes are conceived on a definite plan (to be given later) they are called **unit tubes of induction.** These tubes are *not* endless: they start from a certain positive charge and end at an *equal* negative charge " induced " on the walls of the room or other conductor. In accordance with the conception of strain this fact would mean that the dielectric medium in the tube (or the aether of the medium) was in a state of strain, and that the surface effects associated with this at each end of the tube were equal in magnitude but opposite in sense. Carrying

the analogy with elasticity further, the energy of the electrification, which appears where the end charges are, would on this conception, be taken to lie in the polarised and "strained" medium in the tube.

The conventions actually adopted with regard to unit tubes lead to the fact that, whilst tubes of force are associated with (and, by their number, *measure*) the forces in a field, tubes of induction are associated with the displacement and the charges (and, by their number, they *measure* the induction or the displacement or "strain"), but these points will be more clearly understood from the next chapter: thus the property often referred to, that a tube in air has equal and opposite charges at its ends, is strictly the property of a tube of induction, and is true for tubes of force because *in air* the tubes of force are also the tubes of induction.

Fig. 167 depicts the field of induction in the case of a neutral conductor AB put near the positive ball C. The field in the space now occupied by AB is annihilated and the tubes of induction redistribute themselves as shown. Where the tubes end on A there is −ve electrification: where fresh tubes start from B there is +ve, and where these tubes end on the walls, etc., there is −ve. The number of tubes of induction ending on A (and the −ve charge there) is equal to the number starting from B (and the +ve charge there). If AB be earthed the tubes of induction passing from B to earth disappear and most of those tubes which in Fig. 167 go direct from C to earth now end on A, choosing the shortest distance from C to an earth joined body: a greater −ve charge is now at A.

From the point of view of "strain" in the medium the induction above is said to be a question of the redistribution of strain and strain energy *in the medium* round C consequent upon the annihilation of strain in the space occupied by the conductor AB. The result of this redistribution is that negative electrification is developed on AB at A, and positive electrification at B, and strains in other parts of the field are modified.

Fig. 168 depicts the tubes of induction in the various stages of charging an electroscope by induction, and Fig. 169 indicates the tubes of induction in the process of charging an electrophorus and then allowing the plate to "share its charge" with a conductor C: the student will be able to make out the explanations for himself.

Fig. 170 (*a*) shows the tubes of induction in the case of a charged sphere A inside an insulated hollow conducting sphere B, but not situated at the centre of the latter. The distribution of the induced charge inside is irregular, as shown, but outside the distribution is uniform. The number of tubes of induction leaving the outside surface is equal to the number terminating on the inside surface, *i.e.* is equal to the number emanating from A (B entirely surrounds A here). An additional charge may be given to B, or the outside charge on it may be removed (Fig. 170 (*b*)) without affecting the inside distribution.

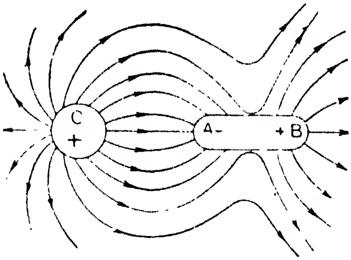


FIG. 167.

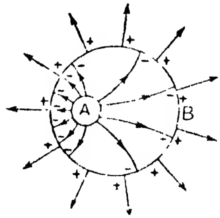


FIG. 170 *a*.

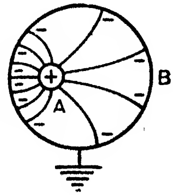


FIG. 170 *b*.

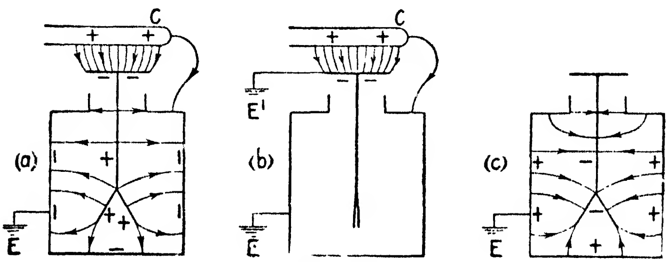


FIG. 168.

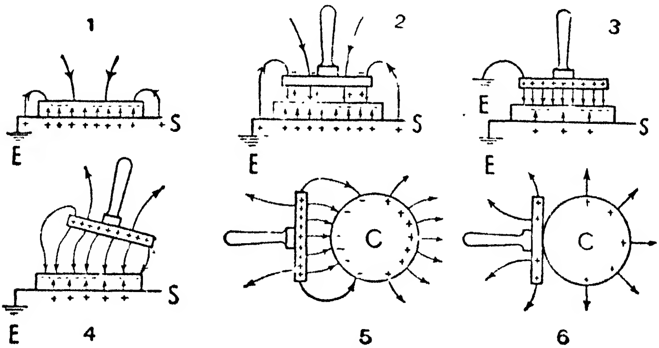


FIG. 169.

11. Electric Screens

The action of conducting sheets and cans as " screens," *e.g.* in wireless receivers and transmitting circuits, and in test-room work involving the use of exceptionally high potentials, etc., referred to in Art. 8, will now be understood. We have seen that there is no electric force inside a charged can for the whole space inside is at a *uniform* potential. Now imagine a metal can *earthed*. If a charge

is *inside,* the electric lines from it *end on the inside* of the can, and there is no force due to it outside so that any apparatus outside is screened from the charge inside. Similarly, if there is a charge (static) somewhere *outside* the earthed can, the electric lines from it end on the outside and there is no force due to it inside, so that any apparatus inside is screened from it. (*Magnetic* fields—due to electricity *in motion*—require *iron* to act as screens, page 89.)

12. Electrical Machines

To quickly obtain large charges many "electrical machines" were invented, and one which is still much used is the *Wimshurst.* Another good machine is the *van der Graaf.* We will, however, deal with the Wimshurst, as it is the one the student will mainly encounter.

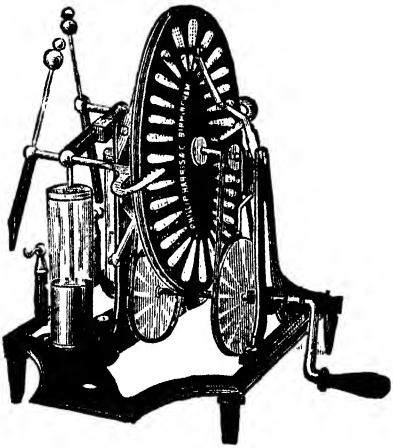


FIG. 171.

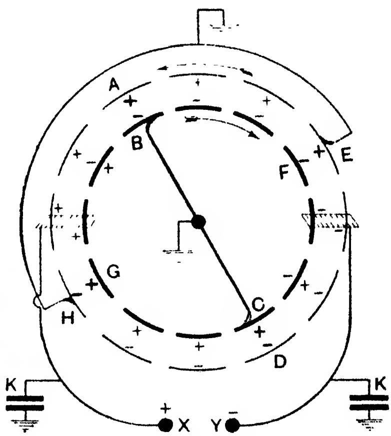


FIG. 172.

(1) THE WIMSHURST.—Fig. 171 shows a simple type of the actual machine, and Fig. 172 will serve to explain its action. It consists of two vulcanite or shellac-varnished glass plates which can be rotated in opposite directions about a horizontal axis. On the sides of the plates remote from each other are fixed a number of metal sectors; in Fig. 172 the sectors on the front plate are represented by the inner broken circle, those on the back plate by the outer and thinner broken circle. Standing at each side is a U-shaped conductor, each carrying on the inside two rows of sharp points (called the combs), facing the plates; these are connected to the "poles" or "dischargers" of the machine X, Y, and to the

inside coatings of two "condensers" KK. (The object of these is to *increase the "capacity" or "capacitance," and make it possible to get heavy charges on* X *and* Y: this is explained in Chapter VI.) Lying across the front plate, at an angle of about 45° with the combs, is a metal rod BC, terminating in wire brushes which graze the sectors as the plate rotates. Across the back plate and about at right angles to BC is a second metal rod EH, terminating in brushes; for convenience this is drawn outside in Fig. 172.

To make the action clear we must deal, in what follows, with one plate at a time, but it must be remembered that *both* plates are moving (in opposite directions): hence when we refer, say, to "the sector H," we mean the sector which happens to be in the position H in the diagram at that instant.

Disregarding for a moment the "earthing" of the rods BC and EH, and imagining the sector A to have a slight +ve charge given to it, the action of the machine is as follows:—A acts inductively (through the two plates) on the conductor BC, causing the sector B to have a −ve charge and C a +ve one. Concentrating our attention on the front plate, B moves to the right with its −ve charge, and on reaching F acts inductively on EH, making the sector E on the back plate +ve and H −ve. Leaving F, the front sector moves into the comb on the right, where it again acts inductively, causing the points to become +*ve and the pole or discharger* Y −*ve.*

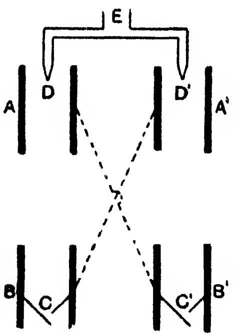


FIG. 173.

The density of the positive charge at the points is very great, so that, as explained in Art 7, page 143, the comb loses its positive charge, and the positive particles streaming away from it come in contact with the sector and neutralise its negative charge. Thus not only does the comb lose its charge, leaving the discharger Y −ve, but the sector comes out uncharged.

When the front sector reaches C it acquires a +ve charge, when it reaches G it acts inductively on the back plate, causing H to become −ve and E +ve, and when it enters the comb on the left, "actions of points" similar to those already explained take place, the result being that *the discharger* X *becomes* +*ve,* and the sector comes out uncharged. At B the sector becomes −ve, and the action is repeated.

Turning to the back plate, the sector E moves to the left with its +ve charge. At A it acts inductively on the front plate, making

the sector at B −ve and that at C +ve; it then enters the comb, with the result that X *acquires a further +ve charge.* On reaching H the sector in question becomes −ve, at D it acts inductively on CB, making C +ve and B −ve: it then enters the comb, with the result that Y *acquires a further −ve charge*, and the action is repeated. *Thus the rotation of both plates results in* X *acquiring a strong +ve charge and* Y *a strong −ve one.*

In the above we have assumed that a small charge is given to A; in practice the small difference of potential between different parts is usually sufficient to start the action. A little consideration will show that matters are improved by having BC and EH earthed.

Compound Wimshurst machines consisting of four, six, eight . . . plates are also on the market, and these larger makes are very powerful. But even an ordinary machine of average size will produce a P.D. of 40,000-50,000 volts. Intense sparks can be obtained between X and Y. Incidentally, as the action of the machine depends on the ionisation, etc., of air at the combs, it would not work in a vacuum.

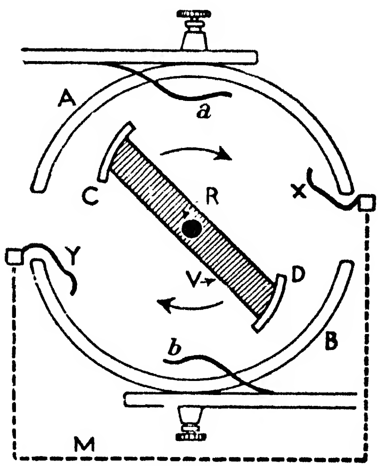


FIG. 174.

(2) THE WATER DROPPER AND REPLENISHER.—Of the other machines brief reference will be made to two—both due to Kelvin—as they specifically illustrate principles dealt with in this chapter.

(*a*) The *Water Dropper* is represented in Fig. 173. It consists of four insulated metallic cylinders A, A′, B, B′, connected in pairs as indicated, the lower ones B, B′ being fitted with funnels C, C′; D, D′ are the fine nozzles of a metal pipe E, earth-connected, from which water is permitted to issue in drops.

Let A (and therefore B′) be given a slight positive charge, and imagine a drop of water just issuing from the nozzle D. Being earth-connected and within a hollow positively charged conductor, the drop acquires a negative charge (Exp. (*b*), page 146) which it gives up to B (and therefore A′) when it strikes the funnel C; this is repeated by each drop issuing from the nozzle D.

Consider now a drop of water just leaving D′. Since A′ is negative the drop acquires a positive charge which it gives up to B′ (and therefore A)

when it strikes the funnel C′; this is repeated by each drop issuing from the nozzle D′. Thus the positive charge on one system (AB′) and the negative charge on the other (BA′) tend to increase until leakage interferes, or B and B′ become so strongly charged that the drops are repelled and do not enter.

(*b*) The *Replenisher* is shown in Fig. 174. A and B, termed the *inductors*, are two portions of a metal cylinder each fitted with a *contact spring* (*a* and *b*). Two other springs X and Y project into A and B and are connected together by a strip of brass (represented by M in Fig. 174). Two part cylindrical metal pieces C and D, called the *carriers*, are fixed to an insulating bar V and capable of rotation about the axis R, in the direction indicated by the arrow.

Suppose A is given a small positive charge and that the spindle is turned until C touches X and D touches Y. The carrier C becomes negative and D positive (inductive effect of A's positive on the connected system of conductors CXMYD). When C touches *b*, D also touches *a* with the result that C "imparts" its negative charge to B and D its positive charge to A. When C and D again touch the springs X, Y, both inductors act so as to charge D negatively and C positively. When D reaches *b* and C reaches *a*, D imparts its negative charge to B and C its positive charge to A, and the actions are repeated as rotation continues. Thus the rotation results in A acquiring a strong positive charge and B a strong negative one.

A practical application of the water dropper is referred to in Chapter VIII. in connexion with atmospheric electricity. The replenisher was really not originally intended to produce large charges, but for use with the Kelvin electrometer (Chapter VII.).

An important electrical machine of comparatively recent introduction is **the Van der Graaf generator,** but space forbids its description in this book.

Other types of electrical machines are on the market.

CHAPTER VI

ELECTROSTATIC UNITS AND THEORY

IN this chapter the main units and fundamental theory in connexion with the subject of electrostatics will be dealt with. It will be noted that a good portion of the work in the earlier part of the chapter is very similar to, and mathematically the same as, that for magnetism, and the two sections should be compared and associated.

1. Quantity of Electricity. Units of Quantity

The method of defining the unit of quantity used in electrostatics—the *absolute* or *C.G.S. electrostatic unit*—is similar to that used in defining the unit pole (page 56). Investigation shows that if two "charges" be concentrated at two points *the force between them varies directly as the product of the charges and inversely as the square of the distance between them.* Thus if Q_1 and Q_2 denote the charges, d the distance apart, and F the force between them:—

$$\text{F is proportional to } \frac{Q_1 \times Q_2}{d^2}; \quad \therefore \ F = \beta\frac{Q_1 \times Q_2}{d^2},$$

where β is a factor depending only on the medium in which the charges are placed, and on the units we decide to adopt.

Now let the charges be in *vacuo* (or what is practically the same from this electrical point of view, in *air*), and let F be in dynes and d in centimetres. As in the corresponding case in Magnetism, it will be convenient to choose the unit charge so that β above is unity, and this will be so if *unit quantity be taken as that quantity which when placed one centimetre in vacuo (or air) from an equal quantity acts on it with a force of one dyne,* for if in the equation above $Q_1 = Q_2 = 1$ (unit quantity), $d = 1$ (cm.), and $F = 1$ (dyne), then $\beta = 1$. Hence adopting the above as our unit quantity, the force *in air* between two charges Q_1 and Q_2 situated d cm. apart is:—

$$F = \frac{Q_1 \times Q_2}{d^2} \text{ dynes} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The above unit is known as the "absolute or C.G.S. electrostatic unit quantity." Hence **the C.G.S. electrostatic unit quantity is such that if placed one centimetre in vacuo (or in air) from an equal and**

like quantity repels it with a force of one dyne. The e.s. unit has no specific name.

Another quantity unit is employed in current electricity. It is known as the "absolute or C.G.S. electromagnetic unit quantity," is equal to 30,000,000,000, *i.e.* (3×10^{10}) electrostatic units, and is defined and explained in Chapter X.: this e.m. unit has no specific name. (Note, in passing, the number (3×10^{10}): we come to it again later.)

When electricity began to be used in a practical way it was found that the absolute units were either too large or too small for the convenient measurement of the various electrical quantities in daily use, and a system of *practical units* was devised, such units being simply related to the corresponding *electromagnetic* units by some power of 10. Thus the practical unit of quantity is known as the **coulomb** (after the scientist Coulomb), and it is $\frac{1}{10}$ (or 10^{-1}) of the C.G.S. electromagnetic unit: thus the coulomb is $\frac{1}{10}$ of (3×10^{10}), *i.e.* (3×10^{9}) electrostatic units. Note that the e.s. unit quantity is small compared with the other two.

The smallest possible charge, viz. the electron, $= 4{\cdot}77 \times 10^{-10}$ e.s. unit $= 1{\cdot}59 \times 10^{-20}$ e.m. unit $= 1{\cdot}59 \times 10^{-19}$ coulomb.

In (1) above the charges are *in vacuo* (or *air*). For any other medium the factor β must be introduced: β is really equal to 1/K where K is the **dielectric constant or permittivity** of the medium: this is defined later. Hence the more general formula is:—

$$F = \frac{1}{K}\frac{Q_1 \times Q_2}{d^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

For a vacuum K is taken as *unity*, *i.e.* the dielectric constant of a vacuum is taken as the unit, and when we say that the dielectric constant of a substance is 20 we mean that it is 20 times the dielectric constant of a vacuum. On this assumption that $K = 1$ for a vacuum, K for air $= 1{\cdot}00058$, which is not so very different from unity. From (1) and (2) above it is clear that the dielectric constant of a medium may be defined as *the ratio of the force between two charges in vacuo (or air) to the force between the same two charges at the same distance in the medium.*

Another point may be briefly noted at this stage. In defining the e.s. quantity unit above the dielectric constant K of vacuo was taken as unity, and on this assumption the *absolute or C.G.S. electrostatic system of units* is based. In defining unit magnetic pole (page 57) the permeability μ of vacuo was taken as unity and on this the *absolute or C.G.S. electromagnetic system of units* is based. It will be seen later, however, that these two assumptions, viz. that *both* K and μ *are unity* for empty space, are really incompatible. In fact, although K and μ are usually regarded merely as numbers and not *physical* quantities, they are associated with a definite *physical property*. This is dealt with later.

As indicated, (1) and (2) above are strictly true only for point charges, *i.e.* two charges concentrated at two points. In the case of two charged spheres it can be shown that *if the charge on each remained uniformly distributed each sphere would act as if its charge were concentrated at its centre,* and the formula would apply, d being the distance between the centres. In practice, however, each charge disturbs the distribution of the other, so that the formula above does not give the actual force; but the error is small if the spheres are small compared with their distance apart.

An interesting (and important) fact about, say, the attraction between electric charges is that the force does not act really instantaneously. Imagine two charged spheres a distance d cm. apart, and that one is suddenly displaced: the force on the other would not alter instantaneously, but after a time d/v sec. where v is the *velocity with which the electrical disturbance travels.* In practice, however, v is great: it is about (3×10^{10}) cm. per sec., and it is interesting to note that this is also the *velocity of light in empty space.* (As a matter of fact light is fundamentally an electro-magnetic phenomena.)

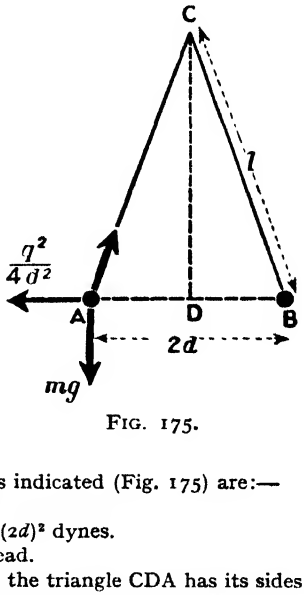


FIG. 175.

Example.—*Two small spheres, each of mass m grm., are suspended from a point by threads, each l cm. long. They are equally charged, and repel each other to a distance of 2d cm. If g = 981, express the charge on each in electrostatic units, electromagnetic units, and coulombs.*

The forces on the sphere A in the directions indicated (Fig. 175) are:—

(1) The weight, mg dynes.
(2) The repulsions, viz. $q^2/(2d)^2$ dynes.
(3) The tension of the thread.

An examination of the figure will show that the triangle CDA has its sides parallel to these forces; hence the forces are proportional to the sides of this triangle to which they are parallel. Thus,

$$\frac{\frac{q^2}{4d^2}}{mg} = \frac{\text{DA}}{\text{CD}} = \frac{d}{\sqrt{l^2 - d^2}}; \quad \therefore q^2 = \frac{4d^3mg}{\sqrt{l^2 - d^2}},$$

$$i.e.\ q = 2d\sqrt{\frac{dmg}{(l^2 - d^2)^{\frac{1}{2}}}} \text{ e.s. units.}$$

In electromagnetic units the charge is $q/(3 \times 10^{10})$, and in coulombs $q/(3 \times 10^9)$.

2. Electrical Potential. Units of Potential

In Chapters I. and V. electrical potential was dealt with in a general sense: we come now to the principles on which the measurement is based, viz. its relation to work and energy.

Consider for convenience a negatively charged brass ball, and imagine a single electron detached from it. This electron will be repelled by all the other electrons forming the "charge" and will set off with a certain initial energy under the repulsive force. Clearly the greater the charge on the ball the more electrons will be repelling the detached one, and the greater will be the initial force on it. Further, if the *same charge* had been on a smaller ball the initial force on the detached electron would have been greater for the electrons would have been more closely packed on the smaller ball, and those on the far side would have been nearer and more effective in the repulsion. Thus the electron acquires more energy in being repelled away from a strongly charged ball than from the same ball with a less charge, and it acquires more energy in being repelled from a small ball than from a large ball with the same charge.

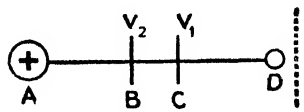


FIG. 176.

We describe the above by saying that a big charge on a body is at a "higher" electrical potential than a small charge, and that a given charge is at a higher electrical potential if it is concentrated on a small body than if it is spread over a large one. Clearly, the "electrical potential" of the charged ball determines the energy with which a portion, say, of its charge would tend to move away from it, and determines also the work which would be necessary to bring that portion back again against the repulsive force.

Of course, remember that the *greater* the *negative* potential the "lower" is the actual potential of the body, *i.e.* the more it is *below* zero: but this is a detail.

Now let A (Fig. 176) be a positive body, no other charge being near. On page 135 it was indicated that the potential falls from its positive value at A to zero at infinity. (In practice we take the earth as zero potential.) Imagine D to be a positive e.s. unit charge at infinity. To bring this unit charge from infinity to C, work must be done against the repulsion of A, more work must be done in bringing it to B, and still more work in bringing it to A. The work so done has its equivalent in the potential energy gained in virtue of the change in relative position of A and D.

If W_1 *ergs of work be done in moving the positive unit from infinity to* C, *then* W_1 *electrostatic units is said to be the potential at this point* C, *due to the charged body* A*: if* W_2 *ergs of work be done in moving the positive unit from infinity to B,* W_2 *electrostatic units is the potential at B, and the potential difference between B and C is* $(W_2 - W_1)$ *electrostatic units: the latter is evidently numerically equal to the work in ergs in moving the positive unit from C to B.* Finally, W_3 *electrostatic units is the potential of the charged body A if* W_3 *ergs of work be done in moving the positive unit from infinity up to it* (compare with page 59).

If the charge at A be negative instead of positive, *then no work would have to be done* in bringing up the unit *positive* charge, but the force of attraction *would do work* in so bringing it up. Hence, the potential at A, B, or C is said to be negative.

To summarise:—*The electrical potential at any point in the field surrounding a charged body is represented numerically by the work done on or by a positive unit charge in moving from infinity, regarded as the zero of potential, to the point in question: the P.D. between two points is represented numerically by the work done on or by a positive unit charge in moving from one point to the other.* Hence **the electrical potential at a point is one C.G.S. electrostatic unit if one erg of work is done on or by a positive electrostatic unit charge in moving from infinity to that point:** similarly *the P.D. between two points is one C.G.S. electrostatic unit if one erg of work is done when a positive e.s. unit charge moves from one point to the other.* This e.s. unit has no specific name.

Another potential unit is employed in current electricity: it is known as the "C.G.S. electromagnetic unit of potential." One electrostatic unit of potential is equal to (3×10^{10}) electromagnetic units. (Note again the $(3 \times 10^{10}$.) The e.m. unit is defined and explained later: it has no specific name.

The practical potential unit is the volt (named after the scientist Volta) which is equal to $\frac{1}{300}$ of an electrostatic unit: thus the volt is $\frac{1}{300}$ of (3×10^{10}), *i.e.* 10^8 electromagnetic units. The volt also is defined later. Note that the e.s. unit of potential is *greater than* the e.m. unit and the volt.

The warning given on page 61 with regard to magnetic potential may be repeated here. The electrical potential is *represented numerically* by the work in ergs in moving unit positive charge: if the work per unit charge is W *ergs*, the "electrical potential" is W *units of potential.*

Again, let V_1 = potential at C (Fig. 176) and V_2 = potential at B; $\therefore V_2 - V_1$ = P.D. between B and C = work done (ergs) in moving a positive e.s. unit charge from C to B. Now let C and B

be close together, and let F dynes be the force on unit charge in the *small* distance CB: then work done (ergs) in moving the positive unit from C to B = F × distance BC. Hence:—

$$F \times BC = V_2 - V_1; \quad \therefore F = \frac{V_2 - V_1}{BC}.$$

Now the right-hand expression is the **potential gradient** between C and B (page 61), and F the force *in dynes* on the positive unit e.s. charge measures the intensity E of the electrical field there in e.s. units as will be seen in Art. 8. Hence writing dv for the small P.D. $V_2 - V_1$ and dx for the very small distance BC we get:—

$$E = -\frac{dv}{dx}, \text{ i.e. } dv = -E dx,$$

the $-$ve sign indicating that the potential V *decreases* as the distance x from the positive charge at A *increases*. Hence (cf. page 61) we can say **the electrical potential at a point is that quantity whose rate of change with distance in any direction is numerically equal to the intensity of the electrical field in that direction.**

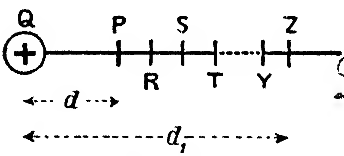


FIG. 177

9. ELECTRICAL POTENTIAL AT A POINT DUE TO A CHARGE.—An expression for this may be obtained in the same way as was done for magnetic potential (page 79). The point P at which the electrical potential is required is d cm. from the charge + Q e.s. units. P, R, S, T . . . Y, Z are points *very close* together, Z being at distance d_1 cm. from Q (Fig. 177). The medium is air (strictly *vacuo*).

$$\text{Force on unit charge at P} = \frac{Q}{OP^2}; \text{ Force at R} = \frac{Q}{OR^2};$$

$$\therefore \text{ Average force between P and R} = \frac{Q}{OP.OR},$$

and thus the work done in moving a positive unit charge from R to P will be given by:—

$$\frac{Q}{OP.OR} \times RP = \frac{Q}{OP.OR} \times (OR - OP) = \frac{Q}{OP} - \frac{Q}{OR}.$$

Similar expressions will be obtained for each of the other short steps between R and Z (see again page 79). On adding all these

together we get the total work done in moving a positive unit charge from Z to P, viz.:—

$$\left(\frac{Q}{OP}-\frac{Q}{OR}\right)+\left(\frac{Q}{OR}-\frac{Q}{OS}\right)+\ldots\left(\frac{Q}{OY}-\frac{Q}{OZ}\right)=\frac{Q}{OP}-\frac{Q}{OZ}.$$

But the work done in moving a positive unit charge from Z to P measures the electrical P.D. between the two points: hence

$$\text{P.D. between P and Z} = \frac{Q}{OP}-\frac{Q}{OZ}=\left(\frac{Q}{d}-\frac{Q}{d_1}\right)\text{ e.s. units} \quad ..(3)$$

If Z be at infinity, Q/d_1 is zero, and the work in moving the positive unit from infinity to P $= Q/d$. But this measures the potential at P: hence for the potential at a point P distance d cm. from a charge $+Q$ e.s. units we have:—

$$\text{Electrical potential at P} = \frac{Q}{d}\text{ e.s. units} \quad \ldots\ldots\ldots(4)$$

If Q be negative the potential at P is $-Q/d$. If the medium is not air or vacuo but one of dielectric constant K, then for the force on unit charge, say at P, we would have $Q/K\,(OP^2)$: making these changes in the mathematics we get for the potential at P the expression $\pm Q/Kd$ e.s. units.

A proof is neater if the Calculus be used (see page 80):—

$$V_2 - V_1 = -\int_{d_1}^{d} E dx = -Q\int_{d_1}^{d}\frac{1}{x^2}dx = -Q\left[-\frac{1}{x}\right]_{d_1}^{d} = Q\left(\frac{1}{d}-\frac{1}{d_1}\right)$$

and if $d_1 = \infty$ then $1/d_1 = 0$, and the potential at distance d is Q/d: the medium is of course air.

In the case of a *uniformly charged conducting sphere* of radius r cm., no other charge being near, the lines of force diverge as if from a point charge *at the centre* (Fig. 161), and in fact the sphere acts at external points as if its charge were concentrated at its centre (Art. 1). Hence if Q be the charge, the potential at an external point distant d cm. *from its centre* is Q/d or Q/Kd, and **the potential of the sphere itself is Q/r or Q/Kr.** Whether the sphere be hollow or solid the latter expressions give the potential not only of the outer surface *but also of the whole interior*, provided, of course, there is no charged body inside in the case of the hollow sphere.

Potential is a scalar quantity; the potential at a point due to a number of charges is simply the algebraic sum of the potentials due to each.

Example.—*Within a spherical vessel of brass 1 cm. thick, the external diameter of which is 14 cm., a brass ball 8 cm. in diameter is hung by a silk thread so that the centres of the two spheres coincide. If the ball is charged with + 36 units of electricity, and if the potential of the vessel is 7 units, what is the potential of the ball?*

The three radii are 4 cm., 6 cm., and 7 cm.

If Q be the charge on the outer surface its potential will be Q/7; but the potential is 7;

$$\therefore \frac{Q}{7} = 7, \textit{ i.e. } Q = 49 \text{ units.}$$

The charge on the ball is + 36, therefore the charge induced on the inner surface of the vessel is − 36. The potential of the ball is the sum of the potentials due to its own charge and the charges on the vessel; thus if V be the potential of the ball—

$$V = \frac{36}{4} - \frac{36}{6} + \frac{49}{7} = 10 \text{ e.s. units.}$$

Note.—In the above the outer vessel has an independent charge. If the outer vessel is merely acted on inductively by the charge on the ball the outer charge would be + 36, the *potential of the ball* would be

$$V = \frac{36}{4} - \frac{36}{6} + \frac{36}{7} = 8\tfrac{1}{7} \text{ e.s. units,}$$

and *the potential of the vessel* would be $36/7 = 5\tfrac{1}{7}$ e.s. units.

If the outer vessel were earthed the charge + 36 on its outer surface would disappear, its potential would, of course, be zero, and the potential of the ball would be

$$V = \frac{36}{4} - \frac{36}{6} = 3 \text{ e.s. units.}$$

FACTORS WHICH AFFECT THE POTENTIAL OF A CHARGED CONDUCTOR.—From the experiments of Chapter V. and the results of the present section it will be seen that the potential of a charged body is affected by several factors. Briefly the main facts are:—(1) The greater the *charge* the "greater" the potential. (2) The less the *size* of the conductor the "greater" will be the potential due to a given charge. (3) The potential depends on the *dielectric medium* being less the greater the value of K. (4) The potential is affected by *neighbouring conductors and charges.*

(*a*) Facts (1), (2), and (3) can also be seen from the expression for the potential of a charged sphere, viz. $V = Q/Kr$.

(*b*) Fact (2) could also be shown experimentally as follows:—Put a *large* metal pot on the cap of an electroscope, charge a metal ball positively, lower it into the pot, letting it touch the inside, and note the divergence. Now discharge the electroscope, put a *small* metal pot on the cap, give the ball an equal charge to what it had before, lower it into the pot, and note the divergence. The small pot gives the bigger divergence, showing that with the same charge the small pot (*plus* leaves, etc.) is at the higher potential.

(c) Fact (3) could also be shown experimentally thus:—Join a charged metal ball to an electroscope by a wire and note the divergence. Now discharge both, give the ball an equal charge, immerse it in paraffin wax ($K = 2$), and join to the electroscope. The divergence will be less, showing that the potential is less.

(d) Fact (4) was indicated experimentally in several cases in Art. 4 of the preceding chapter.

3. Equipotential Lines and Surfaces

An equipotential surface is the locus of all points having the same potential. An equipotential line is the line in which an equipotential surface is cut by a plane; in a diagram the equipotential lines are the lines in which the equipotential surfaces are cut by the plane of the paper.

Imagine a charge to be concentrated at a point A, then all points at the same distance from the charge have the same potential, *i.e.* the equipotential surfaces in this case are a series of concentric *spheres* having their centres at the point A, and the equipotential lines on a diagram are circles with A as centre. The potential at each of the spherical surfaces is different, but for all points on any one surface it is the same; that is, work has to be done to move a positive unit of electricity from one surface to another, but no work is done in moving it from any point on a given surface to another point on the *same* surface. From this it follows that *the lines of force are normal to the equipotential surfaces*, for if no work is done in moving electricity from one point to an adjacent one on an equipotential surface, then the direction of motion must be everywhere perpendicular to the lines of force.

It is convenient in drawing equipotential surfaces to represent them so that unit quantity of work must be done in conveying unit quantity of electricity from any one surface to the next, *i.e.* so that there is unit P.D. between them. When so drawn the distance between consecutive surfaces increases as the distance from the charge increases, for the greater this distance the weaker the force exerted by the charge, and therefore the greater must be the distance through which it has to be overcome to do unit work.

To map the equipotential lines for a positively charged isolated sphere.—Consider the case of a conducting sphere of 1 cm. radius (in air) charged with 12 units; the potential of its surface is ($V = Q/r$) 12 units, and *this is also the potential of all points inside it.* Since $r = Q/V$ the equipotential line 11 will be a circle of radius $r = 12/11 = 1{\cdot}1$ cm. (approx.). The equipotential 10 will be a circle of radius $r = 12/10 = 1{\cdot}2$ cm.; the equipotential 8 a circle

of radius 1·5 cm.; the equipotentials 6, 4, 3, 2, and 1 circles of 2, 3, 4, 6, and 12 cm. radius respectively. The equipotential zero is *strictly* at infinity. Fig. 178 represents this.

Fig. 179 gives the lines for two like charges in the ratio 4 : 1, the dotted curves being the equipotentials and the continuous ones cutting them at right angles the lines of force: these latter are the same as in Fig. 164.

Fig. 180 shows (roughly, to make the idea clear) the general effect on the shape of the equipotentials of bringing an insulated uncharged conductor B into the field of the charged sphere A. The former is acted on inductively and acquires a certain definite potential, assumed V in the figure; thus the whole conductor B forms part of the equipotential V in the figure, and the other equipotentials are distorted as shown. The point *a* has now a potential lower than V_1, whereas before the introduction of B its potential was higher than V_1; this is due to the fact that there is an induced negative charge at X. The point *b* has now a potential higher than V_2, whereas before the introduction of B its potential was lower than V_2; this is due to the fact that there is an induced positive charge at Y. If B in Fig. 180 be earthed it forms part of the equipotential zero, the potential of A is lowered and all the equipotentials between zero and the value for A are crowded in between A and B; the positive charge on B has disappeared and an increased negative charge has appeared at X. (In Fig. 180 the two horizontal lines are merely the top and bottom borders of the diagram.)

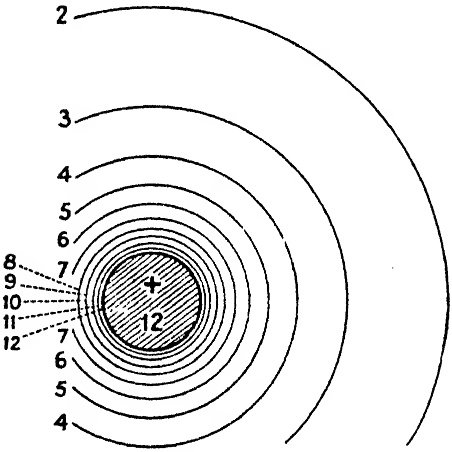


FIG. 178.

It follows that two equipotentials cannot cut each other for the point of intersection would have two different potentials.

A little consideration will show that an equipotential can cut itself, *but only at a null point* (see Fig. 179).

Equipotential lines are similar to the contour lines on a map, for the latter join all points which are at the same height or *level* above the zero level which, of course, is taken as sea-level: a contour line may be said to be an equipotential line from the point of view of gravitational potential.

4. Electrical Capacitance or Capacity. Units of Capacitance

We have seen that the potential produced in a conductor by a given charge varies with the size of the conductor; thus, if a charge of + 20 e.s. units be given to a sphere (in air) of 10 cm. radius, the potential of the sphere will be 20/10, *i.e.* 2 e.s. units, but if the same charge be given to a sphere (in air) of 4 cm. radius the potential will be 20/4, *i.e.* 5 e.s. units. Further, if the same charge be given to any two conductors differing in size or form different potentials will be produced in these conductors.

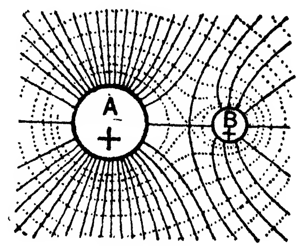


FIG. 179.

These facts are expressed by saying that different conductors have different **electrical capacities** or **capacitances.** Suppose a charge of 100 units causes the potential of a conductor to be 25 units, then 100/25, *i.e.* 4, is said to *measure* the capacitance of the conductor. Expressed in general terms, if a charge Q raises the potential of a conductor to V the capacitance C of the conductor is measured by the expression:—

$$\text{Capacitance or Capacity} = \frac{\text{Quantity}}{\text{Potential}}, \; i.e. \; C = \frac{Q}{V}.$$

Thus the capacitance or capacity of a conductor is measured by the charge given to it divided by the potential which that charge produces. Another way of putting it is to say that **the capacitance or capacity of a conductor is numerically represented by the amount of charge necessary to make it at unit potential.** Thus in the example above, the capacitance of the conductor is 4 units of capacitance, and 4 is also the charge in quantity units which will make the potential one unit of potential.

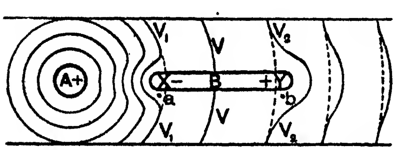


FIG. 180.

In the relation above, if $Q = 1$ e.s. unit and $V = 1$ e.s. unit, then $C = 1$ e.s. unit: hence **a conductor has a capacitance of one absolute or C.G.S. electrostatic unit if the electrostatic unit quantity raises its potential by one electrostatic unit:** this e.s. unit of capacitance has no specific name.

The newer name *capacitance* is in agreement with the British Standard Glossary, and, scientifically, it is better than *capacity*, for

the latter has a large number of other uses. "Capacity" is, however, still widely employed, so that in this book no hard and fast rule will be followed—for simplicity we will take the two terms as synonymous.

Another capacitance unit is known as the "absolute or C.G.S. electromagnetic unit," and it is equal to (9×10^{20}), *i.e.* $(3 \times 10^{10})^2$ electrostatic units: this e.m. unit has no specific name (note again the (3×10^{10})).

The practical unit of capacitance is called the **farad** (after Faraday), and it is equal to 10^{-9}, *i.e.* $1/10^9$, electromagnetic unit: it is therefore equal to $1/10^9$ of (9×10^{20}), *i.e.* (9×10^{11}) electrostatic units. The farad is dealt with later, but it may be mentioned here that **a conductor has a capacitance of one farad if a charge of one coulomb raises its potential by one volt**: thus:—

$$1 \text{ farad} = \frac{1 \text{ coulomb}}{1 \text{ volt}} = \frac{1/10 \text{ e.m. unit}}{10^8 \text{ e.m. unit}} = \frac{1}{10^9} \text{ e.m. unit.}$$

$$1 \text{ farad} = \frac{1 \text{ coulomb}}{1 \text{ volt}} = \frac{(3 \times 10^9) \text{ e.s. unit}}{1/300 \text{ e.s. unit}} = 9 \times 10^{11} \text{ e.s. unit.}$$

Although the *farad* is the true practical unit of capacitance, it is, for many practical purposes, much too large a unit, and another, the **microfarad,** which is one-millionth of the farad, is often employed: thus the microfarad is equal to $(9 \times 10^{11})/10^6$, *i.e.* (9×10^5) or 900,000 electrostatic units. Sometimes a still smaller unit, the *micro-microfarad*, which is one millionth of the microfarad, is used. In practice the symbol F is often used for farads, μF for microfarads (unit often used in wireless), and $\mu\mu$F for micro-microfarads.

Note that the electrostatic unit of capacitance is small compared with the e.m. unit, the farad, and the microfarad. Note, too, that the definition of capacitance does *not* state that capacitance *is* the charge necessary to produce unit potential, but that it is *numerically represented* by the charge.

It is advisable to remember three forms of the algebraic relation given above, viz.:—

$$C = \frac{Q}{V}, \text{ i.e. Capacitance} = \frac{\text{Quantity}}{\text{Potential}} \quad \ldots\ldots\ldots\ldots (5)$$

$$V = \frac{Q}{C}, \text{ i.e. Potential} = \frac{\text{Quantity}}{\text{Capacitance}} \quad \ldots\ldots\ldots\ldots (6)$$

$$Q = CV, \text{ i.e. Quantity} = \text{Capacitance} \times \text{Potential} \quad \ldots\ldots (7)$$

If C be the capacitance of a conductor *in air*, a charge of C e.s. units will raise its potential to 1 e.s. unit *in air*. If the conductor with this charge C be now immersed in a medium of dielectric constant K, its potential will be 1/K of what it was in air (Art. 2). To bring it up to unit potential in this medium the charge must be increased K times, *i.e.* a charge KC e.s. units will be necessary to raise its potential to 1 e.s. unit in this medium. Hence, *if C be*

the capacitance of a conductor in air, its capacitance in a medium of dielectric constant K *is* KC *e.s. units.*

We have seen (Art. 2) that if Q e.s. units be given to a spherical conductor of radius r cm., embedded in a medium of dielectric constant K, the potential is Q/Kr e.s. units. If C denote its capacitance *in this medium* its potential is also given by the expression Q/C; hence $C = Kr$. With air as medium K is unity, and this reduces to $C = r$. Hence **the capacitance of an isolated spherical conductor in air is, in e.s. units, numerically equal to its radius in centimetres** (this means that if the radius is 5 cm. the capacitance is 5 e.s. units of capacitance, *not* 5 cm.): in a medium other than air, the capacitance, in e.s. units, is numerically equal to its radius in centimetres multiplied by K.

It follows from the above that an isolated spherical conductor of 1 cm. radius in air has a capacitance of one electrostatic unit. That the practical unit of capacitance, the *farad,* is a very large unit, will be gathered from the particulars given on page 170: it can also be seen from the fact that a spherical conductor would have to have a radius of about $5\frac{1}{2}$ million miles to have a capacitance of one farad. (Verify this.)

If two conductors charged to different potentials are brought into contact, they take up a common potential and share the combined charge in direct proportion to their capacitances; for example, if two conductors of capacitances 3 and 5 are made to share their charges, the first takes $\frac{3}{8}$ and the other $\frac{5}{8}$ of the combined charge. If two conductors A and B of capacitances C_1 and C_2 be charged to potentials V_1 and V_2 respectively, then on being placed in contact they will take up a common potential V given by

$$C_1V_1 + C_2V_2 = (C_1 + C_2)\,V;$$

$$\therefore \text{ Common potential} = V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2},$$

for the expression $C_1V_1 + C_2V_2$ gives the quantity of electricity before contact, and $(C_1 + C_2)\,V$ expresses the *same* quantity after contact and redistribution of the charge (there is *no loss of charge*). Further, if Q_1 be the charge on A *after* sharing and Q_2 be the charge on B, then the potential of A is Q_1/C_1 and of B it is Q_2/C_2. But the potentials are the same, viz. V: hence:—

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}; \quad \therefore \frac{Q_1}{Q_2} = \frac{C_1}{C_2}; \quad i.e. \ \frac{\text{Charge on A}}{\text{Charge on B}} = \frac{\text{Capacitance of A}}{\text{Capacitance of B}}.$$

That is, the charges are proportional to the capacitances.

Examples.—(1) *A spherical conductor of* 5 *cm. radius and charged to a potential of* 100 *units, is placed inside a larger uncharged spherical conductor of* 10 *cm. radius, and made to touch its inner surface. Find the potential to which the larger conductor is raised (the medium is air).*

Here *all* the charge of the smaller sphere (*i.e.* $5 \times 100 = 500$ units) "passes to the larger." Hence if V = required potential of the larger sphere:

$$10 \times V = 500; \quad \therefore V = 50 \text{ e.s. units.}$$

(2) *The smaller conductor of Example* (1) *after having been discharged by contact with the inner surface of the larger conductor, is taken out and connected with the latter by a long thin wire. Find the common potential which the conductors attain, and the charge on each.*

The total capacitance when they are joined is $(5 + 10) = 15$ units. The total charge is the charge which was originally on the small sphere, viz. $5 \times 100 = 500$ units. Hence if V_1 be the common potential:—

$$15V_1 = 500; \quad \therefore V_1 = \tfrac{500}{15} = 33{\cdot}3,$$

i.e. common potential = 33·3 e.s. units.

Again, if A be the smaller conductor and B the larger we have:—

$$\frac{\text{Charge on A}}{\text{Charge on B}} = \frac{\text{Capacitance of A}}{\text{Capacitance of B}} = \frac{5}{10} = \frac{1}{2};$$

$$\therefore \text{Charge on A} = \tfrac{1}{3} \text{ of total} = \tfrac{1}{3} \times 500 = 166{\cdot}7 \text{ e.s. units.}$$

$$\text{Charge on B} = \tfrac{2}{3} \text{ of total} = \tfrac{2}{3} \times 500 = 333{\cdot}3 \text{ e.s. units.}$$

(3) *Compare the forces between two small spheres charged to the same potentials (a) in air, (b) in a medium of dielectric constant* K.

If Q_1 and Q_2 denote the charges *in air*, and *d* the distance apart—

$$\text{Force in air} = \frac{Q_1 \times Q_2}{d^2}.$$

When in the medium the capacitance of each is increased K times, so that the charge on each must be increased K times to bring it up to the *same potential* as it was in air, *i.e.* the charges must be KQ_1 and KQ_2 respectively. If the spheres were in air with these charges, the force would be—

$$\frac{KQ_1 \times KQ_2}{d^2};$$

but, as they are in a medium of dielectric constant K, the force is 1/K of this,

$$\therefore \text{Force in medium} = \frac{1}{K} \cdot \frac{KQ_1 \times KQ_2}{d^2} = K\frac{Q_1 \times Q_2}{d^2}.$$

Note in (3) above that if the *potentials* of the spheres are the same in the medium as they were in the air the force is *K times greater in the medium.* If the *charges* are the same in the medium as they were in air the force is *K times greater in the air* (Art. 1).

FACTORS WHICH AFFECT THE CAPACITANCE OF A CONDUCTOR.—From the experiments of Chapter V. and the results of the present section it will be noted that:—(1) The greater the *size* the greater

the capacitance. (2) The capacitance depends on the *dielectric medium* being greater the greater the value of K. (3) The capacitance is affected by *neighbouring conductors*.

(*a*) Facts (1) and (2) can also be seen from the expression for the capacitance of a spherical conductor, viz. $C = r$ or $C = Kr$.

(*b*) Fact (1) can also be seen thus:—ABCD is a sheet of tin-foil mounted on a roller (Fig. 181). Charge the sheet and note the divergence. Now roll it up a little so as to lessen the surface area (charge is on the surface): the divergence is greater. Thus the *same* charge produces a *greater* potential when the area is small, i.e. *less* charge is necessary to produce the *same* potential: the capacitance is therefore *less* when the area is less.

(*c*) Fact (2) may also be shown experimentally as follows:—Take two equal brass balls A and B (fitted with insulating handles), join them by a long thin wire, embed B in paraffin wax, and charge A. The charge is shared and they come to the *same potential*. Remove the wire (using insulating tongs) and then bring them in turn into a metal pot on the electroscope. B causes the bigger divergence. B in wax therefore took a bigger charge than A did in the air to bring it to the same potential: hence the capacitance of B is greater when it is in wax than when it is in air.

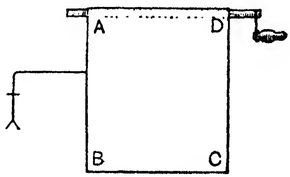


FIG. 181.

(*d*) Fact (3) was indicated by several experiments in Chapter V. Thus in Fig. 140 the potential of C is lowered when AB is brought near, and more so when AB is earthed (Fig. 143); hence C will now require a *bigger charge* to bring it up to the *same potential* as before, *i.e.* the capacitance of C is greater when AB is near.

5. Energy of a Charge

Consider a conductor charged with $+Q$ e.s. units to a potential V e.s. units. By the statement that the "electrical potential" of the charged conductor is V e.s. units we mean that V ergs of work would have to be done to bring a positive e.s. unit of electricity from infinity up to the surface of the conductor *when the conductor is so charged, i.e.* has its *full* charge Q e.s. units. The charged conductor is, of course, a source of "energy": thus if the finger be brought near, a spark is obtained and the conductor is discharged, *i.e.* the original energy of the whole charge is converted into heat and light and part is used in setting up the air vibrations which constitute sound. Now when we speak of the "potential energy of the charge" (or discharge) we are referring to this *total energy* which is the equivalent *not* of the work done in bringing a positive unit from infinity to the surface when the charge Q is already there,

but of all the work done in charging the conductor with a total charge Q units during which process the electrical potential rises from zero to the value V.

An expression for the energy of the charged conductor might be obtained in an approximate elementary way as follows. Imagine a conductor without charge, and picture the charging process to be done by bringing up successive equal *very small* positive charges (or we might say *units*) from infinity until the potential is raised from zero to a value V electrostatic units. The electrical potential at any moment is always directly proportional to the charge which the conductor has at the moment. Hence the average potential during the process is V/2, and this represents by the definition of Art. 2 the average work, in ergs, in putting on a unit charge. If the total charge given to it is Q electrostatic units, the total work done in charging is $Q \times V/2$, *i.e.* $\frac{1}{2}QV$ ergs. This represents the potential energy of the whole charge or the work which can be obtained in discharging; hence, employing electrostatic units, the energy of the charge is given by—

$$\text{Energy} = \tfrac{1}{2}QV \text{ ergs} \quad\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

$$= \tfrac{1}{2}CV \times V = \tfrac{1}{2}CV^2 \text{ ergs} \quad\ldots\ldots\ldots\ldots (9)$$

$$= \tfrac{1}{2}Q\frac{Q}{C} = \tfrac{1}{2}\frac{Q^2}{C} \text{ ergs} \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

In the above Q, V, and C are in e.s. units, and the energy is in *ergs*. If Q be in coulombs, V in volts, and C in farads, the three expressions above will be in *joules* (1 joule = 10^7 ergs). To get the energy in *foot-pounds* multiply the joules by ·7375.

The above is readily seen: thus taking (8) and assuming Q in coulombs and V in volts, we have:—

$$\text{Energy} = \tfrac{1}{2} \times Q \times (3 \times 10^9) \times V \times \tfrac{1}{300} = \tfrac{1}{2}QV \times 10^7 \text{ ergs} = \tfrac{1}{2}QV \text{ joules.}$$

A more exact proof of the energy expressions above is obtained by a simple application of the Calculus. If q be the charge at any instant during the charging and v the potential, then to bring up a further very small charge dq requires work $v.dq$. Hence for the total work in charging (and therefore the energy of the charge—or discharge) we have:—

$$\text{Energy} = \int_0^Q v.dq = \int_0^Q \frac{q}{C}.dq = \frac{1}{C}\int_0^Q q.dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{1}{2}\frac{Q^2}{C}$$

$$\textit{i.e.}\ \text{Energy} = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV \text{ ergs.}$$

Care should be taken to avoid the popular mistake that "electricity" is a form of energy. Just as in order to confer energy upon water some portion must be at higher or lower pressure or "level" than its surroundings, so in order to confer energy upon electricity some portion must be at a higher or lower potential than its surroundings; energy equals $\frac{1}{2}Q$ *multiplied by* V, and if $V = 0$, energy $= 0$, however great Q may be. It should also be noticed that all energy is of the same essential nature whatever form it takes or whatever its source: the energy of a charged metal ball is as truly mechanical as that of a horse or an engine and is measured in the same units—ergs, joules, foot-pounds, etc.

6. Loss of Energy on Sharing Charge

We have seen that if two, say like, charged conductors be at different potentials, then on joining them *electrons* will flow from the one at the lower potential to the other until they come to the same potential: in other words, the "charge" is "shared" and the potentials equalised, but *the total quantity is the same before and after sharing.* On the other hand, *there is always a loss of energy in such cases:* the loss appears as heat in the connecting wire or appears as a spark. First consider a simple numerical example:—

Example.—*A sphere* A *of radius* 5 *cm. and charge* 20 *units shares its charge with a sphere* B *of radius* 10 *cm. Find the loss of energy.*

$$\text{Original energy of A} = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\cdot\frac{400}{5} = 40 \text{ ergs.}$$

$$\text{Final energy of (A + B)} = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\cdot\frac{400}{(10+5)} = 13\tfrac{1}{3} \text{ ergs;}$$

$$\therefore \text{Loss of energy on sharing the charge} = 26\tfrac{2}{3} \text{ ergs.}$$

Now consider the general case. Calling the conductors A and B, let C_1 = capacitance of A and V_1 = its potential: let C_2 and V_2 denote the same for B: let V = the common potential of A and B when joined, and let Q be the "charge" which has "passed" say, from A to B. The "loss" of charge Q has lowered the potential of A from V_1 to V, and the "gain" of Q has raised the potential of B from V_2 to V: hence:—

$$C_1 = \frac{Q}{V_1 - V}; \quad \therefore Q = C_1(V_1 - V).$$

$$C_2 = \frac{Q}{V - V_2}; \quad \therefore Q = C_2(V - V_2).$$

i.e. $C_1V_1 - C_1V = C_2V - C_2V_2; \quad \therefore V = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2}$ (see p. 171).

Now original energy of A and B $= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$.

$$\text{Final energy} = \tfrac{1}{2}(C_1 + C_2)\left(\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right)^2 = \tfrac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{C_1 + C_2}.$$

$$\text{Decrease in energy} = (\tfrac{1}{2}C_1V_1^2 + \tfrac{1}{2}C_2V_2^2) - \tfrac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{C_1 + C_2};$$

$$\therefore \text{ Decrease in energy} = \tfrac{1}{2}\frac{C_1C_2(V_1 - V_2)^2}{C_1 + C_2}.$$

This expression is always positive whatever the sign of V_1 and V_2: hence *there is always a decrease in energy in such cases.*

7. Surface Densities of Uniformly Charged Spheres

Following directly on principles already explained, there is nothing new or difficult in the statements below: we draw attention to them as the expressions are often required in problems.

Consider an isolated sphere charged with Q units: the distribution of charge is uniform. Let r be the radius (cm.) and ρ the surface density: then:—

$$\rho = \frac{\text{Quantity}}{\text{Surface area}} = \frac{Q}{4\pi r^2}; \quad \therefore Q = 4\pi r^2 \rho.$$

Again consider two spheres A and B placed at a considerable distance apart, joined by a long thin wire, and charged. Let Q_1 = charge on A, r_1 = radius of A, ρ_1 = surface density of A, and let Q_2, r_2, and ρ_2 apply to B. Now if V = common potential

$$\frac{Q_1}{Q_2} = \frac{C_1V}{C_2V} = \frac{C_1}{C_2} = \frac{Kr_1}{Kr_2} = \frac{r_1}{r_2}:$$

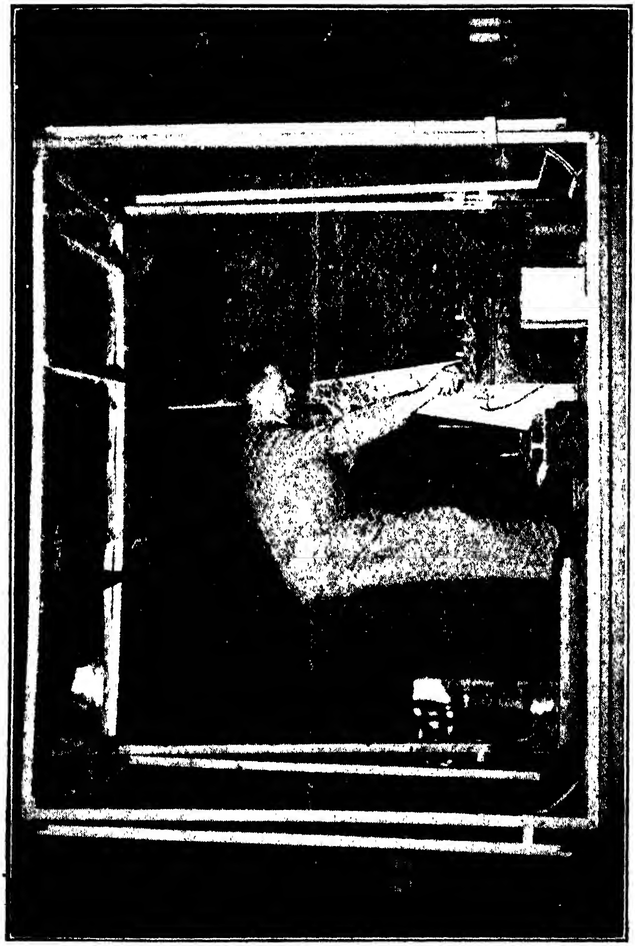
thus *the charges are directly as the radii*, as was to be expected. Further:—

$$\frac{\rho_1}{\rho_2} = \frac{Q_1}{4\pi r_1^2} \div \frac{Q_2}{4\pi r_2^2} = \frac{r_1}{r_1^2} \div \frac{r_2}{r_2^2} = \frac{r_2}{r_1}:$$

thus *the surface densities of the charges are inversely as the radii*, as was also to be expected.

8. Field Strength or Intensity. Unit Electric Field

(1) It has been agreed that *the strength or intensity in electrostatic units of an electric field at any point be represented numerically by the force in dynes on a positive electrostatic unit charge placed at the point*, it being assumed that the positive unit charge itself does not disturb the field: the direction of the field is the direction in which



ELECTRIC SCREENING.

Insulators for the National Electricity Supply Scheme being tested at the National Physical Laboratory (1,000,000 volts). Such high voltage work is carried out in a metal screened enclosure.

the force on the *positive* unit acts. Thus **an electric field has an intensity of one C.G.S. electrostatic unit if it exerts a force of one dyne on a positive electrostatic unit charge,** and a field of intensity E electrostatic units is one which exerts a force E dynes on an e.s. unit charge: if a charge q e.s. units be put in a field of intensity E e.s. units the force F on the charge is given by: $F = Eq$ dynes.

It follows that the intensity (E) at distance d cm. from a point charge $+ Q$ e.s. units, or at a distance d cm. from the centre of a uniformly charged spherical conductor having a charge of $+ Q$ e.s. units (d being greater than the radius of the sphere), is given by

$$E = \frac{Q}{d^2} \text{ e.s. units} \quad or \quad E = \frac{Q}{Kd^2} \text{ e.s. units} \quad \ldots\ldots\ldots (11)$$

(the medium in the first case being air, and in the second one of dielectric constant K), for these are the forces in dynes on the e.s. unit charge (the field inside the sphere is, of course, zero). The e.s. unit field has no specific name.

To find the intensity E at a point due to a number of charges we imagine a unit positive charge placed at the point, calculate the force on this unit positive charge due to each of the others, and then find the resultant by the parallelogram of forces.

The warning given on page 58 may be repeated: the definition means that if the force on unit e.s. charge is, say, 8 dynes, the intensity is 8 C.G.S. *electrostatic units of intensity*, not 8 dynes.

(2) As in the case of the magnetic field and magnetic potential, so here the conception of electrical potential (V) leads to a definition of field intensity (E). On page 163 it was shown that if V_1 = the potential at C (Fig. 176) and V_2 = the potential at B, then if the points are near together and E = field intensity in the space between them, we have (neglecting sign):—

$$E = \frac{V_2 - V_1}{CB}, \quad i.e. \ E = \frac{dv}{dx} \text{ (numerically).}$$

The right-hand side gives the potential gradient between B and C; hence **the intensity at a point in an electrical field is numerically equal to the potential gradient at that point.** It follows that the field intensity is greatest where the potential gradient is steepest (*i.e.* where the rate of variation of potential with distance is greatest) or, what amounts to the same, if the equipotential lines of the field be mapped out, the field intensity is greatest where the lines are nearest together.

Examples.—(1) ABCD *is a square of* 2 *cm. sides. Charges of* + 4, − 4, *and* + 8 *electrostatic units are placed at the points* A, C, *and* D. *The medium is air. Find the intensity of the field at* B (Fig. 182).

(1) Force at B due to A $= \frac{4 \times 1}{2^2} =$ 1 dyne in the direction AB.

Let BG represent this force.

(2) Force at B due to C $= \frac{-4 \times 1}{2^2} =$ 1 dyne in the direction BC.

Let BH represent this force.

(3) Force at B due to D $= \frac{8 \times 1}{(\sqrt{8})^2} =$ 1 dyne in the direction DB.

Let BL represent this force.

The resultant of (1) and (2) is $\sqrt{2}$ dynes represented by BM, and the resultant of this and (3) is represented by BN and is equal to $\sqrt{3}$ dynes (since $BN^2 = BM^2 + BL^2$); hence

Intensity at B $= \sqrt{3}$ e.s. units (and the direction is BN).

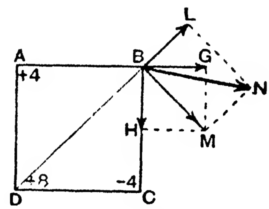


FIG. 182.

(2) *A charge of* + 10 *e.s. units moves along a line of force in a uniform field from a point A to a point B. If the distance* AB *be* 4 *cm. and the work done be* 1500 *ergs, find the P.D. between A and B and the intensity of the field.*

Work in moving 10 e.s. units = 1500 ergs;

∴ ,, ,, ,, 1 e.s. unit = 150 ergs,

i.e. P.D. $= V_2 - V_1 = 150$ e.s. units;

$$\therefore \text{ Potential gradient} = \frac{V_2 - V_1}{4} = \frac{150}{4} = 37 \cdot 5,$$

i.e. Intensity E = 37·5 e.s. units.

Again: Force on the charge 10 e.s. units = 10E dynes.

Work done = Force × Distance = (10E × 4) ergs = 40E ergs,

i.e. 40E = 1500; ∴ E = 37·5 e.s. units.

(3) The conception of electric lines and tubes of force and induction in an electric field referred to on pages 149–154 leads to another method which might be used in defining field intensity: the various unit tubes in electrostatics are dealt with in the next section.

9. Electric Tubes of Force and Induction

In electrostatics the following unit tubes are employed. The student will note that the first two correspond to those dealt with in Magnetism: the third—Faraday tubes—has no exactly corresponding conception in Magnetism but it is most convenient in the study of electrostatics.

(1) MAXWELL UNIT TUBES OF FORCE.—Consider, say, a point charge Q e.s. units in a medium of dielectric constant K, and imagine a spherical surface of radius r cm. drawn in the field, the charge Q at the centre; the intensity of the field at any point on the surface of this imaginary sphere is Q/Kr^2 C.G.S. units. Now picture the electric lines grouped into bundles or unit tubes in such a way that $4\pi Q/K$ unit tubes emanate from the charge Q (or in other words that $4\pi/K$ tubes emanate from unit charge) in this medium; these are called **Maxwell unit tubes of force.**

These tubes all pass through the sphere, and since the area of this spherical surface is $4\pi r^2$ it follows that

$$\left.\begin{array}{c}\text{Number of Maxwell Tubes of}\\ \text{Force per unit area}\end{array}\right\} = \frac{4\pi Q}{K \cdot 4\pi r^2} = \frac{Q}{Kr^2}.$$

But this last expression gives the intensity of the field in electrostatic units; hence, *the intensity* (E) *in C.G.S. units is represented numerically by the number of Maxwell unit tubes of force per unit area.* Thus we can say that **a field has an intensity of one C.G.S. electrostatic unit if it has one Maxwell unit tube of force per unit area** taken perpendicular to the direction of the field. In air (strictly *vacuo*) K is taken as unity and the number of Maxwell tubes from a charge Q is $4\pi Q$, and the number from unit charge is 4π.

Note that the intensity (E) at a point on the sphere, viz. Q/Kr^2, multiplied by the cross-section of the Maxwell unit tube of force at the point in question, viz. $(4\pi r^2 \div 4\pi Q/K)$, *i.e.* Kr^2/Q, is unity, *i.e.* a *constant*, and the same applies to other spherical surfaces. Thus the intensity at any point in a unit tube varies inversely as the cross-sectional area of the tube at that point.

(2) UNIT TUBES OF INDUCTION.—The intensity (E) at any point on the sphere is Q/Kr^2, *i.e.* it involves Q, r, *and the dielectric constant* K. It is convenient (as in Magnetism, page 85) to have another quantity whose component in any direction and in any medium is K times the component of the electric intensity in that direction, and this quantity is termed the **electric induction** (N): thus the induction N at the point considered above is KE, *i.e.* Q/r^2.

Now imagine the electric lines to be grouped into tubes in such a way that $4\pi Q$ unit tubes emanate from the charge Q *whatever the medium* (*i.e.* 4π unit tubes emanate from unit charge); these are called **unit tubes of induction.** They all pass through the surface of the sphere; hence:—

$$\left.\begin{array}{c}\text{Number of Tubes of In-}\\ \text{duction per unit area}\end{array}\right\} = \frac{4\pi Q}{4\pi r^2} = \frac{Q}{r^2} = K\left(\frac{Q}{Kr^2}\right).$$

But Q/r^2 is the induction, so that **the induction (N) is represented numerically by the number of unit tubes of induction per unit area.**

Further, Q/Kr^2 is the intensity (E) which is numerically represented by the number of Maxwell tubes of force per unit area: hence

$$\text{Induction} = K \times \text{Intensity, } i.e. \ N = KE;$$

$$\therefore \left.\begin{array}{c}\text{Tubes of Induction}\\ \text{per unit area}\end{array}\right\} = \left\{\begin{array}{c}K \times \text{Maxwell Tubes of Force}\\ \text{per unit area}\end{array}\right.$$

and *the intensity* (E) *at any point may be said to be represented numerically by the number of tubes of induction per unit area divided by* K. In air (strictly vacuo) K = unity, and Maxwell tubes of force and tubes of induction are identical. (Compare magnetic theory $B = \mu H$: here $N = KE$.)

Note that the dielectric constant K of a medium may be defined as given by the ratio of the induction (N) to the intensity (E)

Note also that the induction (N) at the sphere, viz. Q/r^2, multiplied by the cross-section of the unit tube of induction ($4\pi r^2 \div 4\pi Q$), *i.e.* r^2/Q, is unity, *i.e.* a *constant*, and the same applies to other spherical surfaces. Thus the induction at any point in a unit tube of induction varies inversely as the cross-sectional area of the tube. Similarly the intensity (E) varies inversely as the cross-sectional area.

(3) FARADAY UNIT TUBES. We have seen that the vector quantity *induction* (N) is that quantity whose component in any direction at any point is K times the component of the field intensity (E) in the same direction ($N = KE$). Now it is convenient in electrostatics (as will be seen presently) to have another quantity which is not K times the intensity, but $K/4\pi$ times the intensity, *i.e.* it is closely related to the induction, the numerical measures of the two differing only by the constant factor $1/4\pi$. This quantity is called the **electric displacement** (D) or **polarisation** (this latter is perhaps the better name), so that $D = KE/4\pi$. The term *strain* is sometimes used in connexion with it (from an analogy to the strain in an elastically strained solid).

It is somewhat confusing to a beginner having two vector quantities, *induction* (N) and *displacement* (D), related to each other *so very closely* and differing only by a constant factor: but both are important and must be learned.

Now reverting to the point charge Q at the centre of the spherical surface of radius r, imagine the electric lines grouped into tubes in such a way that Q unit tubes always emanate from a charge Q whatever the medium, and therefore one unit tube from unit

charge. These are called **Faraday unit tubes** (sometimes they are called Faraday tubes of induction: we shall call them simply Faraday tubes to avoid further confusion for the beginner). They all pass through the sphere: hence:—

$$\left.\begin{array}{r}\text{Number of Faraday Tubes}\\ \text{per unit area}\end{array}\right\} = \frac{Q}{4\pi r^2} = \frac{K}{4\pi}\left(\frac{Q}{Kr^2}\right).$$

But Q/Kr^2 gives the intensity (E), hence *the intensity at any point may be said to be represented numerically by the number of Faraday tubes per unit area multiplied by* $4\pi/K$. In air this multiplier becomes 4π. Clearly, also, a Faraday unit tube contains $4\pi/K$ Maxwell tubes of force.

Again $Q/4\pi r^2 = (1/4\pi) \times (Q/r^2) = N/4\pi$ for Q/r^2 is the induction (N): hence *the induction* (N) *is represented numerically by the number of Faraday tubes per unit area multiplied by* 4π. Clearly, also, every Faraday unit tube contains 4π tubes of induction.

Finally, $Q/4\pi r^2 = (K/4\pi)(Q/Kr^2) = (K/4\pi)\,E$ gives the *electric displacement* or *polarisation* (D): hence **the polarisation or displacement (D) is rep ented numerically by the number of Faraday unit tubes per unit area.**

Note that the polarisation (D) at the sphere, viz. $(K/4\pi) \times (Q/Kr^2)$ and $Q/4\pi r^2$, multiplied by the cross-section of the Faraday tube, viz. $4\pi r^2/Q$ is unity, *i.e.* a *constant*. Thus the polarisation, or displacement (or strain) at any point of a Faraday tube varies inversely as the cross-sectional area of the tube. The same applies to the intensity (E).

Summarising, $4\pi Q/K$ Maxwell unit tubes of force, $4\pi Q$ unit tubes of induction, and Q Faraday unit tubes emanate from a charge Q whatever the medium (but see the remarks about tubes *of force* in Art. 17 when *two dielectrics* are concerned). Further, the intensity (E) is measured by the number of Maxwell unit tubes of force per unit area, the induction (N) by the unit tubes of induction per unit area, and the polarisation or displacement (D)—or strain—by the Faraday unit tubes per unit area.

We can treat the above in a slightly different way which is perhaps more satisfying to the mathematically minded. Imagine the lines from the point charge Q at the centre of the imaginary spherical surface of radius r to be grouped into B equal bundles (*where B is an arbitrary number at present*), each subtending the same solid angle ω at Q. Each bundle cuts the same area, viz. $r^2\omega$ at the sphere, so that the area of the sphere is $Br^2\omega$: but the area of the sphere is $4\pi r^2$;

$$\therefore\ Br^2\omega = 4\pi r^2,\ \textit{i.e.}\ B = 4\pi/\omega,$$

and as the cross-section of each bundle at the sphere is $r^2\omega$:—

$$\text{Number of bundles per unit area} = 1/r^2\omega.$$

Now the intensity (E) at the sphere is Q/Kr^2, and *we can arrange that* E *is measured by the number of bundles per unit area* by choosing B so as to make these equal: thus if:—

$$\frac{1}{r^2\omega} = \frac{Q}{Kr^2}, \text{ i.e. if } \frac{1}{\omega} = \frac{Q}{K}, \text{ then } B = \frac{4\pi}{\omega} = \frac{4\pi Q}{K},$$

and the bundles are our *Maxwell unit tubes of force* ((1) above).

Further, the induction (N) is $KE = Q/r^2$, and *we can arrange that* N *is measured by the number of bundles per unit area* by choosing B so that:—

$$\frac{1}{r^2\omega} = \frac{Q}{r^2}, \text{ i.e. } \frac{1}{\omega} = Q; \quad \therefore B = \frac{4\pi}{\omega} = 4\pi Q,$$

and the bundles are our *unit tubes of induction* ((2) above).

Finally, the displacement or polarisation (D) is $KE/4\pi$, and *we can arrange that* D *is measured by the number of bundles per unit area* by choosing B so that:—

$$\frac{1}{r^2\omega} = \frac{KE}{4\pi} = \frac{Q}{4\pi r^2}, \text{ i.e. } \frac{1}{\omega} = \frac{Q}{4\pi}; \quad \therefore B = \frac{4\pi}{\omega} = Q,$$

and the bundles are our *Faraday unit tubes* ((3) above).

Clearly, then, Maxwell (unit) tubes of force are associated with electric force or intensity (E) in the same way as tubes of induction are associated with the induction (N) and in the same way as Faraday tubes are associated with the polarisation or displacement (D) in the dielectric (or the strain).

We have seen that when electric force acts on a conductor, electricity (electrons) continues to move as long as the electric force acts, but that in the case of a dielectric it is assumed that there is merely a slight displacement of the opposite charges in the atoms relative to each other which soon reaches a limiting value (proportional to the force producing it): an opposing influence is thus set up which balances the displacing force. Maxwell used the name "displacement" to denote the "electricity" which crosses *unit area* of a dielectric owing to the electric intensity at that point. Thus considering any section of a tube, it will be "polarised," *i.e.* opposite sides will be oppositely charged. Throughout the tube the face of each section will be neutralised by the opposite charge on the adjoining face of the next section so that only the ends of the tube where it rests on "charged" conductors will exhibit

"free charges," positive at one end, negative at the other. The amount of charge gathering on *unit area* of the conductors, *i.e.* the density ρ, may therefore be taken as a measure of the polarisation or displacement (D) just outside the conductors. In the same way the polarisation or displacement *at any point in a medium* is measured by the density ρ which would appear on a conducting surface placed at that point, it being assumed that the conductor's insertion there does not alter the existing polarisation or displacement. But the charge ρ per unit area is equal to the number of Faraday tubes per unit area; hence the number of Faraday tubes per unit area at any point is a measure of the polarisation, or displacement, at that point, as already stated.

It has been mentioned that the term *strain* is sometimes used in connexion with the displacement or polarisation in a dielectric: further, we saw in Chapter V. that the dielectric medium between charges (or, as it was quite early assumed, the *aether* of the medium) is often said to be electrically "strained" somewhat resembling a strained elastic solid. In the case of an elastically strained solid it is proved in books on mechanics that the *energy stored per unit volume* is given by the expression $\frac{1}{2}$ stress $\times$ strain, the elastic stress being measured by the force per unit area which is applied, and the elastic strain by the distortion produced. Now it will be proved presently that in the case of a dielectric medium, if the medium be the seat of the energy, the *energy per unit volume* is given by the expression $\frac{1}{2}E.D$, where E is the electric intensity (measured by the tubes of force per unit area) and D is the polarisation or displacement (measured by the Faraday tubes per unit area). Assuming, then, an analogy with the elasticity case, if E corresponds to *stress* in the field, D is the corresponding *strain* and is represented numerically by the Faraday tubes per unit area: hence the student will often come across the "omnibus" expression "displacement, polarisation, or strain."

It is interesting to note in connexion with the idea of displacement, etc., in a dielectric that if a number of mica (dielectric) sheets be pressed together between two end metal plates and the latter be strongly charged, one positively the other negatively, so that electric tubes pass along from plate to plate, then on taking the arrangement to pieces *every mica plate* will be found to have one side positive, the other negative.

In the hands of Physicists of the nineteenth century the conception of "aether and strains" proved very fruitful. It explained many facts in all phases of electrical (and magnetic) work, it led

to the prediction and subsequent discovery of wireless, and it led Maxwell to show that light is an electromagnetic phenomena. And although for reasons which cannot be explained in this book, the tendency nowadays is to treat electrical problems in terms of the "charges" (excess and deficit electrons) rather than in terms of *any* medium surrounding them, it is necessary for the student to consider both aspects of the problem and, in some phases at least, to keep an open mind on the question in his early stages.

Before leaving this section note again the following relations which are frequently required in problems: E = intensity, N = induction, D = polarisation or displacement.

$$N = KE; \quad \therefore\ E = \frac{N}{K}; \quad D = \frac{K}{4\pi}E; \quad \therefore\ E = \frac{4\pi D}{K};$$

$$D = \frac{N}{4\pi}; \quad \therefore\ N = 4\pi D.$$

10. Gauss's Theorem

This theorem applied to electrostatics is proved in the same way as the corresponding theorem in Magnetism (page 86). Consider a point P on any closed surface surrounding a point charge Q at O (Fig. 183), and let a be a small area at P containing the point. Let E denote the electrical intensity at P, and let α be the angle between the direction of the force and the *outward* drawn normal to the surface a at the given point. The component of the force along the outward drawn normal is E cos α, and since induction = K × intensity, the *induction* in this direction is KE cos α. The product KE cos $\alpha \times a$ is the *flow of induction* across the small area a. The total flow of induction or the **total normal electric induction** or the total **flux** over the whole closed surface is obtained by supposing the whole surface to be divided up into a very large number of small areas such as a, and summing up the values of KE cos $\alpha . a$ for all these areas, i.e. *the total normal induction is the surface*

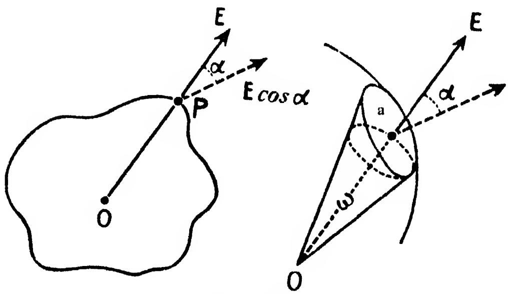


FIG. 183.

integral of the quantity KE *cos* $\alpha . a$ *over the whole surface.* Writing T.N.E.I. for the total normal electric induction we have

$$\text{T.N.E.I.} = \text{K}\Sigma\text{E} \cos \alpha . a.$$

Now considering the area a we have (writing OP = r):—

$$\text{Normal Induction over } a = \text{KE} \cos \alpha . a = \text{K}\frac{\text{Q}}{\text{K}r^2} \cos \alpha . a,$$

$$i.e. \text{ Normal Induction over } a = \text{Q}\frac{\cos \alpha . a}{r^2} = \text{Q}\omega$$

for $\cos \alpha . a/r^2$ = the solid angle subtended at O by the area $a = \omega$. Hence for the whole closed surface we have:—

$$\text{T.N.E.I.} = \Sigma\text{Q}\omega = \text{Q}\Sigma\omega = 4\pi\text{Q} \quad \ldots\ldots\ldots\ldots(12)$$

for $\Sigma\omega$ is the solid angle subtended at O by the whole closed surface and is equal to 4π. To summarise in words:—**The total normal electric induction over a closed surface drawn in an electric field is numerically equal to 4π times the total charge inside; if there is no charge inside the normal induction over the surface is zero.** This is *Gauss's Theorem.*

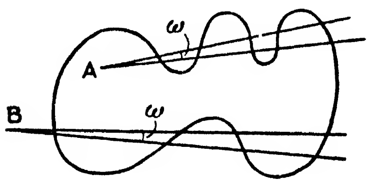


FIG. 184.

If there are several charges inside, some positive and some negative, the "total charge inside" refers to the algebraic sum of the charges inside.

If the charge is *outside* the surface the flux inwards equals the flux outwards, and the T.N.E.I. for the surface is zero.

The theorem also applies to a re-entrant closed surface. If the charge is inside (*e.g.* A, Fig. 184) a cone cuts an *odd* number of times so that for the induction for this cone A we have $+ \text{Q}\omega - \text{Q}\omega + \text{Q}\omega - \text{Q}\omega + \text{Q}\omega = \text{Q}\omega$. If the charge is outside (*e.g.* B, Fig. 184) the cuts are *even* and the contribution of any cone is zero, viz. $- \text{Q}\omega + \text{Q}\omega - \text{Q}\omega + \text{Q}\omega = 0$.

Gauss's theorem states that the *total normal electric induction* over a closed surface (whatever the medium) is numerically 4π times the charge inside. Now induction = K × intensity, and for air K = unity: hence for this medium the reader will come across the statement "the *total normal electric intensity* over any closed surface is numerically 4π times the total charge inside." In another medium, intensity = induction ÷ K, and, therefore, "the *total normal electric intensity* over a closed surface is $4\pi/\text{K}$ times the total charge inside."

Again $N = 4\pi D$, and therefore the induction N must be divided by 4π to give the polarisation or displacement D: hence it follows that **the total normal polarisation or displacement over a closed surface is numerically equal to the total charge Q inside** $(4\pi Q \div 4\pi = Q)$.

11. Applications of Gauss's Theorem

(1) INTENSITY DUE TO A UNIFORMLY CHARGED CONDUCTING SPHERE.—Let Q be the charge on the sphere and P an *external* point distance d from the centre at which the electric force or intensity of the field is required (Fig. 185). Take as the closed surface a sphere through P concentric with the charged sphere. By symmetry the intensity E is the same at every point of this surface and it is along the outward normal. Hence:—

$$\text{T.N.E.I.} = KE \times \text{Area} = 4\pi d^2 KE = 4\pi Q \text{ (by Gauss)};$$

$$\therefore \text{Intensity at } P = E = \frac{Q}{Kd^2}.$$

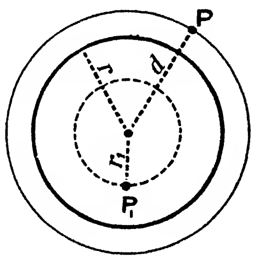


FIG. 185.

Thus the field at an external point is the same as if the charge on the sphere were concentrated at the centre. The potential at P is thus Q/Kd and the potential of the sphere itself is Q/Kr.

To get the field inside, say at P_1 (distance r_1 from centre), take as the closed surface a concentric sphere through P_1. As before, T.N.E.I. $= 4\pi r_1^2 KE_1$, where E_1 = intensity at P_1. But T.N.E.I. $= 4\pi Q = 0$ for there is no charge *inside* the dotted surface (it is on the outside of the sphere). Equating: $4\pi r_1^2 KE_1 = 0$; $\therefore E_1 = 0$, *i.e.* the field inside is zero.

For a point *just* outside the sphere, i.e. *indefinitely near* the surface—

$$E = \frac{Q}{Kr^2} = \frac{4\pi r^2 \rho}{Kr^2} = \frac{4\pi\rho}{K},$$

where ρ is the surface density. This is a special case of **Coulomb's Law,** viz. that the intensity *just* outside a charged surface *of any form* is $4\pi/K$ times the surface density.

(2) INTENSITY DUE TO A LONG CHARGED CYLINDRICAL CONDUCTOR.—Suppose the cylinder (Fig. 186) has a charge *q per unit length* and that it is so long that there are no end effects, so that the intensity at any point is radial from the cylinder. If P is the point, distant r cm. from the axis of the cylinder, at which the

intensity E is required, consider a concentric cylindrical surface (dotted) with P on its curved surface, *i.e.* a cylinder of radius r. Let l be the length of this cylinder, and imagine the surface closed by two plane ends perpendicular to the axis of the cylinder.

The charge inside the dotted cylindrical surface is ql. The flow of induction over the flat ends is zero for the intensity is parallel to them. The normal induction (T.N.E.I.) over the curved surface is $KE \times \text{Area} = KE \times 2\pi rl$. Applying Gauss's theorem

$$2\pi rlKE = 4\pi \text{ (charge inside)} = 4\pi ql;$$

$$\therefore \text{ Intensity} = E = \frac{2q}{Kr}.$$

Note particularly that q is the charge *per unit length* of the charged cylindrical conductor.

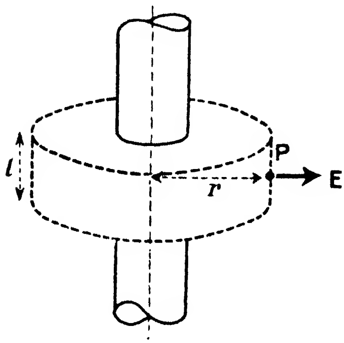


FIG. 186.

For a point *just* outside the cylinder we have, if R = radius of cylinder, $q = 2\pi R\rho$; $\therefore$ $E = 4\pi R\rho/KR = 4\pi\rho/K$. This is another special case of *Coulomb's Law* referred to in (1) above (ρ = surface density).

(3) INTENSITY INSIDE A HOLLOW CHARGED CONDUCTOR.—In this case consider the normal induction (T.N.E.I.) over *any* closed surface *entirely within* the hollow conductor (Fig. 187). With the usual lettering apply Gauss's theorem to this surface:—

$$\text{T.N.E.I.} = \Sigma KE \cos \alpha . a = 4\pi Q = 0,$$

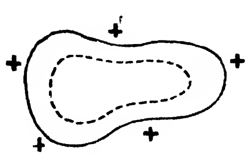


FIG. 187.

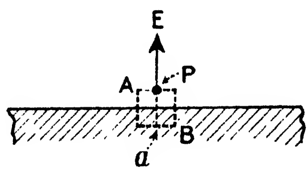


FIG. 188.

for Q, the charge, is *outside* the surface considered (it is all on the outside of the hollow charged conductor). But all the factors on the left-hand side (*i.e.* $KE \cos \alpha . a$) of this equation *except E* are definitely finite, and therefore as the result is zero:—

$$E = \text{zero},$$

and since the field inside is everywhere zero there is no difference of potential inside.

(4) INTENSITY NEAR A CHARGED PLANE CONDUCTOR.—Suppose ρ is the surface density of the charge, and P (Fig. 188) the point at which the intensity is required, the distance of P from the charged plane being small compared with the size of the plane. Imagine a narrow right cylindrical surface (dotted) of cross-section a with its axis perpendicular to the plane and passing through P, and let one end contain P and the other end be inside the conducting material. The charge inside this cylindrical surface is ρa.

Now there is no flux or flow of induction over the curved side of this cylindrical surface, for the intensity is perpendicular to the plane at the point considered, *i.e.* parallel to the sides of the cylinder. The flux over the end A is K$\text{E}a$. The flux over the end B is $\text{KE}'a =$ zero since E′ the intensity inside the conductor is zero. Hence applying Gauss's theorem:—

$$\text{T.N.E.I.} = \text{KE}a = 4\pi\rho a;$$

$$\therefore \text{ Intensity at P} = \text{E} = \frac{4\pi\rho}{\text{K}} \quad \ldots\ldots\ldots\ldots (13)$$

This is another case of *Coulomb's Law* referred to in (1) and (2).

(5) COULOMB'S LAW.—Special cases of this have already been referred to. The law states that **the electric force at any point infinitely close to the surface of *any* charged conductor is equal to $4\pi\rho/\text{K}$, where ρ is the density of the charge at the point,** and its direction is, at every point, normal to the surface of the conductor at that point. The surface of the conductor is an equipotential surface, and therefore the direction of the force at any point must be normal to the surface at that point.

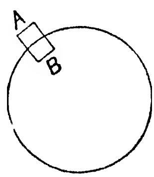


FIG. 189.

To determine the force at a point (Fig. 189), infinitely close to *any* charged surface, we first imagine a very small closed surface placed as AB, with its ends A and B infinitely close to and parallel to the surface at the point considered, one end inside and the other outside the charged surface and bounded laterally by a tubular surface determined by lines drawn normal to the charged surface through all points on the boundary of each of the ends. Then on applying Gauss's theorem to the small closed tubular surface the proof that $\text{F} = 4\pi\rho/\text{K}$ follows immediately as in (4) above.

(6) INTENSITY AT VARIOUS POINTS OF A TUBE.—It has been shown (Art. 9) that the intensity at any point of a tube is inversely as the cross-section taken at right-angles to the tube, and the fact can also be proved by Gauss's theorem. Let S_2 and S_1 be two normal cross-sections (Fig. 190) and E_2 and E_1 the intensities at these sections. Then normal induction at $S_2 = KE_2S_2$ (outwards): normal induction at $S_1 = KE_1S_1$ (inwards): and the curved sides are formed by lines of force, so that the induction *normal* to these is zero. Hence

$$\text{T.N.E.I. for portion of tube shown} = KE_2S_2 - KE_1S_1.$$

$$\text{But T.N.E.I.} = 4\pi Q = 0 \text{ for charge inside is zero;}$$

$$\therefore\ KE_2S_2 - KE_1S_1 = 0, \ \textit{i.e. } E_2S_2 = E_1S_1;$$

$$\therefore\ \frac{E_1}{E_2} = \frac{S_2}{S_1}.$$

Example.—*Show that the negative charge on which a tube terminates is equal to the positive charge from which it starts.*

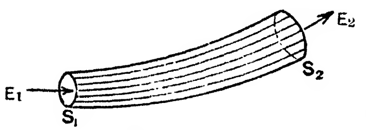


FIG. 190.

Let the tube be "cut off" just outside the charges.

By Coulomb's Law, if ρ = density at the positive end $E_1 = 4\pi\rho/K$, *i.e.* $KE_1 = 4\pi\rho$. If S_1 be the sectional area at this end

$$KE_1S_1 = 4\pi\rho S_1.$$

Similarly, at the negative end $KE_2 = -4\pi\rho_1$;

$$\therefore\ KE_2S_2 = -4\pi\rho_1S_2,$$

where E_2, S_2, ρ_1, denote the intensity, sectional area, and density at the negative end.

But from (6) above, $KE_2S_2 = KE_1S_1$; $\therefore\ 4\pi\rho S_1 = -4\pi\rho_1S_2$;

$$\therefore\ \rho S_1 = -\rho_1S_2,$$

and ρS_1 is the charge at the positive end, and $-\rho_1S_2$ that at the negative end.

12. Mechanical Force on Unit Surface of a Charged Conductor

Every portion of the charge on a conductor is repelled by all the remaining charge, *i.e.* the charge on any small area experiences a force (outwards) due to the charge on the remainder of the conductor. The charge is, of course, "bound" to the material of the conductor, so that every portion of the latter experiences a force (outwards): that this is so may be seen experimentally by gradually charging a soap bubble when it will be seen that the bubble increases in size. The problem then is to find an expression for the magnitude

of the (outward) force on, say, unit surface area of a charged conductor due to the charge on the remainder of the surface.

Consider a very small element of surface, say a (Fig. 191). The intensity E (measured by the force F dynes on a unit + charge) at a point P *just outside a* is $4\pi\rho/K$, where ρ is the surface density at a. Now this force F (or intensity E) may be regarded as made up of two parts, viz. a force F_1 (outwards) due to the charge on a, and a force F_2 (outwards) due to the charge on the rest of the surface:

$$\therefore\ F_1 + F_2 = 4\pi\rho/K \quad \ldots\ldots\ldots\ldots (a)$$

Consider a point Q *just* inside a. The total force F (or intensity E) is zero, but we can still look upon this as being due to two forces, F_1 and F_2, the former due to the charge on a and the latter due to the charge on the rest of the surface. Now Q is not very far from P since both points are quite close to a, but now F_1 is *reversed* in direction (inwards) and the resultant is zero, *i.e.*

$$-F_1 + F_2 = 0; \quad \therefore\ F_1 = F_2 \ldots (b)$$

and therefore from (a) $F_2 = 2\pi\rho/K$.

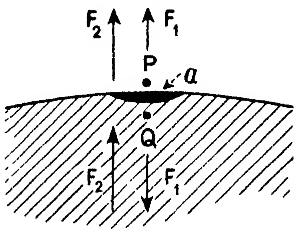


FIG. 191.

Again, the charge on the area a is $a\rho$, so that the total force experienced by this *due to the charge on the rest of the surface* is $\rho a \times F_2 = \rho a \times 2\pi\rho/K = 2\pi a\rho^2/K$ and therefore the force on *unit area* due to the rest of the charge $= 2\pi\rho^2/K$. Since the surface is an equipotential one the direction of the force is outwards along the normal at the point considered.

Further, since (Coulomb's law) $E = 4\pi\rho/K$ we have $\rho^2 = (KE)^2/(4\pi)^2$ and therefore $2\pi\rho^2/K = KE^2/8\pi$: and since $E = 4\pi D/K$ we also have $KE^2/8\pi = 2\pi D^2/K$. Summarising, then, the surface of a charged conductor is subject to an outward force, the *force per unit area* where the surface density is ρ being given by:—

$$\left.\begin{array}{c}\text{Force (outwards)}\\ \text{per unit area}\end{array}\right\} = \frac{2\pi\rho^2}{K} = \frac{KE^2}{8\pi} = \frac{2\pi D^2}{K} \text{ dynes per sq. cm.} \ldots (14)$$

where, of course, E = intensity is numerically equal to the Maxwell tubes of force per unit area, and D = polarisation or displacement —or strain—is numerically equal to the Faraday tubes per unit area.

The greatest value of ρ possible in air is about 8 in C.G.S. units, so that the maximum value of $2\pi\rho^2$ is about 400 dynes per square centimetre, or about the $\frac{1}{2500}$th part of an atmospheric pressure; if these be exceeded the charge begins to escape into the air.

13. Tension and Pressure in Electric Tubes

We have seen that in order to explain the attraction and repulsion between charges at a distance, the electric lines and tubes in the field were assumed to possess certain mechanical properties which would make them the *cause* of the action between the charges. The properties to be assumed were (see again the Faraday idea—pages 150-3) that in each tube there was a longitudinal force, a tension acting along the tube which tended to make it shorter, and a lateral pressure so that the tube resisted " sideways " compression and was able to exert pressure on its neighbouring tubes. We have seen (Chapter V.) how tubes endowed with these properties *could* account for the facts of attraction and repulsion. The problem now is to find expressions for the " magnitudes " of these tensions and pressures in the tubes if they are to account for the facts: we will use Faraday tubes.

It has been proved that the outward force on unit area at the surface of a charged conductor is $2\pi D^2/K$ dynes (D = polarisation). Now from this unit area D Faraday tubes emanate, and if we imagine each tube to exert a pull equal to $2\pi D/K$ the required tension will be obtained. Hence, we might regard the outward force at the surface of a conductor as being due to the fact that the Faraday tubes originating at the surface are in a state of tension, the magnitude of the " pull " at any point in a tube being given by the value of $2\pi D/K$ at that point; but there are D tubes per unit area at that point, so that the tension (force *per unit area*) at a point in the Faraday tubes where the electric force is E is given by

Longitudinal tension

$$= \frac{2\pi D}{K} \times D = \frac{2\pi D^2}{K} = \frac{KE^2}{8\pi} \text{ (dynes per sq. cm.).}$$

Since there is a tension along the tubes each tube must also exert a lateral pressure on its neighbours, otherwise the tubes joining, say, two opposite charges would shrink into straight lines between the charges and there would be no tubes in other parts of the medium. It can be shown that in order to maintain equilibrium in the medium the lateral pressure, *i.e.* the force per unit area at right angles to the tubes, is also given by the expressions above,

$$\textit{i.e.}\text{ Lateral pressure} = \frac{2\pi D^2}{K} = \frac{KE^2}{8\pi} \text{ (dynes per sq. cm.).}$$

For a more exact and full investigation of these tensions and pressures in Faraday tubes the student should refer to *Advanced*

Textbook of Electricity and Magnetism. Incidentally, these tubes may also be considered to possess *mass* (a rather strange statement to the beginner), as will be seen later.

14. Energy per Unit Volume of the Medium

A further development of the theory that the electrical effects in the space or field surrounding a charged body are the results of stresses and strains in the field itself, leads to the conclusion that the energy of electrification, the energy which is expended in separating electrons from their parent atoms (and which is converted into other forms—mechanical work, heat, light, sound—when the body is discharged, *i.e.* the electrification neutralised) must be stored *in the electric field* rather than in the electrified bodies themselves. The case might, in fact, be again roughly compared to that of a stretched elastic cord where the energy expended in stretching the cord is stored up in the elastic itself: in the electrical case the energy is assumed to exist as energy of strain in the medium. An expression for the magnitude of the energy per unit volume of the medium is obtained as indicated below.

In Art. 12 it has been shown that the force on unit area of a charged surface is $KE^2/8\pi$ dynes (outwards) where E is the intensity which is measured by the number of Maxwell tubes of force per unit area. Imagine this unit area to be moved in the opposite direction to this force by a small amount, dx; the work done is $(KE^2/8\pi) \times dx$ ergs; and the increase in the volume of the field, *i.e.* the volume swept out by the unit area, is dx c.cm. The work done in producing this volume of field is, therefore, $(KE^2/8\pi) \times dx$ ergs; hence the work done in producing unit volume of field, which is the energy per unit volume of the medium, is $KE^2/8\pi$ ergs, *i.e.*

$$\text{Energy per unit volume} = \frac{KE^2}{8\pi} \text{ (ergs per c.cm.)},$$

or, since $E = 4\pi D/K$, where D is the number of Faraday tubes per unit area, we have

$$\text{Energy} = \frac{KE^2}{8\pi} = \frac{2\pi D^2}{K} = \tfrac{1}{2}ED \text{ (ergs per c.cm.)} \quad \ldots .(15)$$

From analogy with the corresponding problem in elasticity, viz. energy per unit volume $= \frac{1}{2}$ stress $\times$ strain, if E be of the nature of "stress," D is the corresponding "strain," as already indicated.

In Art. 5 it was shown that the energy of a charged body was given by the expression $\frac{1}{2}QV$ ergs: the connexion between the

results of Art. 5 and those of this section would be appreciated, however, if we associated the energy of the medium with the Faraday tubes; for this, see *Advanced Textbook of Electricity and Magnetism.*

15. A Field with Two Dielectric Media. Boundary Conditions

So far we have assumed that the dielectric medium round about our charges has been *one homogeneous medium* of dielectric constant K. We come now to the case where a field contains more than one dielectric medium, and first we will find in a general way the conditions which *must* hold at a point on the boundary of two media of different dielectric constants K_1 and K_2.

Let AB (Fig. 192) be a *small* portion of the boundary between the upper medium of dielectric constant K_1 and the lower medium of dielectric constant K_2. Now whatever the magnitude and direction of the electric force (or intensity) in each medium at a point in the boundary, it can be resolved into two components, one parallel to or tangential to the surface at the point and the other normal to the surface at the point. And the first condition that must hold is that at any point in the boundary **the tangential component of the force in the K_1 medium must be equal to the tangential component of the force in the K_2 medium**: this can be shown as follows:—

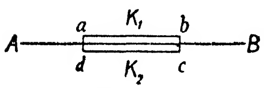


FIG. 192.

Consider an infinitely small cycle indicated by *abcd*, and imagine a charge of electricity moved round this cycle. As the cycle is infinitely narrow we can assume that the work in going along *da* and *bc* is zero. Now if the tangential component of the force in K_1 is greater than the tangential component in K_2, the work done, say, *on* the charge in going along *ab* will be greater than the work done *by* the charge in going along *cd*. Thus it would be possible to obtain an infinite supply of energy by suitably moving a charge round and round the cycle. This is impossible: hence the first condition that must hold at the boundary is that if at a point in it T_1 and T_2 be the *tangential* components of the *electric forces* in the two media, then:—

$$T_1 = T_2.$$

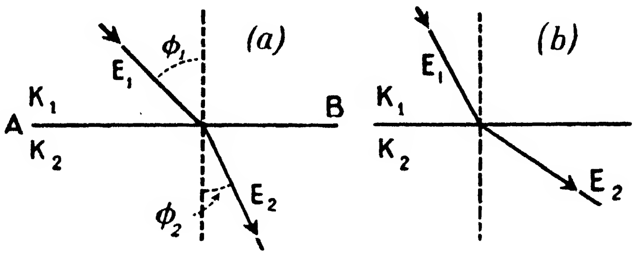
Again, as there is no free charge at the surface of separation of the two *dielectric* media, it is evident from the definition of displacement or polarisation that **at any point in the boundary the normal polarisation in one medium must be equal to the normal polarisation**

in the other medium. Thus, if F_1 and F_2 are the *normal components of the electric force* in the upper and lower media, then the corresponding normal polarisations are $K_1F_1/4\pi$ for the K_1 medium and $K_2F_2/4\pi$ for the K_2 medium, and the second condition which must obtain at the bounding surface is given by the relation:—

$$\frac{K_1}{4\pi}F_1 = \frac{K_2}{4\pi}F_2 \text{ (or, of course, } K_1F_1 = K_2F_2\text{)}.$$

16. Refraction of Faraday Tubes at the Boundary

When a tube passes from one medium to the other there is often a bending or refraction at the separating surface (Fig. 193) and the law of its refraction can be determined from the conditions specified above. Let E_1 and E_2 denote the magnitudes of the electric force (intensity) in the two media, and let the directions of these forces make angles ϕ_1 and ϕ_2 with the normal to the surface. Then the



(a) (b)

FIG. 193.

tangential components in the two media are $E_1 \sin \phi_1$, $E_2 \sin \phi_2$, and the normal components are $E_1 \cos \phi_1$, $E_2 \cos \phi_2$, and the necessary relations between these quantities are

$$E_1 \sin \phi_1 = E_2 \sin \phi_2$$

$$\frac{K_1}{4\pi}E_1 \cos \phi_1 = \frac{K_2}{4\pi}E_2 \cos \phi_2;$$

$$\therefore \frac{\tan \phi_1}{K_1} = \frac{\tan \phi_2}{K_2}, \text{ i.e. } \frac{\tan \phi_1}{\tan \phi_2} = \frac{K_1}{K_2} \quad \ldots\ldots\ldots(16)$$

a relation which determines the refraction of a tube in passing from a medium of dielectric constant K_1 to one of dielectric constant K_2. From this relation it is evident that if K_1 is greater than K_2, then ϕ_1 is greater than ϕ_2, that is, when a tube passes from one medium

to another of smaller dielectric constant the tube is bent towards the normal, as in Fig. 193 (*a*): if it passes to one of higher K value it is bent away from the normal (Fig. 193 (*b*)).

This question of refraction of the tubes may also be associated with the distribution of energy in the media of the field. It has been shown that, for a given polarisation, the energy per unit volume of a medium is smaller the greater the value of K (Energy $= 2\pi D^2/K$). Hence since the energy in the

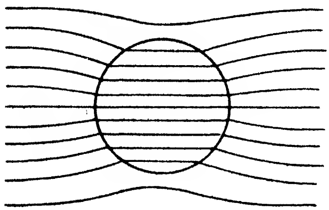


FIG. 194.

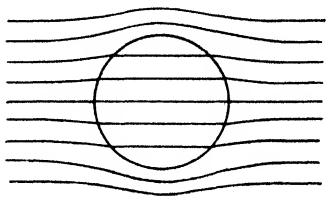


FIG. 195.

field always tends to a minimum, the tubes will pass as far as possible through the media of greatest K value, and the law of refraction from one medium to another is that each tube bends so as to take the path of least potential energy possible for it.

Fig. 194 illustrates the preceding for the case of a ball of sulphur or other dielectric of high dielectric constant placed in the initial uniform field in air between two charged parallel plates. Fig. 195 shows the case when K for the ball is less than K for the surrounding dielectric medium. For comparison the case of a conducting sphere in a uniform field is indicated in Fig. 196.

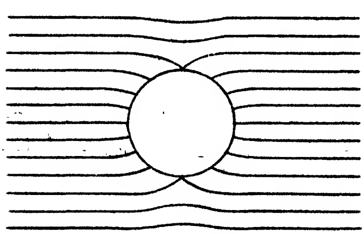


FIG. 196.

The fuller treatment of problems connected with electric fields containing two or more different *dielectric* materials is beyond the scope of this book, but the following elementary consideration of a *few* points about *one special case* is interesting and important.

17. Dielectric Slab at Right Angles to a Uniform Air Field

In Fig. 197 suppose A and B are two parallel metal plates, large compared with their distance apart, and that A is charged positively to a density $+\rho$ whilst B is charged negatively to density $-\rho$.

A slab of dielectric with parallel faces C and D and of dielectric constant K is placed as shown, the rest of the medium between A and B being air. By symmetry the electric lines are all perpendicular to the plane faces A, C, D, and B in this special case—there is no "bending."

The number of Faraday tubes from a charge Q is Q *whatever the medium*, and the charge on unit area of A is ρ: hence as the lines are normal to the surfaces in Fig. 197 the Faraday tubes per unit area in the air spaces AC and DB and also in CD is ρ. Again, the tubes of induction from a charge Q is $4\pi Q$ *whatever the medium:* hence the tubes of induction per unit area in AC, DB, and CD is $4\pi\rho$. The surfaces C and D are *uncharged*, and it may be taken as an absolute fact that Faraday tubes and tubes of induction never start from, or end at, an uncharged surface: these tubes always start from a positive charge and end at a negative charge—they never start or stop in any other way—but, of course, they may be more crowded together (*i.e.* more per unit area) in some parts of their journey than in other parts (*e.g.* in Figs. 194, 195).

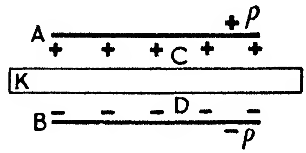


FIG. 197.

Coming to the *Maxwell unit tubes of force*, however, the number from charge Q is $4\pi Q/K$: hence the Maxwell tubes of force per unit area in the space AC is $4\pi\rho$ whilst in space CD it is $4\pi\rho/K$, so that $4\pi\rho - 4\pi\rho/K$, *i.e.* $4\pi\rho\,(1 - 1/K)$ Maxwell tubes of force have evidently terminated at each unit area of surface C although there is no free charge there. Again, the Maxwell tubes of force per unit area in the air space DB is again $4\pi\rho$, so that $4\pi\rho\,(1 - 1/K)$ tubes have started from each unit area of surface D although there is no free charge there.

If M Maxwell tubes of force end on, or start from a surface in the way indicated above, then M divided by 4π is sometimes called the "apparent charge" on that surface. Thus C above, where some Maxwell tubes of force *end*, has an apparent charge *per unit area* (density of charge) of $-\rho\,(1 - 1/K)$ and D, where some *start*, has an apparent charge of density $+\rho\,(1 - 1/K)$, *i.e.*

$$\left.\begin{array}{c}\text{Apparent charge per}\\ \text{unit area on C}\end{array}\right\} = -\rho\left(1 - \frac{1}{K}\right);$$

$$\left.\begin{array}{c}\text{Apparent charge per}\\ \text{unit area on D}\end{array}\right\} = +\rho\left(1 - \frac{1}{K}\right).$$

If K is greater than 1 (the other medium is air in this case where K = 1) the apparent charge on C is −ve (as is seen from its expression $-\rho\,(1 - 1/K)$ for unit area), and on D +ve just as the real charges induced on a conducting slab put there would be (but, of course, the apparent charges are *less in amount*). If K were infinite the apparent charges on the dielectric slab would be $-\rho$ and $+\rho$ per unit area, *i.e.* the same both in sign and amount as those induced on a conducting slab. In fact, a conducting slab is equivalent (electrostatically) to a dielectric slab of infinite dielectric constant.

In the above we have dealt with a simple special case of a dielectric slab normal to a uniform air field, but the general principles can be extended to more complex cases. As a practical illustration, if a glass ball (K for glass = say 6) be suspended (in air) and a strongly electrified body be brought near so that the ball is hanging in a non-uniform field, the glass ball, although uncharged, is attracted just as a brass ball would be but to a less extent: the ball is "apparently" charged, and as K for the ball (glass) is greater than the K value of the surrounding medium (air) the "apparent charges" have the same sign as the induced charges on the brass ball would have. Mathematics shows that the force on a dielectric ball in an electric air field is approximately equal to the force on a conducting ball multiplied by $(K - 1)/(K + 2)$.

If K for the dielectric ball were less than 1 (*i.e.* less than the K of the surrounding medium) the sign of the apparent charges would be the other way about and the ball would tend to move the other way in the non-uniform field, *i.e.* be "repelled"—move from strong to weak parts of the field.

Note that if, say a metal sphere with charge $+Q$ be suspended at the centre of a larger metal sphere, the charges on the inner and outer surfaces of the latter are $-Q$ and $+Q$. If the large sphere be replaced by a spherical shell of dielectric constant K, the "apparent" charges on its inner and outer surfaces are $-Q\,(1 - 1/K)$ and $+Q\,(1 - 1/K)$.

CHAPTER VII

CONDENSERS, ELECTROMETERS, AND ELECTROSTATIC MEASUREMENTS

THE condenser is a very important appliance both in laboratory and test-room work and in the various phases of applied electricity, both heavy and light electrical engineering work: the essential principle underlying the action has really been given by several of the experiments of Chapters V. and VI.

A—CONDENSERS

1. The Principle of the Condenser

In Fig. 198 A is a metal plate joined to the positive pole of a Wimshurst or of a battery and fully charged, *i.e.* charged up to the same potential, say, V, as the pole so that no more "charge" can be accumulated on it.

Now place another plate B near: it is acted on inductively.

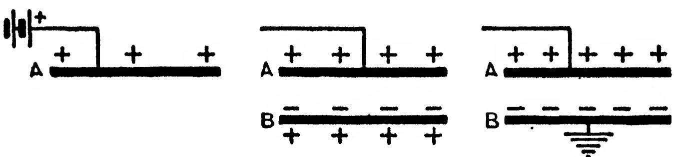


FIG. 198.

The negative on B lowers A's potential, and the positive on B raises it. The negative is nearer and has the advantage, so that A's *potential is lowered a little.* A further "charge" can now be given to A by the Wimshurst or battery to get its potential up to V.

Earth B: its positive disappears (and more negative appears). The negative cuts down A's potential, and as there is no positive on B to counteract this, *the potential of* A *is considerably weakened,* and a much bigger "charge" can now be accumulated to bring the potential once more up to the value V of the pole.

But the more "charge" a body takes in order to make it at a certain potential the greater is the *capacitance* of the body. Thus the capacitance of A is increased when B is near and more so when B is earthed. Such an arrangement of conductors which makes the capacitance greater and enables larger charges to be accumulated

is called a **condenser**: the conductors are called the *plates* or *coatings*, and the insulator in between them (air in this case) is called, of course, the ***dielectric of the condenser***.

A big conductor takes more charge in coming to a certain potential than a small one, so the larger the plates the greater will be the capacitance. It is clear, too, that the nearer the plates are together the better will be the result, *i.e.* the greater will be the capacitance. If a solid dielectric of glass, wax, mica, etc. (or a liquid dielectric) be put to fill the space between the plates of (*c*), then, since the electric intensity in the solid dielectric is only 1/K of what it is in the air dielectric, the potential difference between the plates will be reduced in the same ratio: thus the potential of A is reduced in the same way and the capacitance in consequence increased. Again, not only is the capacitance increased by the introduction of these substances, but owing to their greater mechanical rigidity there is less tendency to rupture and loss of charge by sparking between the plates. Note, of course, that solid dielectrics

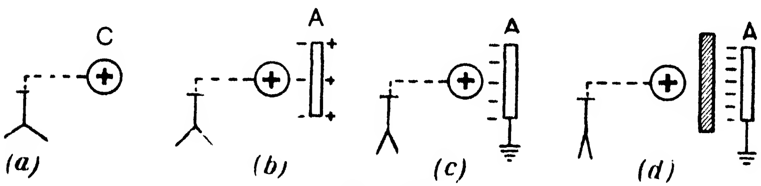


FIG. 199.

such as mica are permanently damaged by the passage of a spark through them as a permanent "puncture" is produced, but gaseous or liquid dielectrics recover their insulating properties.

In practical everyday electrical work condensers are usually "charged" by means of batteries (or the supply mains), A being joined to the positive pole and B to the negative pole of the battery. This simply means that whilst A takes the positive potential of the positive pole, B, instead of being zero potential, takes the ***negative potential*** of the negative pole, and there is a big positive charge on A and a big negative charge on B. The action and explanation are the same as the above.

Most of the points mentioned above have been shown by simple experiment (Chapter V.): we give one or two more. In Fig. 199, at (*a*) C and the electroscope are positively charged and at a positive potential. At (*b*) the metal plate A is brought near and the leaves collapse a little showing that the potential is less and the capacitance therefore greater, for more charge must be given to C to produce the same potential (*i.e.* same divergence). At (*c*)

the divergence is again less and more so at (*d*) showing further reductions in potential and therefore increases in capacitances.

In Fig. 200 A is a plate positively charged, and B is negative. They are a good distance apart and the electroscopes indicate their potentials. Bring them nearer: divergences are less so that potentials are less and capacitance greater. Raise one plate upwards so that the effective plate areas exactly facing each other are *less*: leaves diverge *more* showing *greater potentials* and therefore *less capacitance*. Interpose a slab of dielectric: less divergence follows showing increased capacitance.

2. Capacitance of a Condenser

The capacity or capacitance of a body has been defined as numerically represented by the quantity of electricity necessary to raise it to unit potential. Now the device of the *two* plates of Fig. 198 constitutes the condenser, and the " capacitance of the condenser " is defined similarly, only in this case it is the P.D. " between the coatings " which is involved in the definition: thus *the capacitance of a condenser is numerically represented by the quantity of electricity which must be given to it to establish unit potential difference between the coatings.* If one coating be earthed the capacitance of the condenser will be measured by the quantity of electricity necessary to raise the other coating to unit potential, *i.e.* the capacitance of the condenser is numerically the same as the capacitance of the plate A (Fig. 198) *if* B *is earthed.* Hence:—

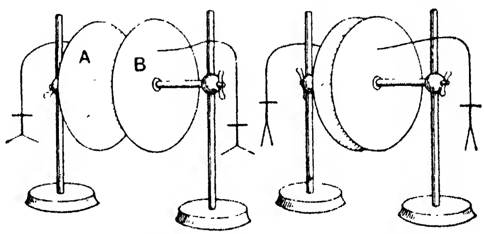


FIG. 200.

A condenser has a capacitance of one C.G.S. electrostatic unit if the electrostatic unit quantity produces a P.D. of one electrostatic unit between its coatings.

The practical unit is the farad; **a condenser has a capacitance of one farad if a charge of one coulomb produces a P.D. of one volt between its coatings.**

$$\text{One farad} = 9 \times 10^{11} \text{ electrostatic units.}$$

$$\text{One microfarad} = 1/10^6 \text{ farad} = 9 \times 10^5 \text{ electrostatic units.}$$

Note particularly that **the capacitance of a given condenser is constant,** depending only on its dimensions and the nature of the dielectric (this will be more fully realised presently), but the

capacitance of A (given by charge on A divided by the *actual potential of A*) is affected by the electrical condition of B, and is only numerically the same as the capacitance of the condenser when B is earthed, for then the potential of A is the potential difference between the coatings, since B is at zero potential.

It has been stated above that the capacitance of a given condenser is *constant*. It depends in fact on:—(1) The size of the plates —the greater the size the greater the capacitance. (2) The distance between the plates—the greater the distance the less the capacitance. (3) The constant of the dielectric—the greater the dielectric constant the greater the capacitance.

3. Dielectric Constant or Permittivity

We saw in Chapter VI. that the force between two charges in air was given by the expression $F = Q_1Q_2/d^2$, whereas at the same distance in another medium the expression became $F = Q_1Q_2/Kd^2$ where K was the "dielectric constant" of the medium: from this K might be defined as measured by *the ratio of the force between two charges in air to the force between the charges at the same distance in the medium.* Again, since $N = KE$ where $N =$ induction and $E =$ intensity in an electric field (pages 180, 181), K might be defined as *the ratio of* N *to* E.

Further, it has been shown that the capacitance of a conductor is increased if it is embedded in another dielectric in place of air. *If the capacitance of a conductor be increased* K *times when it is embedded in another dielectric in place of air,* K *measures the "dielectric constant" or "permittivity" of the dielectric.*

The preceding may be put into a more convenient form for experimental purposes, and this is usually given as the definition of K. Imagine two precisely equal condensers, one with air the other with D for dielectric, and let their capacitances be compared experimentally. *The ratio of the capacitance of the condenser with dielectric* D *to that of the equal air condenser gives the dielectric constant or permittivity of D, i.e.*—

$$\frac{\text{Capacitance of condenser with dielectric D}}{\text{Capacitance of equal air condenser}} = \text{Dielectric constant of D} = \text{K};$$

$$\therefore \left\{\begin{matrix}\text{Capacitance of condenser}\\ \text{with dielectric D}\end{matrix}\right\} = \text{K}\left\{\begin{matrix}\text{Capacitance of an equal}\\ \text{air condenser}\end{matrix}\right\}.$$

In less "mathematical" form we might say, therefore, that the extent to which the capacitance of a condenser is increased when

the space between the plates is filled with another dielectric material in place of air is a measure of the dielectric constant of that material.

The reason why the capacitance depends so much on the kind of dielectric material between the plates follows directly on what has been explained about displacement, induction, and the electron theory of matter. First imagine the two parallel plates A and B of a condenser equally and oppositely charged, A +ve and B −ve. Now suppose an insulated metal plate placed midway between them. It will have a −ve charge induced on its surface facing A and a +ve charge on the surface facing B. The result of this is practically to bring the opposite charges nearer together, which is equivalent to lessening the distance between the plates: the P.D. between the plates is thus reduced and the capacitance increased. A somewhat similar effect takes place if a slab of good dielectric takes the place of the metal plate or if the whole space between A and B be filled with the dielectric (say glass or wax or mica, etc.). As already explained electrons do not leave their atoms and " flow " through the dielectric as they do in the metal plate, but the +ve and −ve charges in the atoms are " displaced " relative to one another. The individual atoms become " polarised," their −ve towards A, and the result is similar to that indicated above. Air is polarised to some extent, but polarisation is much more pronounced with glass, mica, etc.

The dielectric constant used to be known as the *specific inductive capacity.* The newer name is better but by no means ideal, for the " constant " varies with the conditions of measurement: thus if alternating current be used the value obtained for K varies with the frequency. Displacement is not instantaneously complete; it takes a certain time to reach its full value when the displacing influence is " put on " and a certain time to disappear when it is " taken off," so that with a given stress the displacement is less when it is increasing than when it is decreasing, *i.e.* the displacement " lags ": there is a certain similarity with magnetic hysteresis, and the phenomena is often called dielectric hysteresis. The still newer name for K, viz. *permittivity,* emphasises its similarity to μ, the permeability in magnetism.

In selecting a substance for the dielectric of a condenser attention must be paid not only to its K value and general insulating properties, but also to its ability to stand the application of a high P.D. without being punctured or breaking down. The ability to withstand electrical " pressure " is spoken of as the **dielectric strength.** The break-down P.D. is usually taken to be proportional to the thickness of the dielectric (modern work disputes the *exact* proportionality), so dielectric strength is usually measured by the

P.D. in kilovolts necessary to break through one millimetre thickness: as examples:—glass 60, mica 100, paraffin wax 40, air 3 (kilovolts per mm. in each case). In engineering practice the dielectric strengths are often given in terms of volts per mil (1 mil = ·001 inch): as examples:—micanite plate 1012 volts per mil, oiled asbestos 320 volts per mil, red fibre 307 volts per mil.

In connexion with dielectric "polarisation" it may be noted that some substances are composed of charged *ions* which are "displaced" when the substance is put between the plates of a condenser. These substances also increase the capacitance but at the same time *they themselves get slightly longer* in the direction plate to plate. Quartz is a good example of this, and this property of quartz is used in tuning wireless circuits and securing constant frequencies for "radio" transmission at transmitting stations.

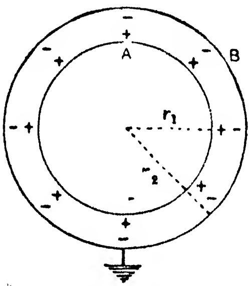


FIG. 201.

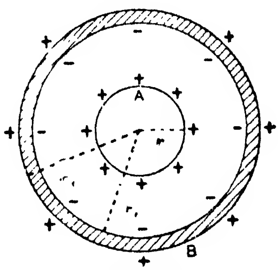


FIG. 202.

4. Capacitance of a Spherical Condenser

If a conducting sphere A (Fig. 201) of r_1 centimetres radius be concentric with another conducting hollow sphere B of r_2 centimetres radius, the space between being occupied by a dielectric of dielectric constant K they form, of course, a condenser, and an expression for the capacitance is readily obtained.

First let B be earthed and a charge $+ Q$ e.s. units be given to A which therefore induces $- Q$ e.s. units on the inner surface of B. The potential at *all points inside B* due to B's charge only is $- Q/Kr_2$ so that this is the potential of A due to B's charge. The potential of A due to its own charge is $+ Q/Kr_1$. Hence the actual potential (V) of the sphere A is given by:—

$$V = \frac{Q}{Kr_1} - \frac{Q}{Kr_2} = Q\frac{1}{K}\left(\frac{r_2 - r_1}{r_1 r_2}\right) \text{ e.s. units,}$$

and as B is earthed and at zero potential this is *the potential difference*

between the coatings of the condenser. Hence for the capacitance (C) of the condenser we have:—

$$\text{Capacitance} = C = \frac{\text{Charge on A}}{\text{P.D. between A and B}}$$

$$\therefore\ C = \frac{Q}{V} = K\frac{r_1r_2}{r_2 - r_1}\ \text{e.s. units} = \frac{r_1r_2}{r_2 - r_1}\ \text{e.s. units} \dots\dots (1)$$

the second expression applying to an air condenser. Note that the expressions for the capacitance *of the condenser* involve only dimensions and dielectric medium. To obtain the capacitance in farads divide by (9×10^{11}) and in microfarads by (9×10^{5}).

Now consider the case where B is insulated so that a charge $+ Q$ is induced on the outer surface, and let r_3 be the radius of this outer surface (Fig. 202). For simplicity let the medium be air. For the potential of A we now have $Q/r_1 - Q/r_2 + Q/r_3$ (see example, page 166), and for the potential of B we have $+ Q/r_3$. Thus the potential difference *between the coatings of the condenser* is $Q/r_1 - Q/r_2$, and the capacitance *of the condenser* is Q divided by $(Q/r_1 - Q/r_2)$, that is, $r_1r_2/(r_2 - r_1)$ e.s. units as before. (Note that whilst the *condenser capacitance* is the same, *the capacitance of* A—got by dividing Q by the new potential of A—is not the same.) This second case, however, used as indicated, would not be effective in obtaining large *charges* on A for the potential of A is not sufficiently reduced by the presence of B.

Using simple Calculus the capacitance of a spherical condenser is quickly determined. If E = intensity at a point between the spheres distant r from the centre, $E = Q/Kr^2$. Hence:—

$$\text{P.D. between spheres} = V_A - V_B = \int_{r_2}^{r_1} - E\,dr = - \int_{r_2}^{r_1} \frac{Q}{Kr^2}dr;$$

$$\therefore\ \text{P.D.} = \frac{Q}{Kr_1} - \frac{Q}{Kr_2} = Q\frac{1}{K}\left(\frac{r_2 - r_1}{r_1r_2}\right);$$

$$\therefore\ \text{Capacitance of condenser} = \frac{Q}{\text{P.D.}} = K\frac{r_1r_2}{r_2 - r_1}\ \text{e.s. units.}$$

5. Capacitance of a Parallel Plate Condenser

(1) Let A sq. centimetres be the surface area of plate A (Fig. 198), t centimetres the distance between the plates, and K the dielectric constant of any dielectric *completely filling* the space between them; the capacitance can be shown to be—

$$C = K\frac{A}{4\pi t}\ \text{electrostatic units} \dots\dots\dots\dots\dots (2)$$

if the plates are large compared with their distance apart so that there is no disturbance due to edge distribution, and the field is uniform between the plates and normal to them.

A proof of this may be obtained by considering a spherical condenser of inner radius r_1, which is exceedingly large, separated from the outer sphere by a small radial distance t, the medium having a dielectric constant K. Since $r_2 = r_1 + t$, we have—

$$C = K\frac{r_1(r_1 + t)}{t} = K\frac{(r_1^2 + r_1t)}{t} \text{ e.s. units.}$$

Now, if r_1 be imagined infinitely great, so that any small portion of the sphere is practically plane, and t is small, r_1t may be neglected in comparison with r_1^2, and the expression becomes practically $C = Kr_1^2/t$. Multiplying numerator and denominator by 4π:—

$$C = K4\pi r_1^2/4\pi t \text{ e.s. units,}$$

the numerator, $4\pi r_1^2$, being the area of the whole inner sphere. Since this is true for the whole, it will also be true for any part; hence if A sq. cm. be the area of any part—practically plane—

$$C = K\frac{A}{4\pi t} \text{ e.s. units} = \frac{A}{4\pi t} \text{ e.s. units (air condenser).}$$

If we again assume the plates large compared with their distance apart so that there is no edge disturbance we can establish the formula as follows:—Let the density of the charges on A and B be $+\rho$ and $-\rho$. A positive unit in between the plates will be repelled by A with a force $2\pi\rho/K$, and it will be attracted by B with an equal force; hence the force on the positive unit will be $4\pi\rho/K$, and the work done in moving it from B to A will be $(4\pi\rho/K) \times t$. But this work in ergs measures the P.D. in e.s. units; also the total charge on A is $A\rho$. Hence

$$\text{Capacitance} = \frac{\text{Charge on A}}{\text{P.D. between A and B}} = \frac{A\rho}{4\pi\rho t/K} = K\frac{A}{4\pi t}.$$

(2) In the above the dielectric of dielectric constant K *completely fills* the space between the plates. Consider now a slab of dielectric between the plates the remaining dielectric medium being air (Fig. 203). If d be the thickness of the slab of dielectric constant K and t be the distance between the plates as before, the capacitance of the condenser can be shown to be

$$C = \frac{A}{4\pi\left(t - d + \frac{d}{K}\right)} \text{ e.s. units} \quad \dots\dots\dots\dots(3)$$

Before proceeding with this proof the student should again read Art. 17 of Chapter VI. We again assume the plates large compared with the distance apart so that all lines are perpendicular to the planes A, B, and the faces of the slab. Let the surface densities of the charges on A and B be $+\rho$ and $-\rho$.

The intensities (forces on unit charge) in the three regions P, Q, and R (Fig. 203) are $4\pi\rho$, $4\pi\rho/K$, and $4\pi\rho$ respectively. The total work done in conveying unit quantity from B to A is therefore:

$$\text{Work} = 4\pi\rho y + \frac{4\pi\rho}{K}d + 4\pi\rho x = 4\pi\rho\left(x + y + \frac{d}{K}\right).$$

and this measures the P.D. The charge on A is $A\rho$. Hence:—

$$\text{Capacitance} = \frac{A\rho}{4\pi\rho\,(x + y + d/K)} = \frac{A}{4\pi\,(t - d + d/K)},$$

from which it follows that the capacitance of this condenser is the same as it would be if the dielectric of thickness d were replaced by a layer of air of thickness d/K (*i.e.* a *less* thickness of air).

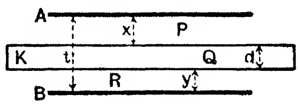


FIG. 203.

6. Capacitance of a Cylindrical Condenser. Cables

A cylindrical condenser consists of two coaxial cylinders. For simplicity consider an air condenser. If the inner cylinder is charged and the outer one connected to earth, the induced charge on the outer cylinder will be equal and opposite to that on the inner one for all the tubes of induction emanating from the inner cylinder must terminate on the inner surface of the outer one. Let the charge *per unit of length* on the inner cylinder be Q units. Then, as shown on page 188, the electric force at a point at a distance r from the axis of the cylinders is $2Q/r$. Let the radii of the inner and outer charged cylindrical surfaces be a and b cm. respectively. Then the work done in conveying unit quantity of electricity from the outer to the inner surface is given by

$$\text{Work} = -\int_b^a \frac{2Q}{r}dr = 2Q\log_e\frac{b}{a}, = \text{P.D. between them.}$$

Hence the capacitance of the condenser *per unit length* is—

$$\frac{Q}{PD} = Q/2Q\log_e\frac{b}{a} = 1/2\log_e\frac{b}{a}, \text{ and for a length } l \text{ cm.:—}$$

$$\text{Capacitance} = \frac{l}{2\log_e(b/a)} \text{ or Capacitance} = K\frac{l}{2\log_e(b/a)} \text{ e.s. units} \quad (4)$$

Common logarithms (of base 10) may be converted into Napierian logarithms (of base $e = 2{\cdot}71828$) by multiplying by $2{\cdot}3026$: hence:—

$$\text{Capacitance} = K \frac{l \text{ (cm.)}}{2{\cdot}3026 \times 2 \log_{10}\frac{b}{a}} \text{ e.s. units} \quad \ldots\ldots (5)$$

and, of course, as b/a is a mere ratio, b and a may be in any length unit provided that the same be used for both.

Every overhead telegraph line and every wireless aerial is a condenser of small capacitance, the conductor forming one coating, the ground and neighbouring earth-joined bodies the other coating, and the intervening air the dielectric. Concentric and submarine cables are condensers of decided capacitance, the surface of the copper conductor being one coat, the water or metal sheath the other, and the insulating material—gutta-percha, india-rubber, diatrine, vulcanised bitumen, air-paper, etc.—the dielectric. The following practical capacitance formulae are quoted for reference: for "proofs" see *Advanced Textbook of Electricity and Magnetism.*

(*a*) **Submarine or Concentric Cable.** This is merely the case established above. If the length of the cable be l cm., the radius (or diameter) of the inner conductor r_1 cm., the outer radius (or diameter) of the insulating material r_2 cm., and K the dielectric constant of the insulation:—

$$C = K \frac{l}{2 \log_e \frac{r_2}{r_1}} = K \frac{l \text{ (cm.)}}{2{\cdot}3026 \times 2 \log_{10}\frac{r_2}{r_1}} \text{ e.s. units,}$$

$$\textit{i.e.}\ C = K \times \frac{2{\cdot}413}{10^7} \times \frac{l \text{ (cm.)}}{\log_{10}\frac{r_2}{r_1}} \text{ microfarads.}$$

(*b*) **Two Parallel Wires.** If two isolated parallel wires are d cm. apart and each is of radius r cm. and length l cm.

$$C = \frac{l}{4 \log_e \frac{d}{r}} \text{ e.s. units} = \frac{l \text{ (cm.)}}{900000 \times 4 \log_e \frac{d}{r}} \text{ microfarads.}$$

(*c*) **Telegraph Wire and Horizontal Wireless Aerial.** If l = length (cm.), r = radius (cm.), and h = height above the ground (cm.):—

$$C = \frac{l}{2 \log_e \frac{2h}{r}} \text{ e.s. units} = \frac{l \text{ (cm.)}}{900000 \times 4{\cdot}6052 \times \log_{10} \frac{2h}{r}} \text{ microfarads.}$$

The capacitance of the vertical part of an aerial may be calculated by assuming it to be equal to that of the same wire stretched horizontally at the same *mean* height. It will be understood that the above is only an approximate expression for an actual aerial in practice, although it is near enough for most purposes. For an accurate calculation several factors have to be taken into account—effect of masts and buildings, etc.

AN X-RAY PHOTOGRAPH.

Compound comminuted fracture of upper arm—an injury received during the last war (1914-1918). The photograph also shows numerous small particles of bone and metal. Radiograph by Dr. James F. Brailsford, Radiologist to the Queen's Hospital, Birmingham.

(*d*) **Flat Circular Disc.** The capacitance of an isolated thin circular disc of radius r cm. is:—

$$C = \frac{2r}{\pi} \text{ e.s. units} = \frac{r}{450000\pi} \text{ microfarads.}$$

(*e*) **Example.**—*Determine the capacity of one nautical mile of submarine cable of which the following particulars are known: Diameter of copper conductor =* ·16 *inch; outside diameter of gutta-percha =* ·40 *inch; dielectric constant of gutta-percha =* 4·03; $\log_{10} 2 = \cdot 30103$.

$$C = K \frac{l}{900000 \times 4{\cdot}6052 \times \log \frac{r_2}{r_1}} \text{ microfarads.}$$

$$l = 2029 \text{ yd.} = (2029 \times 3 \times 12 \times 2{\cdot}54) \text{ cm.} \quad K = 4{\cdot}03$$

$$\log_{10} \frac{r_2}{r_1} = \log_{10} \frac{40}{16} = \log_{10} \frac{10}{4} = \log_{10} 10 - \log_{10}(2)^2$$

$$= \log_{10} 10 - 2 \log_{10} 2 = 1 - {\cdot}60206 = {\cdot}39794.$$

Substituting in the formula, $C = {\cdot}45$ m.f. per nautical mile.

7. Force of Attraction between the Plates of a Condenser

Since the A plate of a condenser is positively charged and the B plate is negative there will be attraction between them, and the magnitude of this force is easily found. Let $+\rho$ and $-\rho$ be the densities of the charges on A and B respectively, A sq. cm. the area of each plate (supposed large compared with the distance apart) and first let the medium be air.

Each unit of charge on A is attracted by B with a force $2\pi\rho$ (cf. pages 88, 191). Now unit area of A contains a charge ρ: hence for the force of attraction on unit area of A we have:—

$$\text{Force on unit area} = 2\pi\rho \times \rho = 2\pi\rho^2 \text{ dynes} \quad \ldots\ldots(1)$$

and the force on the whole area A if we assume the charge uniform over the whole surface $= 2\pi A\rho^2$ dynes.

Now suppose the space filled with a dielectric of dielectric constant K. The force on each unit of charge on A due to the attraction is now $2\pi\rho/K$: hence for the force of attraction on unit area of A now we have:—

$$\text{Force on unit area} = \frac{2\pi\rho}{K} \times \rho = \frac{2\pi\rho^2}{K} \text{ dynes} \quad \ldots\ldots(2)$$

and the force on the whole area A is $2\pi A\rho^2/K$ dynes.

It is important to remember that the force of attraction between the plates of a condenser is $2\pi\rho^2/K$ ***dynes per unit area.*** Further, if E

be the *intensity* in the field between the two plates $E = 4\pi\rho/K$ (see pages 88, 189): hence $\rho = KE/4\pi$, and we can write:—

$$\text{Force on A} = \frac{2\pi\rho^2}{K} \times A = \frac{KE^2}{8\pi}A.$$

There is a further point to note in connexion with the above. When the dielectric (K) was inserted between the plates it was assumed that the *charges on the plates still remained the same* so that the density on A was still ρ, and (1) and (2) show that in this case (*charge* constant) *the force of attraction on* A *when the dielectric is inserted is only* $1/K$ *of the force on it when the medium is air.* When the dielectric is inserted the *potential* of A decreases, so that a larger charge must be given to it to bring it to the same potential as before. It is important, then, to determine the force between the plates if A be given this further charge when the dielectric is inserted so as to make the *P.D. between the plates the same* as before (this would happen if the condenser remained "joined up" to the charging device all the time).

Since the intensity (E) between the plates of a condenser is $4\pi\rho/K$, the work done in taking unit quantity from B to A $= (4\pi\rho/K) \times t$ ergs. But this measures the P.D., say V, between the plates: hence

$$V = \frac{4\pi\rho t}{K}; \quad \therefore \rho = \frac{KV}{4\pi t}.$$

First take again the air condenser and we have for the force on A (since $K = 1$):—

$$\text{Force on A} = 2\pi A\rho^2 = 2\pi A\left(\frac{V}{4\pi t}\right)^2 = \frac{AV^2}{8\pi t^2} \quad \ldots\ldots\ldots(3)$$

It is important to note that the attraction between the plates of a condenser is proportional to the square of the P.D. between them.

Now take the condenser with dielectric inserted, and let the charge be increased so that *the potential difference* V *is the same as before.* Let ρ_1 be the new density:—

$$\text{Force on A} = \frac{2\pi A\rho_1{}^2}{K} = \frac{2\pi A}{K}\left(\frac{KV}{4\pi t}\right)^2 = K\frac{AV^2}{8\pi t^2} \quad \ldots\ldots(4)$$

and we have [by (3) and (4)] that if the potential difference between the plates be the same in the two cases *the force of attraction on* A *when the dielectric is inserted is* K *times greater than when the medium is air.* Summarising, when the dielectric K replaces air:—

Force is $1/K$ of what it is with air if charges are the same.

Force is K times what it is with air if P.D.s are the same.

Again, since the *energy* of a condenser is $Q^2/2C$, it follows that *if* Q *be constant*, the energy is *inversely* as the capacitance, *i.e.* it is less with a medium of dielectric constant K than it is with air. Further, since the *energy* is also $\frac{1}{2}CV^2$, it follows that *if* V *be constant*, the energy is *directly* as the capacitance, *i.e.* it is greater with the medium of dielectric constant K than it is with air. In the above K is, of course, assumed greater than unity.

In the above the dielectric completely fills the space between the plates. If it is a slab, say midway between the plates, then, if the *charge* be the same, the force on A is *unaltered* when the slab is inserted, for the plate is still in contact with air; but if the *potential* be made the same the force on A is greater (again assuming $K > 1$). We leave the full proof as an exercise for the student. (See also *Advanced Textbook of Electricity and Magnetism.*)

8. Condensers in Parallel and in Series

A parallel group is indicated in Fig. 204; it is equivalent to forming one large condenser, the area of whose coatings is equal to

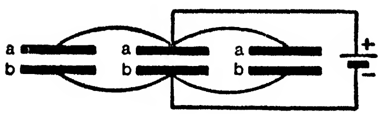


FIG. 204.

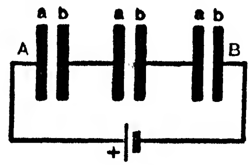


FIG. 205.

the sum of the areas of the coats of the individual condensers. If C be the joint capacitance, C_1, C_2, C_3 separate capacitances,

$$C = C_1 + C_2 + C_3 \quad \dots\dots\dots\dots\dots\dots (1)$$

If the condensers are alike, say each C_1, the joint capacitance is $3\,C_1$, and if there are n condensers the combined capacitance is nC_1.

Again, if V be the P.D. between the coatings, the charge given to each is C_1V and the total charge is CV, *i.e.* nC_1V.

Finally, the energy of each is $\frac{1}{2}C_1V^2$ and the accumulated energy of the battery is $\frac{1}{2}CV^2$, *i.e.* $n\frac{1}{2}C_1V^2$.

Summary of *n* equal Condensers in Parallel.

(1) Joint capacitance = n times the capacitance of one condenser.
(2) Total charge = n times the charge of one condenser used alone.
(3) Total energy = n times the energy of one condenser used alone.

A series or cascade is indicated in Fig. 205. Since the outflow from one condenser passes to the next, the charge Q on the positive coating of each must be the same and equal to that communicated

to the first condenser from the battery. If V be the P.D. between A and B, V_1, V_2, V_3 the P.D.'s between the coatings of the separate condensers, C_1, C_2, C_3 their capacitances, and C the joint capacitance,

$$V = V_1 + V_2 + V_3;$$

$$\therefore \frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}, \ i.e. \ \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad \ldots\ldots\ldots (2)$$

If the capacitances are the same, say each C_1, the above becomes $1/C = 3/C_1$, *i.e.* $C = \frac{1}{3}C_1$. Similarly if there are n equal condensers each of capacitance C_1, the joint capacitance is C_1/n.

The charge and energy expressions may be worked out as before, and we get:—

Summary of *n* equal Condensers in Series.

(1) Joint capacitance $= 1/n$ of the capacitance of one condenser. (2) Total charge = same as on one condenser used alone. (3) Total energy $= 1/n$ of the energy of one condenser used alone.

Note that joining condensers in series *lessens* the capacitance: the capacitance of 5 equal condensers in series is $\frac{1}{5}$ of the capacitance of one. Joining in parallel *increases* the capacitance: the capacitance of the 5 in parallel would be 5 times that of one of them. (The rule is the exact opposite to that for the *resistances* of conductors in series and parallel—see page 295.)

9. Residual Charges

A condenser designed by Musschenbrock at Leyden nearly 200 years ago and known as the *Leyden jar* played an important part in the growth of electrical theory, but, of course, it is " out-of-date " as a practical condenser. It is a glass jar, the bottom and sides (to about three-quarters of the height) being coated inside and outside with tin-foil, the tin-foil forming the two " plates " and the glass the dielectric. Contact with the inner coating is made by a brass rod carrying a brass chain at its lower end, the chain resting on the inside foil. To charge the jar the outside coating is held in the hand (earthed) and the rod made to touch the pole of the Wimshurst: to discharge, the outer and inner coatings are joined by an insulated wire. The jar as ordinarily constructed has not a very great capacitance, but it can withstand high P.D.s without a breakdown.

It was early discovered that if a Leyden jar be charged to a given P.D. and then be allowed to stand insulated for a time, the P.D. diminishes (although there is no leakage), so that a further

small charge is necessary to bring the P.D. up to the original value: this may even be repeated. Again, if a jar be suddenly discharged so that the two coatings are brought to the same potential, and then be allowed to stand insulated for a time, it will be found to gradually acquire a potential of the same sign as at first, but smaller, and a second small discharge can be obtained: with some Leyden jars four or five successive discharges can be obtained in this way, the jar being allowed to rest insulated after each discharge. These effects are known as *residual effects*, the small extra charges being called *residual charges* (*or discharges*).

Faraday assumed the above phenomena to be due to what was termed *electric absorption*—in simple language, that the charges *soaked* into the dielectric. On applying the P.D., *i.e.* on charging, the penetration was not instantaneously complete—after the first main penetration it gradually increased for a time: similarly, on discharging the charges did not all come out at the first main discharge. We have seen, however, that when the P.D. is applied "displacement" occurs *in the dielectric atoms*, *i.e.* they are "polarised." This displacement depends on the kind of dielectric—it is much more pronounced with glass than with air (hence the greater K value): further, it is not *quite* complete immediately the P.D. is applied. Moreover, when the condenser is discharged the polarisation or displacement does not completely disappear at once—the medium *nearly* recovers and then completes its recovery gradually. Adopting the conception of "strain," it would be said that when the jar is charged the glass experiences a severe electric strain. On discharging, the potentials are equalised, but *the glass does not at once completely regain its original unstrained state*: there still remains a small amount of strain. During the minute or two of waiting, the glass picks up its overtaxed power of righting itself, and as it does so *applies additional potential to the inner coat, so that the potential of the latter becomes positive*, and all the rest follows.

The following analogy may assist the student in connexion with this matter. If a glass fibre be fixed at one end and the other end be twisted, the twist will extend along the whole fibre up to the fixed end. If the twisting couple be constant the fibre twists practically immediately through a certain angle, say α, and then gradually increases this by a small amount β, so that after a time the twist is $(\alpha + \beta)$. If the twisted end be now released the fibre untwists practically at once through an angle α. If it is then immediately clamped and after a time again released, it will turn through a further small angle β_1 very nearly equal to β.

The case of the Leyden jar is somewhat similar. In charging, a P.D., say V, is applied, and the jar takes a charge, say Q: if the potential be kept at V for a time the jar takes a further small charge q. In discharging suddenly, the jar at once loses an amount practically equal to Q: if allowed to stand for a time a further small charge q_1 can be obtained and sometimes another q_2—the residual discharges. Of course, the sum of all the output charges is equal to the total input charge.

Residual effects are not apparent with air condensers: they are only small with mica, quite noticeable with paraffin-wax paper, but most noticeable with glass.

It is evident that an experimental determination of the capacitance of a condenser may be affected by the phenomena referred to, *i.e.* the result obtained may depend on the time of charging. Thus in the case of the Leyden jar mentioned above, if the capacitance be measured at once we obtain $C = Q/V$, but if we keep it joined to the charging battery (say) for a time and then make the observations we get $C_1 = (Q + q)/V$, *i.e.* a larger result. Hence capacitance might be more exactly defined as *measured by the instantaneous quantity required to produce unit P.D. between the coatings.*

Note that if a Leyden jar be charged, then after a short interval be discharged, then again charged and then discharged, and if this alternate charge and discharge be repeated the glass becomes heated. Charge Q enters at potential Q/C and charge Q leaves at potential Q/C_1 and C_1 is greater than C so that Q/C_1 is less than Q/C. There is therefore a loss of energy at each double operation, and this lost energy is spent in heating the glass. It is evident that care must be exercised in selecting a suitable dielectric for a condenser if it is to be used in engineering practice on an A.C. circuit.

10. Various Types of Condensers in Practice

Two metal plates placed parallel to each other a short distance apart in air form a simple parallel plate air condenser. Sometimes a number of plates are arranged in this way (Fig. 206), alternate plates being joined together and connected to the two terminals of the condenser: this increases the effective area of the "two plates." In engineering practice the two sets of interlaced plates are often immersed in insulating oil: this increases the capacitance owing to the higher dielectric constant of the oil, and improves insulation. Another type is constructed of alternate sheets of tin foil (thin lines of Fig. 207) and paraffined paper or mica (thick lines), the odd conducting sheets, as before, being bunched together and connected to one terminal A, the even numbers being joined to terminal B; the arrangement is thus equivalent to two large plates separated by a thin and good dielectric. (The sheets, of course, rest *on* the paper

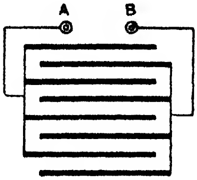


FIG. 206.

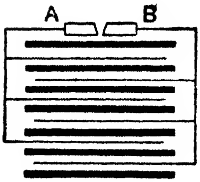


FIG. 207.

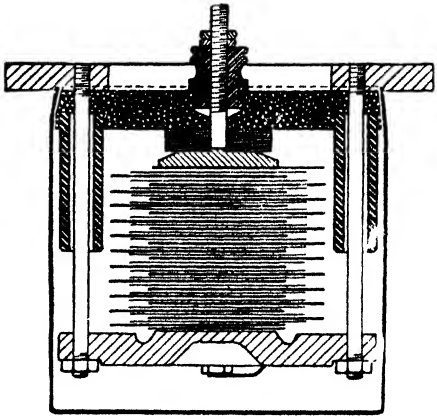


FIG. 208.

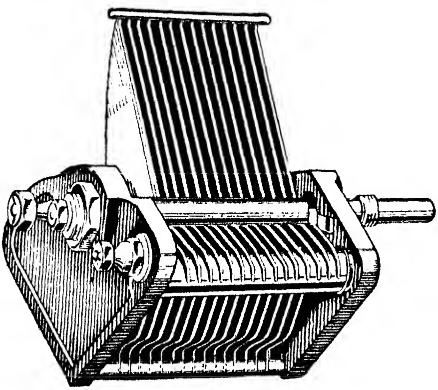


FIG. 209.

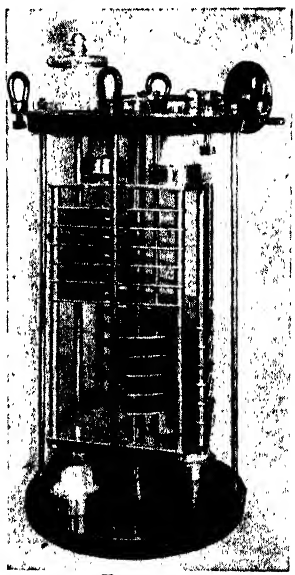


FIG. 210.

or mica: the small "fixed condensers" used in wireless are made on this principle.) In engineering practice the nest of plates, etc., are also often oil immersed: in one form the plates with intervening micas are firmly clamped between heavy end-plates and the whole enclosed in a containing vessel of insulating oil (Fig. 208).

Fig. 209 shows a type of "variable air condenser," largely used in wireless. One "coating" consists of a number of fixed metal plates arranged parallel with air spaces between them: these are all joined together and to one terminal of the condenser. The other "coating" consists of another set of parallel metal plates attached to a spindle and capable of being rotated "into" or "out of" the spaces between the fixed plates. Clearly the more the movable plates are turned "into" the fixed, the greater is the

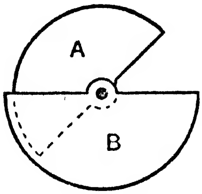


FIG. 211.

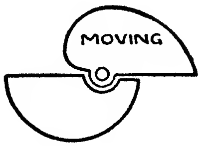


FIG. 212.

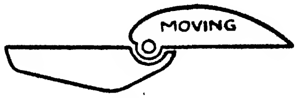


FIG. 213.

capacitance, for it means that the effective areas of the two plates facing each other are greater. For work on an "engineering" scale variable condensers are also often oil-immersed (Fig. 210).

There is quite a different type of condenser which is of importance particularly where very large capacitances are required for relatively low voltage circuits (it is now widely used in wireless). This is the **electrolytic condenser** which depends upon electrolytic action for its properties. It is described in Chapter XIII.

It may be noted that in condensers used in wireless of the type shown in Fig. 209 the plates are sometimes semi-circular (Fig. 211): this is known as a *straight-line capacitance condenser*, for the rotation is more or less proportional to the capacitance. Sometimes the shape is as shown in Fig. 212: this cam-shaped moving coating gives a *straight line wave-length condenser*, which is more useful for wireless into which "wave-length" enters so much. Yet another type is shown in Fig. 213 known as a *straight-line frequency condenser*, which is even more important in wireless receivers, since stations are separated from each other by a definite frequency. We cannot *explain* here the full "theory" of these shapes.

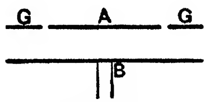


FIG. 214.

There are two condensers which are much used in accurate test-room work. In a simple parallel plate condenser the density is greatest round the edges of the plate and the field is not everywhere uniform as is assumed in developing the formula $A/4\pi t$. This is remedied in the **guard-ring condenser** (Fig. 214) by surrounding the plate A by a metal ring G, the two being in the same plane.

If A and G are in conducting communication and charged, the effect of G is to prevent this irregularity at the edge of A. The second plate B of the condenser is fixed to an insulating support and its distance from A can be adjusted by a micrometer screw. The capacitance of the condenser is given by the formula $A/4\pi t$, A being the mean of the areas of A and the circular opening in G. The reason for taking this mean will be gathered from Fig. 215.

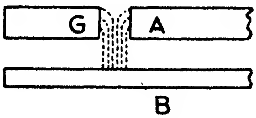


FIG. 215.

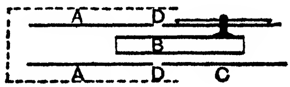


FIG. 216.

The principle of the **sliding condenser** is shown in Fig. 216. A and C are two metallic cylinders in line and separated by a small air gap D, whilst B is an inner coaxial cylinder carrying a slider which moves on C. A is insulated and B and C are earthed. If B be caused to slide into A by an additional distance l, the *change* in the capacitance is $l/2 \log_e (b/a)$ (Art. 6). B is supported on vulcanite rests inside C and metal covers are arranged to protect A from external disturbing influences. It will be noted that it is the *change* in capacitance due to a sliding of B through a certain distance which is observed: if, however, the capacitance be determined experimentally for one position of B the capacitance for any other position will be known.

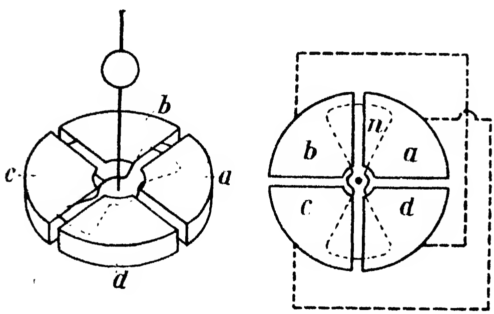


FIG. 217.

B—ELECTROMETERS

11. The Quadrant Electrometer

We have seen that the gold-leaf electroscope measures, and its action depends on, *potential*, but for more accurate determinations, particularly of small potentials, some form of **electrometer** is generally employed.

The **quadrant electrometer** consists of a shallow cylindrical metal box divided into four quadrants, each quadrant being supported on an insulating pillar, and opposite quadrants being connected by fine wires, viz. *a* to *c*, and *b* to *d*, as indicated in Fig. 217. A light spindle-shaped vane or needle *n* of aluminium is suspended by a fine silver wire, so that it hangs inside the box symmetrically along one of the lines of separation of the quadrants. In Kelvin's original pattern the needle was connected by a platinum wire to the inside coating of a Leyden jar (which has sulphuric acid for its inner coating, and tin-foil for its outer coating), situated at the bottom of the instrument, and was kept charged to a *constant high* potential. This charge was kept up by means of a small induction machine (the *replenisher*, page 157), and the *constancy* of the potential was ascertained by means of a *gauge*, which was really an attracted disc electrometer (Art. 13). If now the opposite pairs of quadrants be connected to two bodies at different potentials, the high potential needle will be deflected towards that pair of quadrants with the lower potential, and the magnitude of the deflection will depend on the P.D. between the two pairs of quadrants, *i.e.* upon the P.D. between the two bodies joined to them. The torsion of the suspension, of course, furnishes the controlling couple. Deflections are read by the "lamp and scale" method. The Kelvin instrument, however, has been almost entirely superseded by the Dolezalek.

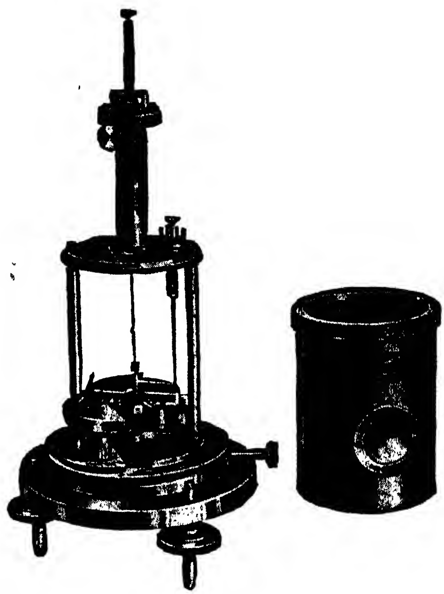


FIG. 218.

The **Dolezalek quadrant electrometer** is of more slender construction and much more sensitive than the Kelvin: it is an important instrument, and is shown in Fig. 218. The needle is of silvered paper and is suspended by a quartz fibre which is made conducting by dipping it into a solution of calcium chloride. The quadrants

are made of brass and are well insulated by being supported on amber pillars. Access to the needle is rendered easy by mounting two adjacent quadrants on a brass piece so that they can be swung to one side and returned again to the working position as desired. There is no Leyden jar, the needle being charged by a battery to about 80 volts; the spot of light moves about a metre on a scale at a metre distance when the potential difference between the quadrants is 1 volt. The instrument is contained in a brass case provided with a window for the light to pass to and from the mirror. The working capacity of the instrument is about 50 e.s. units. It is extensively used in measurements connected with radio-activity, conduction of electricity through gases, etc.

The **Compton quadrant electrometer** is similar to the Dolezalek, but it is smaller, and arrangements are such that the vane can be tilted and one of the quadrants raised or lowered, thus varying the sensitivity of the instrument. In **Lindemann's electrometer** the needle consists of gilded glass threads, and is attached to the *middle* of the suspension fibre of silvered quartz, both ends of the latter being fixed to a quartz frame. This frame is surrounded by a metal case provided with a window for observation purposes, such observations (of one end of the needle) being made by a microscope fitted with an eye-piece scale. The needle moves between *four rectangular plates* cross connected.

12. Theory of the Quadrant Electrometer

It will considerably help the student to get clear ideas on this if we first merely summarise the essential formulae which we use with the instrument. In fact the beginner may omit, on a first reading, the "proofs" of these formulae which follow this summary.

If θ be the deflection, V, v_1, and v_2 the potentials of the needle and the two pairs of quadrants, it can be shown that:—

$$\theta = k_1\left(V - \frac{v_1 + v_2}{2}\right)(v_1 - v_2) \quad \ldots\ldots (1)$$

where k_1 is a constant. If V be very large compared with v_1 and v_2, then (1) may be written:—

$$\theta = k_1 V (v_1 - v_2) \quad \ldots\ldots (2)$$

Thus the deflection of the spot of light is proportional to the potential of the needle and the potential difference between the two pairs of quadrants, or, since V is constant, *θ is proportional to the P.D. between the two pairs of quadrants*, *i.e.* between any two points joined to them.

Frequently one pair of quadrants is earthed and the other pair joined to a body, the potential of which is required: in this case, if $v_2 = 0$, equation (2) becomes:—

$$\theta = k_1 V v_1 \quad \text{.........................} (3)$$

i.e. *the deflection is proportional to the potential of the insulated pair of quadrants and that of the body joined thereto.*

In measuring large differences of potential the needle, instead of being separately charged, is frequently joined to one pair of quadrants: in this case, putting, say, $v_1 = V$ in (1)—

$$\theta = k_2 (V - v_2)^2 \quad \text{.......................} (4)$$

i.e. *the deflection is proportional to the square of the potential difference between the two pairs of quadrants*: if the needle and one pair of quadrants be connected together and to a source at potential V, and the other pair be earthed:—

$$\theta = k_2 V^2 \quad \text{..........................} (5)$$

This is independent of the sign of V, so that an alternating potential may in this way be measured.

The essential facts then are as follows:—If the needle be at a constant high potential *the deflection is proportional to the potential difference between the two pairs of quadrants,* and if one pair be earthed (zero potential), and the other pair connected to an electrified body, *the deflection will be proportional to the common potential of the body and the quadrants to which it is joined;* in both these cases, *i.e.* when the needle and the two pairs of quadrants are all at different potentials, the instrument is said to be used **heterostatically.** If the needle and one pair of quadrants be connected together, and at a much higher potential than the other pair, *the deflection is proportional to the square of the potential difference between the two pairs of quadrants;* in this case the instrument is said to be used **idiostatically.**

We can now establish the above relationships as follows:—Let V, v_1, and v_2 denote, as above, the potentials of the needle and the two pairs of quadrants. Let θ be the deflection, and T the corresponding couple exerted on the needle by its torsion fibre. It will be seen that the quadrant-needle system constitutes a double condenser, each pair of quadrants forming a condenser with the part of the needle that lies within them.

Imagine the deflection increased from θ to $\theta + \alpha$, α being a very small angle. Then we may assume, for an ideally symmetrical arrangement, that the capacitance of one condenser is increased and that of the other decreased by a definite equal amount proportional to α.

Let c denote the change of capacitance of each condenser for unit angular displacement of the needle. Then, for the displacement α in a direction causing an increase of capacitance in the condenser formed by the needle and the quadrants at potential v_2,

the *change in the energy* of the condenser system is given by (energy = $\frac{1}{2}$ capacitance × (P.D.)²):—

$$\tfrac{1}{2}ca\,(V - v_2)^2 - \tfrac{1}{2}ca\,(V - v_1)^2,$$

$$i.e.\ \tfrac{1}{2}ca\,\{(V - v_2)^2 - (V - v_1)^2\},$$

$$i.e.\ ca\left\{V - \frac{v_1 + v_2}{2}\right\}\{v_1 - v_2\}.$$

This change of energy is equal to the work done against the couple T during the extra displacement a. Hence:—

$$Ta = ca\left\{V - \frac{v_1 + v_2}{2}\right\}(v_1 - v_2);$$

$$\therefore\ T = c\left(V - \frac{v_1 + v_2}{2}\right)(v_1 - v_2).$$

But T is the torsion moment for a twist θ, and is equal or nearly equal to $\kappa\theta$, where κ is a constant depending on the wire or other suspension. Hence we have:—

$$\kappa\theta = c\left(V - \frac{v_1 + v_2}{2}\right)(v_1 - v_2);$$

$$\theta = \frac{c}{\kappa}\left(V - \frac{v_1 + v_2}{2}\right)(v_1 - v_2) \quad \ldots\ldots\ldots\ldots\ldots\ldots (p)$$

If the potential of one pair of quadrants be the same as that of the needle, for example, if $v_1 = V$, we get (from (p))

$$\theta = \frac{c}{\kappa}\left(\frac{V - v_2}{2}\right)(V - v_2);\quad \therefore\ \theta = \frac{c}{2\kappa}(V - v_2)^2 \quad \ldots (q)$$

and the other expressions quoted above can be similarly deduced.

Note that in the summarising note at the beginning of this section the constants are merely abbreviated by using k: the k_1 corresponds to c/κ and the k_2 to $c/2\kappa$.

13. The Attracted Disc or Absolute Electrometer

The *absolute* measurement of difference of potential can be effected by the attracted disc electrometer. It has been shown that in the case of a charged disc the density is not uniform, but is greatest round the edge; this defect in the early forms of this instrument is remedied by the employment of a "guard-ring."

The principle of one type of the electrometer will be gathered from Fig. 219. The attracted disc, C (of area, say, S sq. cm.), is

surrounded by a guard-ring, BB, forming the bottom of a metal box, AA, which protects the disc from external influences. The disc is suspended from the top of the box by springs, and its normal position is slightly above the plane of the guard-ring bottom, BB. The distance between the disc and the lower plate, PP, can be adjusted so that the attraction on the disc (when there is a potential difference between it and the plate PP) is just sufficient to bring it exactly into the plane of the guard-ring BB, and as the force in dynes necessary to do this is readily found by direct experiment (by placing weights on C), the force of attraction on the disc for a known distance between the plates is determined. PP is raised or lowered by a micrometer screw attached to its supporting pillar.

In the arrangement of Fig. 219, let the disc C be charged to potential V, and let the surface density of the charge be denoted by σ. If PP be now connected to earth it remains at zero potential, but a charge of opposite sign to that on C is induced on it, and the surface density of this charge will be $-\sigma$. The difference of potential between the plates is therefore V, and on page 206 it is shown that $V = 4\pi\sigma d$, that is $\sigma = V/4\pi d$, where d is the distance between C and PP. Again, the force F on the plate C, drawing it down to the level of BB, is shown on page 210 to be given by the expression $F = 2\pi S\sigma^2$; hence:—

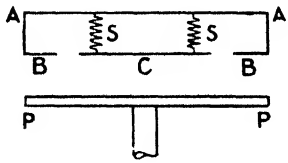


FIG. 219.

$$F = 2\pi S\sigma^2 = \frac{2\pi SV^2}{16\pi^2 d^2} = \frac{SV^2}{8\pi d^2}; \quad \therefore\ V = d\sqrt{\frac{8\pi F}{S}}.$$

A delicate gauge is provided to indicate when C is exactly on a level with BB, and the force F required to do this is found by a preliminary experiment with known weights, so that the factor $\sqrt{(8\pi F/S)}$ is known. (P, B, and C are all earthed during this preliminary observation.) In practice C and BB are usually kept at a constant potential and PP joined to the body to be tested: the method will be gathered from the following:—

The disc and the guard-ring are maintained at a constant potential, and the lower plate is first connected to earth and adjusted until the disc is in the plane of the ring. The reading of the micrometer screw attached to the pillar supporting the lower plate is then taken.

The plate is next disconnected from earth and joined to the conductor whose potential has to be measured, and its position is again adjusted so that C is level with BB.

Now, if V denote the constant potential at which the disc and guard-ring are maintained, and v the potential to be measured, then we have, since the potential of the lower plate for the first adjustment is zero,

$$V = d_1 \sqrt{\frac{8\pi F}{S}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where d_1 denotes the distance between BCB and PP when the first adjustment is made. Similarly we have

$$V - v = d_2 \sqrt{\frac{8\pi F}{S}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

where d_2 denotes the distance between the plates when the second adjustment is made. Hence, subtracting (2) from (1), we get

$$v = (d_1 - d_2) \sqrt{\frac{8\pi F}{S}},$$

where v denotes the required potential, and $(d_1 - d_2)$ is the distance the lower plate is moved in making the second adjustment.

The factor $\sqrt{(8\pi F/S)}$ may be determined once for all as a constant of the instrument by finding F and S. Imagine all parts at zero potential. Suppose M grm. must be placed on C to bring it to the level of B, then the force F = Mg dynes. S is known (it should include half the gap area—see Fig. 215). Note that C and BB are *metallically* connected.

14. String Electrometers. Condensing Gold-Leaf Electroscope

(1) The **simple string electrometer** (Fig. 220) consists of a quartz fibre (silvered) suspended under tension between two parallel plates whose distance apart can be varied, and the whole is enclosed in a metal case to protect the parts from external electric fields. The fibre is joined to the body whose potential is to be measured, whilst the two plates are kept at a constant potential difference. Another method consists in keeping the fibre at a constant high potential, joining one plate to earth, and connecting the other plate to the potential to be measured. The deflection of the fibre depends on the potentials (and, of course, the tension), and is observed by a microscope: a moving photographic film may also be used for recording as the instrument responds at once to potential changes.

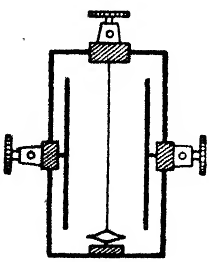


FIG. 220.

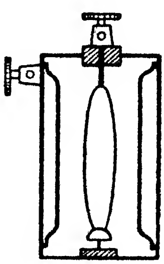


FIG. 221.

A modified form is the **bifilar string electrometer** (Fig. 221). The moving part is a loop of platinum wire suspended under tension. The loop hangs between two plates as before,

and when a P.D. is applied between the loop and plates the two sides of the loop move away from each other: observations are made by a microscope.

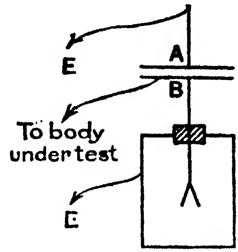


FIG. 222.

(2) The ordinary gold-leaf electroscope (Fig. 133) is not sensitive enough for the detection of the electrification on a body which is only at a low potential but has a large capacitance compared with the electroscope. In such cases a useful device is to increase the capacitance of the electroscope while it is in contact with the body to be tested, so that it takes a very large charge: it will not, however, rise to a high potential, for the resulting common potential will be lower than the original low potential of the body under test. The electroscope *is then disconnected from the body*, and then its capacitance is reduced to its own lower value, with the result that the large charge it has taken causes the potential of the leaves to rise, and they diverge. An electroscope constructed on this principle is called a **condensing electroscope.**

The electroscope has a large disc B (Fig. 222) and a movable plate A, the latter being earthed—thus AB forms a parallel-plate condenser, the plates being separated by air or by a coat of insulating varnish, with A close to B. B is connected momentarily to the charged body to be tested *and then disconnected.* A is then removed to a distance, and the big charge B has taken will then raise the potential and the leaves will diverge.

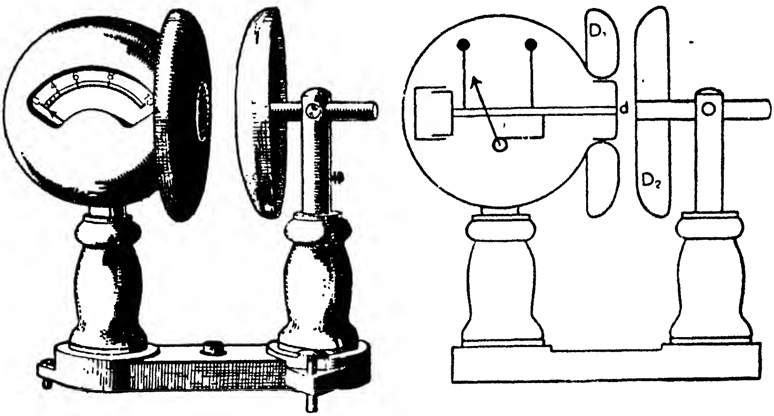
As an illustration the P.D.s which occur in a cell or even a battery of three or four cells are very small compared with the potentials met with in electrostatics. A sensitive condensing electroscope may, however, be able to give results with the P.D. at the terminals of a single cell and certainly with a battery of three or four cells. For accurate work with a single cell, however, an electrometer should be used.

15. Electrostatic Voltmeters

Commercial instruments for the measurement of potential differences in practical units (volts, etc.) are termed *voltmeters.* They depend for their action on various electrical principles and are dealt with in detail in later Chapters: at this stage we only briefly glance at one or two which are based on electrostatic principles.

In electrostatic voltmeters the attractive force between two bodies charged to different potentials is used to measure the P.D. between them. A simple form is shown in Fig. 223. The two

vertical copper discs D_1 and D_2 facing each other form, as it were, the two plates of a parallel plate air condenser. The central part *d* of the disc on the left is separate from the rest of the disc (guard-ring principle) and is free to move horizontally a short distance to and from the other disc, any such movement being communicated to the instrument pointer by a suitable mechanism. If the two discs be connected to two points at different potentials, the central moving part is attracted towards the disc on the right. The force of attraction depends on the P.D. (it is proportional to the *square of the P.D.*) and the instrument is graduated so that the deflection gives the P.D. direct in volts. As a rule D_2 is mounted on a slide bar so that it can be clamped at various distances from D_1 and the



(*a*) FIG. 223. (*b*)

range of the instrument thereby altered. They give full scale deflections with voltages ranging from 10,000 to 250,000 volts.

The *multicellular electrostatic voltmeter* (Fig. 224) is rather like the variable condenser (Fig. 209) in construction. It consists of a fixed set of triangular brass plates Q arranged parallel to each other, and a set of light aluminium paddle-shaped vanes V fixed parallel on a vertical spindle S, and capable of moving into and out of the spaces between the plates Q. When the fixed and moving sets are joined to two points at a P.D. the set V is attracted into the set Q and the movement is communicated to a pointer moving over a scale graduated to read the voltage direct. The controlling influence is the torsion of the suspending wire. The bottom of the spindle S carries a disc D which moves in oil and prevents undue

oscillation of the moving system, *i.e.* makes the instrument more or less "dead beat," as it is termed. They are used for smaller voltages—about 20 to 600 volts. Electrostatic voltmeters can be employed on either A.C. or D.C. circuits (see Chapter XVIII.).

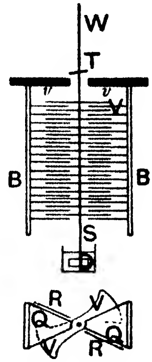


FIG. 224.

C—ELECTROSTATIC MEASUREMENTS

16. Comparison and Determination of Capacitances

Most laboratory methods—including the most satisfactory methods—for the comparison and determination of capacitances are based on principles dealt with in current electricity, and these are described in Chapter XV. In this section only those methods involving principles already dealt with are discussed.

(1) To compare the capacitances of two condensers. Connect up the apparatus as indicated in Fig. 225, where A and B are the two condensers of capacitances C_1 and C_2 respectively, D is a convenient battery, E a quadrant electrometer (used heterostatically), and K_1, K_2, K_3 well insulated keys; it will be noted that one pair of quadrants, one terminal of the battery, and one plate of each condenser are earthed at *e*.

Close K_2, thus charging A to a certain potential. Next close K_1, and note the deflection, d_1, indicated by the electrometer; d_1 is proportional to the potential, say V_1. Open K_2, thus breaking the battery circuit. Close K_3, thus allowing A to share its charge with B, the two taking up a common potential V_2, lower than V_1. Note the deflection, d_2, indicated by the electrometer; d_2 is proportional to V_2.

If Q denote the charge given to A in the first case, then, neglecting the capacitance of the electrometer, $Q = C_1V_1$. Similarly, in the second case,

$$Q = (C_1 + C_2)\, V_2;$$

$$\therefore\ C_1V_1 = (C_1 + C_2)\, V_2,\ i.e.\ \frac{C_1}{C_1 + C_2} = \frac{V_2}{V_1} = \frac{d_2}{d_1};$$

$$\therefore\ \frac{C_1}{C_2} = \frac{d_2}{d_1 - d_2}.$$

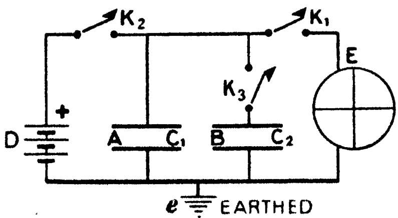


FIG. 225.

If A be a standard condenser of known capacitance C_1, the capacitance C_2 can be determined from the above.

It has been assumed that the capacitance of the electrometer is so small in comparison with the others that it can be neglected. If this be taken into account—call it *c*—we have $Q = (C_1 + c)\, V_1$ and $Q = (C_1 + C_2 + c)\, V_2$;

$$\therefore\ (C_1 + c)\, V_1 = (C_1 + C_2 + c)\, V_2;$$

$$\therefore\ C_2 = \frac{d_1 - d_2}{d_2}(C_1 + c),$$

in which C_1 and c are supposed known, and d_1 and d_2 are the deflections.

(2) To determine the capacitance of a quadrant electrometer. Let two spherical conductors of capacitances C_1 and C_2, measured by their radii, be charged to the same potential, and their potentials compared when joined successively to the quadrants of the electrometer. Let c denote the capacitance of the quadrants of the instrument, V the common initial potential of the two conductors, and V_1 and V_2 the potentials assumed by these conductors when connected with the quadrants; then, if d_1 and d_2 denote the deflections corresponding to these potentials, we have

$$V_1 (C_1 + c) = VC_1 \quad \text{and} \quad V_2 (C_2 + c) = VC_2;$$

$$\therefore\ \frac{V_1 (C_1 + c)}{V_2 (C_2 + c)} = \frac{C_1}{C_2}. \quad \text{But } \frac{V_1}{V_2} = \frac{d_1}{d_2};$$

$$\therefore\ \frac{d_1}{d_2} \cdot \frac{C_1 + c}{C_2 + c} = \frac{C_1}{C_2}, \text{ i.e. } c = \frac{C_1C_2 (d_2 - d_1)}{d_1C_2 - d_2C_1},$$

in which C_1 and C_2 numerically equal the radii and c is therefore determined.

A more convenient laboratory method is as follows. Arrange apparatus as in Fig. 226 where C is a condenser of known capacitance C (preferably a guard-ring type so that C is accurately calculated from dimensions), E is the electrometer (its needle n is drawn off to one side for simplicity), B is a battery, K_1 and K_2 are keys, and X is the battery for charging the needle. Note the electrometer connexions.

Close K_1 thus raising one pair of quadrants to potential v_1 (the other pair is earthed), and note the deflection d_1. If c be the electrometer capacitance the charge on the electrometer is cv_1. Now open K_1 thus disconnecting B, and close K_2. The electrometer shares its charge with C and they come to a common potential v_2. Let d_2 be the deflection. Then $cv_1 = (c + C)\, v_2$.

$$\therefore\ c = C\frac{v_2}{v_1 - v_2} = C\frac{d_2}{d_1 - d_2}.$$

17. Determination of Dielectric Constant

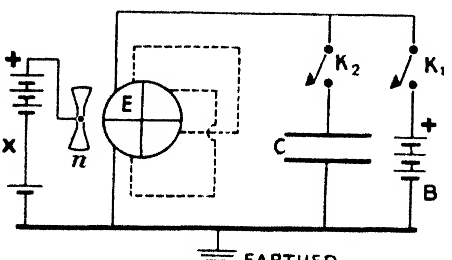


FIG. 226.

As in Art. 16 only those methods based on principles already dealt with will be considered at this stage.

(1) HOPKINSON'S METHODS FOR SOLIDS AND LIQUIDS.—The principle will be gathered from Fig. 227. In the figure B is a battery, the middle point

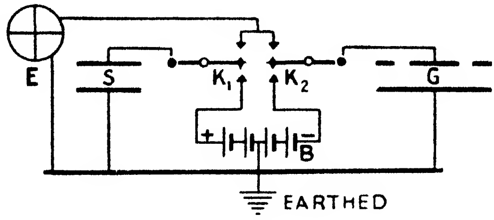


FIG. 227.

of which is earthed (zero potential), in which case, as will be seen later, if $+V$ be the potential of the positive pole the negative pole will have the equal but opposite potential $-V$. S is a sliding condenser. G is a guard-ring condenser, the movable plate of which is earthed; E is an electrometer, and other earth connexions are as indicated.

On bringing the keys, K_1 and K_2, into contact with the lower studs, S is charged to a positive potential and G to an equal negative potential. Since $Q = CV$, if the capacitances of S and G are equal their charges will be equal but of opposite sign. Hence on raising K_1 and K_2 to the upper studs the charges will "mix" and being equal and opposite will neutralise so that the electrometer will not be deflected. The sliding condenser S is adjusted, until on performing these operations there is no deflection, in which case the capacitances of S and G are equal.

A slab of the solid dielectric, whose dielectric constant K is required, is next placed on the movable plate of G, thus increasing the capacitance of this condenser. S is kept fixed, and the movable plate of G is lowered until, on repeating the experiment, there is again no deflection. Now

$$\text{Capacitance of G in Case 1} = A/4\pi t,$$

where A = area of G, and t = distance between the plates.

$$\text{Capacitance of G in Case 2} = \frac{A}{\{4\pi\,(t - d + x) + d/K\}}$$

where d is the thickness of the slab and x the distance the movable plate is lowered (Art. 5). Now these capacitances are equal, both being equal to that of S, *i.e.*

$$\frac{A}{4\pi\,\{(t - d + x) + d/K\}} = \frac{A}{4\pi t}; \quad \therefore K = \frac{d}{d - x}.$$

In dealing with liquids Hopkinson used a special cylindrical condenser, a section of which is shown in Fig. 228; this takes the place of the guard-ring condenser above. With air as dielectric in the special condenser, the sliding condenser S is adjusted for no deflection and the position of the sliding tube of S is noted. The

special condenser is then filled with the liquid, S again adjusted for no deflection and the position of the sliding tube of S again noted. The capacitance of the special condenser is, in each case, equal to the capacitance of S, and these are (Arts. 6, 10)

$$(1)\ C_1 = \frac{l_1}{2\log_e \frac{b}{a}}.\quad (2)\ C_2 = \frac{l_2}{2\log_e \frac{b}{a}};\quad \therefore\ K = \frac{C_2}{C_1} = \frac{l_2}{l_1}.$$

In practice S is graduated so that its capacitance is known for any position of the inner tube inside the outer.

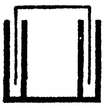


FIG. 228.

(2) BOLTZMANN'S METHOD FOR GASES.—The condenser consisted of two metal plates P and Q, (Fig. 229), surrounded by an enclosure of brass, B, the whole being contained in a glass receiver. The whole could be exhausted or filled with any desired gas. Connexions between the condenser plates P, Q, and the battery, electrometer, and earth were made by wires insulated from the walls of the chambers by passing through shellac plugs. A battery of 300 cells was employed to charge the condenser.

With the enclosure exhausted, and k_1, k_2 closed, the plate P is charged to a potential 300V, where V is the potential due to one cell. Q is at zero potential.

The keys are opened and the gas admitted. If K be its dielectric constant the potential of P falls to 300V/K. Q is still at zero.

P is again joined to the battery by closing k_1, an additional charge passes to P raising its potential again to 300V. Q now acquires a potential due to this extra charge which has passed to P and a deflection α is obtained on the electrometer. α depends on the amount by which the potential of P has altered; hence:—

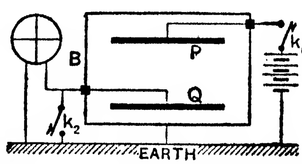


FIG. 229.

$$\alpha \propto \left(300V - \frac{300V}{K}\right) \propto 300V\left(1 - \frac{1}{K}\right).$$

An additional cell is added, thus altering the potential of P by an amount V. If β be the change in the deflection, $\beta \propto V$. Thus:—

$$\frac{\alpha}{\beta} = \frac{300V\,(1 - 1/K)}{V} = 300\left(1 - \frac{1}{K}\right);\quad \therefore\ K = \frac{300\beta}{300\beta - \alpha}.$$

(3) MISCELLANEOUS.—Faraday used two spherical condensers A and B exactly alike, the outer sphere of B being in two halves. A contained air and the lower half of B the dielectric (sulphur), their capacitances were compared, and it was found that $C_b = 1{\cdot}6C_a$ where C_a = capacitance of A and C_b = capacitance of B. As B is only half full of sulphur, and its capacitance when the sulphur **is** *not* there is C_a the same as A, we have:—

$$C_b = \left(K\frac{C_a}{2} + \frac{C_a}{2}\right) = \frac{K+1}{2}C_a;$$

$$\therefore\ 1{\cdot}6C_a = \frac{K+1}{2}C_a,\ i.e.\ \frac{K+1}{2} = 1{\cdot}6;\quad \therefore\ K = 2{\cdot}2.$$

It has been stated (page 198) that the action of an external charge on a suspended *dielectric sphere* is practically $(K - 1)/(K + 2)$ of its action on an equal conducting sphere. The *field* alters at the surface of the dielectric sphere so that it is "apparently" charged, the apparent charges being of the same sign as, but smaller than, the induced charges on the conducting sphere; hence the force on the dielectric sphere is of the same nature as (attraction), but smaller than, the force on the other sphere. Boltzman compared the forces on the two equal spheres and determined K for the dielectric sphere. Other methods of determining K are given later.

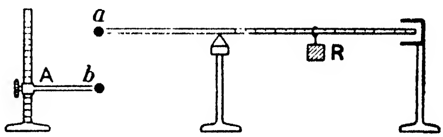


FIG. 230.

In the case of gases K increases with the pressure: for air at normal pressure $K = 1{\cdot}00059$, but at 100 atmospheres $K = 1{\cdot}054$. For some substances K slightly decreases with rise in temperature, but with others there is an increase, *e.g.* with glass and vulcanite. Note that the amount by which K for air at 100 atmospheres exceeds 1 (*i.e.* ·054) is *less* than 100 times the amount by which it exceeds 1 at 1 atmosphere (*i.e.* ·00059): this is because the displacement or polarisation of the atoms is interfered with by the greater number of collisions among the atoms at the higher pressure.

18. Proving the Law of Inverse Squares

(*a*) PRINCIPLE OF LABORATORY METHOD.—The fact that the force between two charges varies inversely as the square of the distance between them can be approximately verified by using a balance such as is shown diagrammatically in Fig. 230. A small brass ball *a* is attached by vulcanite to the end of a long rod suspended on a knife-edge, and the rod is balanced horizontally by the rider R. The distance of another ball *b* from *a* can be adjusted by raising or lowering the insulating arm A which carries it. The

balls are charged (say unlike) so that a is attracted, R is adjusted so as to bring the rod into the horizontal again, and the force can be measured by the amount R has to be moved. By repeating with a and b at different distances apart, the law can be approximately verified.

(*b*) THE CAVENDISH EXPERIMENT AND PROOF.—It has been indicated that the potential is uniform and therefore the electric force zero inside a charged hollow conductor, and this is really a proof—the best proof—of the law of inverse squares, for it can be shown that **this result follows from the law of inverse squares but does not follow from any other law.** For simplicity we will prove this for a charged hollow *sphere*, but it applies to a charged hollow conductor of any shape.

Consider a sphere uniformly charged positively, and take any point, X, within it (Fig. 231). Through X draw lines forming two small cones, the apex of each being therefore at X, the small solid angles at X being θ, and let the two small areas intercepted on the sphere be S and S_1 respectively. Consider now a right section of the cone at S. Its area is $r^2\theta$ and (since the angle between two straight lines is equal to the angle between their perpendiculars) it makes an angle a (Fig. 231) with S; thus $r^2\theta = S\cos a$; $\therefore S = r^2\theta/\cos a$, and, similarly, $S_1 = r_1{}^2\theta/\cos a$. If ρ be the surface density the charge on S will be $S\rho$, *i.e.* $r^2\rho\theta/\cos a$, and the charge on S_1 will be $r_1{}^2\rho\theta/\cos a$. Now if the force varies inversely as the pth power of the distance the force at X due to S will be $\dfrac{r^2\rho\theta}{r^p\cos a}$, and the force at X due to S_1 will be $-\dfrac{r_1{}^2\rho\theta}{r_1{}^p\cos a}$, and these will be in the same straight line but in opposite directions. The following cases arise:—

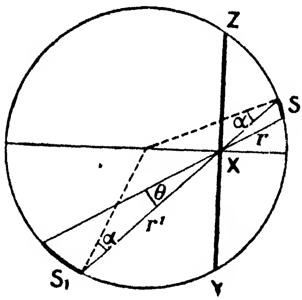


FIG. 231.

Case 1. If $p = 2$ these become $\dfrac{\rho\theta}{\cos a}$ in each case; thus the field at X, due to the charges on S and S_1, is zero, and, since the whole sphere can be divided into cones in this way, the total field at X, due to the charged sphere, is zero. This is so in practice; hence we infer that $p = 2$.

Case 2. If $p > 2$ these become $\dfrac{\rho\theta}{r^{p-2}\cos\alpha}$ and $\dfrac{\rho\theta}{r_1^{p-2}\cos\alpha}$ respectively. Since $r_1 > r$, the former expression is greater than the latter; hence, taking the whole sphere, the force due to the part of it on the right of the plane, ZXY, will be greater than the force due to the part of it on the left of ZXY, and there will be a resultant force at X acting towards the left, *i.e.* towards the centre. This is not so in practice.

Case 3. Similarly, if $p < 2$ it can be shown that there is a resultant force at X acting towards the right, *i.e.* away from the centre. This is not so in practice.

Thus, from the fact that the force inside such a charged sphere is everywhere zero, we deduce that p must equal 2.

Cavendish placed a sphere, B, inside another, A. A was charged positively, joined to B for a moment by a wire, then the two were disconnected. From the preceding B would have no charge if $p = 2$, it would be positive if $p > 2$, and negative if $p < 2$. In all cases B was without charge; hence $p = 2$. Maxwell in repeating the experiment tested B with a quadrant electrometer with which he could have detected a charge on B if p had differed from 2 by as little as $\pm \frac{1}{20000}$: no charge was found.

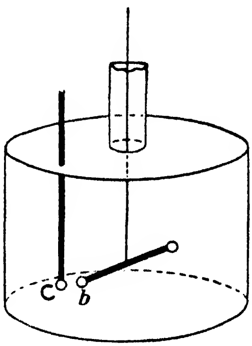


FIG. 232.

(*c*) COULOMB'S TORSION BALANCE.—This instrument is not used to any extent in the laboratories of to-day, and it does not prove the law with the accuracy of the preceding method: it is, however, of historical interest, and a brief consideration is necessary. Before proceeding further the student should again read pages 114-16.

The balance, as used in Electrostatics, is similar to that described in Magnetism. In place of the magnet suspended horizontally from the torsion head at the top, a light rod of insulating material with a small conducting ball at each end is used (Fig. 232). Through a hole in the cover an insulating rod with an equal small conducting ball at its lower end can be inserted vertically, and the apparatus is adjusted so that this latter ball is level with and just touches a suspended one when the suspension is without torsion and the end of the suspended rod is opposite the zero of the scale

etched on the glass. A charge is given to the ball (call it *c*) on the end of the vertical rod, the rod is inserted, with the result that the charge is shared with the suspended ball (call it *b*), and the latter is repelled until the couple due to the torsion balances the repulsion between the balls.

To verify the Law of Inverse Squares by Coulomb's Method.—If the angle of deflection be small it may be assumed that the *distance* between the balls is reduced to one half if *the angle* is reduced to one half. Coulomb's method was, therefore, as follows: On inserting *c* the ball *b* was repelled through an angle of 36°. The twist on the wire was, therefore, 36°, and this balanced the repulsion. The torsion head was then turned through 126° in the opposite direction to the deflection to reduce the deflection to 18°. The twist on the wire was, therefore, (126° + 18°), *i.e.* 144°, and this balanced the repulsion. Now, 144 = 4 × 36; thus the force of repulsion was increased fourfold, which verifies the law, if we assume the distance was halved.

To verify by Coulomb's method that the force is proportional to the product of the charges.—On inserting *c* the ball *b* was repelled through a certain angle $\theta°$, so that the twist on the wire ($\theta°$) balanced the repulsion. The ball *c* was removed, its charge reduced to one-half by allowing it to touch an equal ball, and it was again inserted (without touching *b*). The deflection was less, and the torsion head was then turned through a certain angle $\alpha°$, *in the same direction*, until the angular distance between the balls was again $\theta°$. The twist on the wire was now ($\theta° - \alpha°$), and this balanced the repulsion. ($\theta° - \alpha°$) was found to be one-half of $\theta°$, so that the repulsion was halved when the charge on *c* was halved.

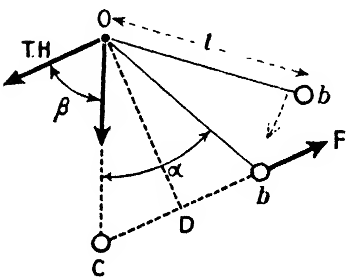


FIG. 233.

For more accurate work the more exact theory of the torsion balance must be applied. A formula for the instrument may be developed thus:—Call the ball inserted vertically *c* and the suspended one *b*. Let $2q$ be the charge given to *c*. On inserting it the charge is equally shared, so that the charge on each is q, and *b* is repelled. Let the torsion head be now turned through an angle, β, in the opposite direction, so as to reduce the angle through which *b* is repelled to α. Let F denote the force between the balls at this distance (Fig. 233). The twist on the wire is $(\alpha + \beta)$, and this balances the repulsion; hence:—

$$\text{Couple due to torsion} \propto (\alpha + \beta) = C\,(\alpha + \beta),$$

where C is a constant for the wire. Again:—

$$\text{Couple due to repulsion} = F \times OD = Fl \cos\frac{\alpha}{2};$$

$$\therefore\ Fl \cos\frac{\alpha}{2} = C(\alpha + \beta),$$

$$\textit{i.e.}\ Fd^2 = \frac{C(\alpha+\beta)}{l \cos \alpha/2} d^2 = \frac{C(\alpha+\beta)}{l \cos \alpha/2} \times \left(2l \sin\frac{\alpha}{2}\right)^2,$$

$$\textit{i.e.}\ Fd^2 = 4lC(\alpha+\beta) \sin\frac{\alpha}{2} \tan\frac{\alpha}{2}, \qquad [d = \text{distance } cb]$$

or, since l and C are constants, putting $4lC = \angle =$ a constant,

$$Fd^2 = \angle(\alpha+\beta) \sin\frac{\alpha}{2} \tan\frac{\alpha}{2} \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\text{Further, } F = q^2/d^2; \quad \therefore\ q^2 = \angle(\alpha+\beta) \sin\frac{\alpha}{2} \tan\frac{\alpha}{2} \ldots\ldots (2)$$

If the law of inverse squares be true $F \propto 1/d^2$, *i.e.* Fd^2 is constant. Hence to prove the law by applying the formulae the above would be repeated, *i.e.* the torsion head would be turned through a further angle giving a different value for β and α, and this would be continued for several readings. It should be found that for each pair of readings of β and α the expression $(\alpha + \beta) \sin(\alpha/2) \tan(\alpha/2) =$ a constant.

Examples.—(1) *In an experiment with the torsion balance the lever was deflected through* 10°. *How much must the T.B. be turned to reduce this deflection to* 5°, *supposing the charges on the balls to remain constant.*

If the deflection be reduced from 10° to 5°, say, halved, the force between the balls will be quadrupled, and, therefore the torsion must be quadrupled. Hence if the *torsion head* be turned through $\beta°$ to reduce the deflection to 5°, the twist on the wire will be $(\beta° + 5)$, and we must have

$$\beta° + 5 = 4 \times 10 = 40 = \text{torsion}; \quad \therefore\ \beta° = 40 - 5 = 35.$$

Thus the torsion head must be turned through 35°, and the torsion on the wire is 40°.

(2) *In an experiment with the torsion balance the lever is deflected through* 90°. *Find the torsion necessary to reduce this deflection to* 60°.

In this example the deflections are too large to admit of the application of the method of the preceding example. With the usual notation we have—

$$\text{Case (1)—}F_1 d_1^2 = \angle(\alpha_1 + \beta_1) \sin\frac{\alpha_1}{2} \tan\frac{\alpha_1}{2};$$

$$\text{Case (2)—}F_2 d_2^2 = \angle(\alpha_2 + \beta_2) \sin\frac{\alpha_2}{2} \tan\frac{\alpha_2}{2};$$

and, since we are to assume the inverse square law, $F_1 d_1^2 = F_2 d_2^2$, *i.e.*

$$(\alpha_2 + \beta_2) \sin \frac{\alpha_2}{2} \tan \frac{\alpha_2}{2} = (\alpha_1 + \beta_1) \sin \frac{\alpha_1}{2} \tan \frac{\alpha_1}{2}.$$

But from the problem $\alpha_1 = 90°$, $\beta_1 = 0°$, $\alpha_2 = 60°$;

$$\therefore\ (60° + \beta_2) \sin 30° \tan 30° = 90° \times \sin 45° \tan 45°,$$

$$i.e.\ (60° + \beta_2) = 90° \times \frac{\sin 45° \tan 45°}{\sin 30° \tan 30°} = 90° \times \sqrt{6} = 220{\cdot}5°;$$

$$\therefore\ \beta_2 = 220{\cdot}5° - 60° = 160{\cdot}5°.$$

Thus the torsion head must be turned through 160·5°, and the torsion on the wire is 160·5° + 60°, *i.e.* 220·5°.

CHAPTER VIII

TERRESTRIAL MAGNETISM AND ATMOSPHERIC ELECTRICITY

WE have seen that the earth behaves like a magnet, the whole region on and around its surface being a magnetic field. The north magnetic pole is in the vicinity of Boothia Felix in the far north of North America, about lat. 70° 40′ N., long. 96° 5′ W.: the south magnetic pole is in the southern hemisphere, probably about lat. 71° 10′ S., long. 150° 45′ E. The poles, however, are not *fixed*: their position is *very slowly* changing, and in addition, each day they describe an oval several miles across.

A—TERRESTRIAL MAGNETISM

The Magnetic Elements of the Earth

The *magnetic elements* at any place are (1) the *declination* or variation, (2) the *dip* or inclination, (3) the *horizontal intensity* of the earth's field (H of Chapter IV.).

Consider Fig. 234, where L is a point at any place. Suppose LG is the true (geographical) south and north line: then the vertical plane LGG′L′ is part of the plane called the **geographical meridian,** so that *the geographical meridian at any place is the vertical plane passing through that place and the north and south geographical poles.* Suppose a compass at L sets with its magnetic axis along LM so that LM is the magnetic south and north line: then the plane LMM′L′ is a part of the **magnetic meridian,** i.e. *the magnetic meridian at any place is the vertical plane in which the magnetic axis of a compass comes to rest,* or the vertical plane containing the magnetic axis of a freely suspended magnetic needle. The angle GLM (α) is the **declination** (or *variation*):

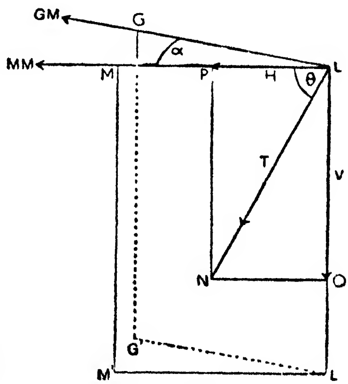


FIG. 234.

hence *the declination at any place is the angle between the magnetic meridian and the geographical meridian at that place* (see Fig. 22).

Now the earth's total magnetic force acts in the plane LMM′L′, but at most places in some sloping or dipping direction such as LN. The angle MLN (θ) is the dip (or *inclination*), so that *the dip at any place is the angle between the direction of the earth's total magnetic force and a horizontal line in the magnetic meridian at that place.* A freely suspended magnetic needle would come to rest with its magnetic axis in the magnetic meridian, as stated above, but this axis would be lying along LN. A "dip needle" is a magnetised needle mounted on a horizontal axis so that it can move in a vertical plane (Fig. 235), and if such a needle be placed so that it can *move in the magnetic meridian* it will come to rest with its magnetic axis along LN: hence *dip is the angle between the magnetic axis of a dip needle able to move in the magnetic meridian and a horizontal line in the meridian.*

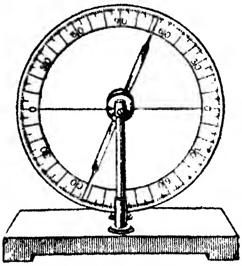


FIG. 235.

Again, let LN represent both the direction *and* the magnitude (T gauss, say) of the earth's total field. Then LN can be replaced by two components, viz. LP horizontally, which represents the *horizontal component* (H) *of the earth's field,* and LQ vertically, which represents *the vertical component* (V). θ is the dip and—

$$\frac{H}{T} = \cos\theta; \quad \therefore T = \frac{H}{\cos\theta}; \qquad \frac{V}{H} = \tan\theta; \quad \therefore V = H\tan\theta.$$

In practice H and θ are usually found by experiment, and then V and T are calculated from the above relations.

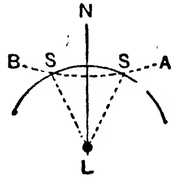


FIG. 236.

In this country the values of the elements at present are taken to be as follows: Declination = 11° 1·4′ W. Dip = 66° 43·2′. H = ·185, and V = ·43. These records are now made at Abinger, near Dorking, instead of at Greenwich as formerly, and there is a slight difference between Abinger and Greenwich values. Of course, it will be evident that the values will vary in different parts of the country. Thus speaking generally, declination is between 11° and 12° W. (see above), but it reaches about 16° W. on the west coast of Scotland. Dip increases as we travel from south to north: speaking generally

it is between 66° and 67° in the south (see above) but reaches about 71° at the Orkneys off the north of Scotland. The values of all the elements are in fact (apart from geographical position) affected by local circumstances. Thus there are at various places magnetic rocks some of which are on the surface like the Malvern Hills, while others are underground as, for example, near the Wash and near Reading: at the Malverns the declination at certain points on the west is from 8′ to 25′ less than the normal, and at certain points on the east from 12′ to 22′ greater than the normal. Masses of magnetic rock are also somewhat pronounced in Skye, Mull, Antrim, North Wales, and the Scottish coalfield.

2. Determination of the Magnetic Elements

(1) DECLINATION.—To determine the declination it is necessary to find (*a*) the geographical meridian, (*b*) the magnetic meridian, and to measure the angle between the two.

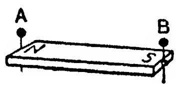


FIG. 237.

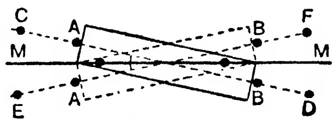


FIG. 238.

(*a*) *Geographical meridian.* An approximate method of finding this is by observing when the shadow of a vertical stick or plumb-line is shortest: it is shortest about noon when the sun is in the geographical meridian of the place of observation and the shadow then lies geographical north and south.

Thus let L (Fig. 236) be the position of the stick and suppose B is the end of the shadow some time before noon. Continue marking the end, *e.g.* S, S, until some time after noon when it is, say, at A. Join the points by a curve BSSA and with L as centre describe a circle cutting the curve in two points, say S, S. The bisector LN of the angle SLS is the geographical meridian, for it is the line along which the least shadow lay.

The above is not workable to any degree of accuracy. A better method is to find from the Nautical Almanac the exact time at which the sun is due south on any given day, and the shadow of the plumb-line or stick *at that moment* is the geographical meridian. The accurate method consists in observing the image of the sun (produced by a mirror) in passing the cross-wires of a carefully adjusted telescope: from the time of the image passing the

cross-wires, the longitude of the place where the observation is made, and the equation of time, the exact direction of the sun at the time of observation is known and the true geographical north is determined. (See *Advanced Textbook of Electricity and Magnetism.*)

(*b*) *Magnetic Meridian.*—Assuming the true north and south line to be marked on the table, the magnetic meridian may be found by suspending a *thin* magnet so that it can move in a horizontal plane above this line. When at rest, place two brass pins vertically in the table so that the pins and the magnet are in line. The line through the two pins is the magnetic meridian, and the angle between the two lines is the declination.

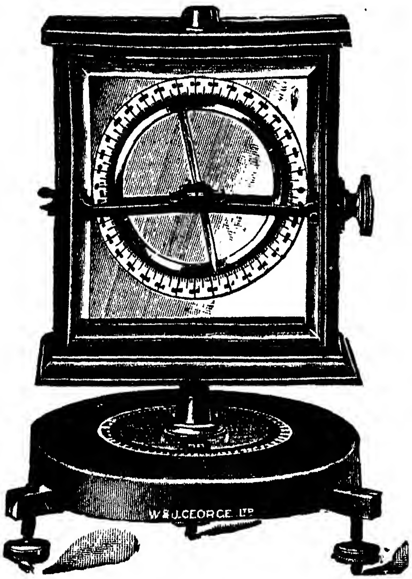


FIG. 239.

In the above the magnet is assumed to be *thin*, and the magnetic axis to run from the centre of one end to the centre of the other end. In most cases, however, the position of the magnetic axis of the magnet is not known with certainty, and we proceed thus:—Fasten two pins (with wax) to the magnet (Fig. 237) —or use cross-wires—and suspend it. Place two pins C, D in the table so that CABD are in line. Turn the magnet over (pin heads downwards), suspend again, and place two pins E, F so that EABF are in line. Join CD and EF and bisect the angle between them. The bisecting line MM is the magnetic meridian, for it is the line along which the magnetic axis lay. Fig. 238, where we assume the magnetic axis runs from corner to corner, will show the principle.

(2) DIP.—If a dipping needle is set down *so that it can move in the magnetic meridian*, it will come to rest in the direction of the earth's total force (LN in Fig. 234), and the angle between its magnetic axis and the horizontal is the dip. For accurate work an instrument called a dip circle is used. It consists (Fig. 239) of

a vertical graduated circle at the centre of which is a horizontal axle on which the dipping needle turns. There is also a horizontal graduated circle at the bottom of the instrument, from the centre of which projects a vertical axle supporting the upper part of the apparatus. The vertical scale—in fact the whole upper part—can be turned round on this vertical axle over the fixed horizontal scale.

Let this page of this book represent the magnetic meridian, the north being on the right, and *suppose* the figure shows the position taken up by the needle *when the plane of the vertical circle is in the magnetic meridian, i.e.* in the plane of the page. Then the dip is simply read off on the vertical circle: it is the angle between the horizontal and, say, the N end of the needle.

The magnetic meridian may not be known exactly, however, and the horizontal circle enables us to find it. Imagine the vertical circle set in a plane *perpendicular* to the magnetic meridian. The horizontal component of the earth's force, being *in* the meridian, simply tends to tilt the needle against the plane of the vertical circle, and this it cannot do by reason of the way the needle is mounted; this part of the earth's force has, therefore, no effect except to cause a pressure on the supports. But the *vertical* component tends to pull the needle into a vertical position, and to this the mounting offers no opposition. *Accordingly the needle sets vertically.* (Fig. 240 will help to make this clear.) Moreover it will not set vertically if the vertical circle be in any other plane. We therefore begin by turning the upper framework of the instrument until the needle sets vertically at the reading 90°; the circle is then perpendicular to the magnetic meridian, and all we have to do is to turn it through 90° over the horizontal circle at the bottom, when it will be *in* that meridian, and we can then read the dip straight away.

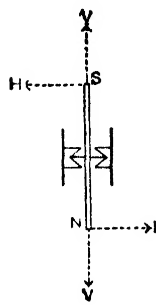


FIG. 240.

Several errors, however, may exist in the apparatus, and for accurate work these are eliminated by taking the following readings:—With the instrument set up as above, a reading is taken for each end of the needle. The vertical circle is then turned through 180° over the horizontal scale, and each end is again read. The needle is then removed from its bearings, turned over and replaced, and the four readings repeated. Finally, the needle is reversed in polarity, *i.e.* magnetised in the opposite direction, and the eight readings repeated. The mean of the sixteen readings gives the true dip.

The possible errors which are corrected by taking the above readings are as follows. (In the diagrams the full line represents the needle, the dotted lines the *true* horizontal and *true* line of dip, and the errors are exaggerated to make the idea clear.)

Error 1. The needle may not be pivoted at the centre of the vertical circle: to correct, read both ends. Thus in Fig. 241 the angle read from the N end is too large and from the S end too small, so that taking the mean of the two will correct. (2 readings.)

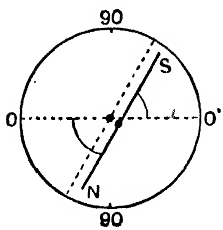


FIG. 241.

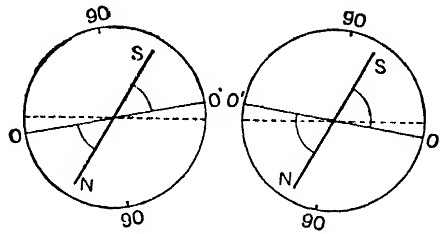
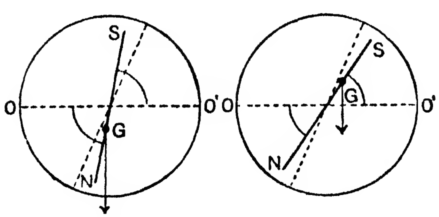


FIG. 242.

Error 2. The zero line of the vertical circle may not be exactly horizontal. To correct, turn the vertical circle through 180° over the horizontal scale and repeat the readings. Study Fig. 242 for this. (2 more readings.)

Error 3. The magnetic axis of the needle may not run from end to end of it. To correct, take the needle out, turn it over, and repeat. This is the same error as occurred in finding the declination. (4 more readings.)

Error 4. The needle may not be supported at its centre of gravity G. If G be along the neutral line off to one side the method in (3) will correct. If G is nearer one end, then to correct, the needle must be remagnetised the other way about. In Fig. 243 (*a*) G is in the north half and dip angles are too large, whilst after remagnetising G is in the south half and dip angles are too small (Fig. 243 (*b*)): taking the mean corrects. (8 more readings.)



(*a*) (*b*)

FIG. 243. Vertical arrows show "weight" acting at G.

The Dip is sometimes determined from observations of the apparent dip taken in *any two vertical planes at right angles*. Let PA and PB (Fig. 244) be the traces in a horizontal plane of these two planes, making angles α_1 and α_2 with the magnetic meridian PM. The components of H along PA and PB are H cos α_1 and

$H \cos \alpha_2$, and V, the vertical component at P, is the same for both planes. Hence if δ_1 and δ_2 denote the apparent dip in the planes of PA and PB we have (since $V/H = \tan \theta$):—

$$\frac{V}{H \cos \alpha_1} = \tan \delta_1 \text{ and } \frac{V}{H \cos \alpha_2} = \tan \delta_2.$$

Since α_1 and α_2 are complementary, we may write $\sin \alpha_1$ for $\cos \alpha_2$ and the above may be put in the forms:—

$$\frac{H \cos \alpha_1}{V} = \cot \delta_1; \quad \frac{H \sin \alpha_1}{V} = \cot \delta_2; \quad \therefore \cot^2 \delta_1 + \cot^2 \delta_2 = \left(\frac{H}{V}\right)^2.$$

But $H/V = \cot \theta$ where θ is the *true dip*: hence:—

$$\cot^2 \theta = \cot^2 \delta_1 + \cot^2 \delta_2.$$

(3) HORIZONTAL COMPONENT.—The method of determining H, the horizontal component of the earth's field, has been fully explained in Chapter IV. (pages 116-120). Knowing H and the dip $\theta°$ the total intensity T is found from $T = H/\cos \theta$ and the vertical component V from $V = H \tan \theta$.

It may be noted, and it will be seen later, that very accurate determinations of H can be made by observations based on the magnetic effects of electric currents flowing in conductors: so, too, the dip and V can be determined by methods involving the use of electric currents.

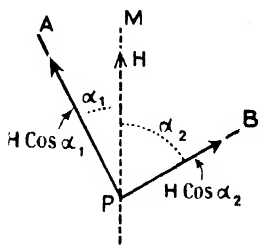


FIG. 244.

The *Kew magnetometers* used in the determination of declination and horizontal intensity, and the *Kew dip circle* are very elaborate forms of apparatus for accurate work, but their description and operation cannot be given in this book. (See *Advanced Textbook of Electricity and Magnetism.*)

3. Variations in the Magnetic Elements at any Place

The magnetic elements vary from time to time *at the same place*, and they also vary from place to place on the earth's surface (see Art. 5). The following are the chief facts in connexion with the former:—

(1) *Secular Changes.*—The elements are found to be undergoing gradual changes extending over a very long period of time, and these are referred to as the secular changes. Thus the declination at London was 11° 15′ E. in 1580, *i.e.* the north pole of the compass pointed east of the true north. Year by year the declination

became less, and in 1657 the compass pointed geographically north. The declination then became west, reached its westerly limit of 24° 38′ W. in 1818, and is now decreasing at the rate of about 11′ per year, and somewhere about the year 2130 it should again be zero. The magnetic system is, in fact, slowly rotating as shown in Fig. 245, the time of a complete revolution being about 950 years. The angle of dip is decreasing at the rate of about 1·1′ per year, and H is increasing at the rate of about ·00002 unit per year. (In recent years, however, there has been a slight variation from this general change, dip showing a slight increase.)

(2) *Daily Changes.*—Periodic daily variations in the elements are also observed. In England the north pole of the compass starting from its mean position moves to the west from about 10 a.m. to 1 p.m., at which time the declination has increased about 5′. The north pole then moves to the east, crosses its mean position about 7 p.m., moves still to the east (but very slowly) until the early hours of the morning, then moves rapidly to the east until between 7 a.m. and 8 a.m., at which time the declination has decreased about 4′. The north pole then moves to the west, reaching the mean position again about 10 a.m. This is shown by the curve D in Fig. 246. On the same figure the curve I gives the daily changes in dip; the maximum is reached between 10 a.m. and 11 a.m. and the minimum between 7 p.m. and 8 p.m. The curve H gives the changes in the horizontal force; the maximum is between 7 p.m. and 8 p.m. and the minimum between 10 a.m. and 11 a.m. The changes are less in winter than in summer.

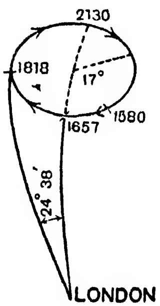


FIG. 245.

(3) *Annual Changes.*—Periodic annual variations also take place; thus the declination has its greatest (westerly) value in February and its least (westerly) value in August. There is a small periodic monthly variation in the elements also.

(4) *Pulsations.*—By using very small magnets with small moments of inertia it is seen that the curves such as are shown in Fig. 246 are really composed of very small undulations due to the magnetic system being constantly subject to vibrations of very small amplitude; these are spoken of as pulsations.

(5) *Irregular Changes.*—Sometimes the recording instruments indicate sudden, irregular, and often large disturbances in the

elements, such disturbances occurring more or less simultaneously all over the world; these are referred to as *magnetic storms*, and frequently accompany volcanic eruptions, earthquakes, earth currents, and abnormal displays of the aurora. Magnetic storms often accompany the appearance of unusually large sun spots, and the "sun spot eleven-year period" coincides with a similar period of change in the extent of the daily variations.

The daily changes in the elements seem to indicate that there is some association with the sun and with the rotation of the earth; probably radiation from the sun brings about an alteration in the electrical conductivity of the upper atmosphere and therefore of any currents in it, which alteration will vary with the time of day and, of course, affect the earth's magnetism. The eleven-year period referred to above also supports the idea.

4. Recording the Variations in the Elements at any Place

Instruments known as **magnetographs** are used for the continuous recording of the changes in the elements at any place, the three elements selected being the declination, the horizontal component, and the vertical component. In each case the moving magnet system carries a mirror, from which a beam of light from a lamp is reflected on to sensitised paper or photographic film, the latter being attached to a revolving drum; thus any movement of the system is indicated by the trace on the steadily moving film.

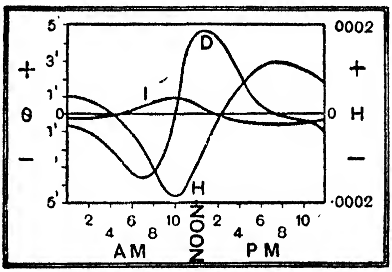


FIG. 246.

(1) *Declination.*—The magnetograph consists of a magnet suspended by a long fibre, and carrying a mirror (usually concave). Just below this mirror is a second one fixed to the base of the instrument. Light from a lamp passes through a slit, falls upon these two mirrors, and is reflected therefrom through a plane cylindrical lens to the sensitised paper on the revolving drum, the paper moving parallel to the axis of rotation of the magnet. The spot of light produced by reflection from the fixed mirror traces a straight line on the paper, which serves as a zero or base line. The spot of light produced by reflection from the magnet mirror traces a curve, the distance of which from the base line will vary with

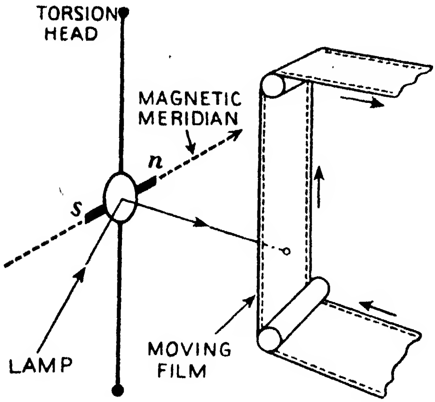


FIG. 247.

any variation in the position of equilibrium of the magnet, *i.e.* with any variation in the declination.

The torsion magnetometer of Fig. 126 may be used for the purpose, a mirror being attached to the suspended magnet. The torsion head is turned at the outset until the magnet rests in the meridian. Fig. 247 roughly indicates the idea, the fixed mirror being dispensed with.

(2) *Horizontal Component.*—One form of magnetograph consists of a magnet with a bifilar suspension, and arranged so that *the magnet is at right angles to the meridian.* In this case (see page 63) the couple due to the earth is MH, and this is balanced by the couple due to the bifilar suspension; hence any change in H will result in a slight rotation of the magnet which is recorded by a trace on the moving film as before. Fig. 248 roughly indicates the general principle using the simpler apparatus.

(3) *Vertical Component.*—In this case the instrument is arranged with the *suspension horizontal and in the meridian.* The N pole of the needle will, of course, dip, but the torsion head is turned until the magnet lies horizontally. The forces (mV) acting on each pole are as shown in Fig. 249, so that any change in V will result in a slight rotation of the magnet-mirror system, the change being recorded as before.

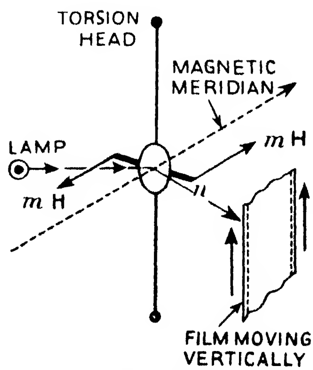


FIG. 248.

Watson's apparatus is more complex and accurate. In it (Fig. 250) the magnets NS, N′S′, are arranged as above to move about a horizontal axis, the plane of rotation being the magnetic meridian. Q is a plate of fused quartz, the projecting arms of which carry the magnets, and the

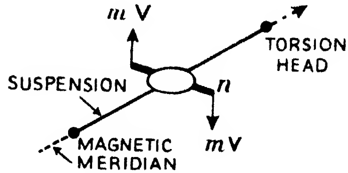


FIG. 249.

upper surface of which acts as a mirror. The fibres qq are of quartz, one being attached to the spring S, and the other to the torsion head T. The counterpoise C is adjusted until the ends SS′ are brought below the horizontal, and T is then turned until the magnets are brought into the horizontal. The totally reflecting prism P enables light in a horizontal direction to be utilised. As before, any variation in V will result in a movement of the system, which is recorded photographically. The construction of Watson's eliminates temperature errors.

Variations in dip *can* be continuously recorded, but the arrangement is troublesome, so that these variations are usually estimated from those of V and H ($V/H = \tan \theta$).

5. Variations in the Elements from Place to Place. Magnetic Maps

The magnetic elements have different values at different places on the earth's surface, and this variation is indicated by **magnetic maps,** *i.e.* maps on which lines are drawn joining those places which have the same value for the various elements: thus we have maps of the following:—

(1) *Isogonals, i.e.* lines joining places at which the declination is the same: the isogonals which pass through points having zero declination are termed *agonic lines.*

(2) *Isoclinals, i.e.* lines joining places at which the inclination or dip is the same: the line which passes through points of zero dip is called the *aclinic line* or *magnetic equator.*

(3) *Isodynamics, i.e.* lines joining places at which the horizontal force is the same. Lines joining points having equal values for vertical force and for total force are also used.

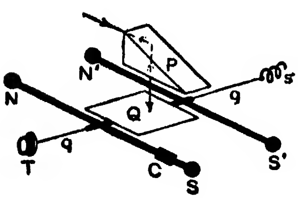


FIG. 250.

In addition to the above we have *Duperrey's lines* which indicate the directions of the magnetic meridians. In all cases the lines are drawn as smooth curves, but in reality they are very irregular, due to local circumstances.

Fig. 251 shows the *isogonals* and the *agonic lines*, the thick lines denoting

the agonic lines, the continuous lines the isogonals on which the declination is west, and the dotted lines those on which the declination is east. An examination of the map will show that the isogonals converge towards four points, viz. the two geographical poles and the two magnetic poles of the earth.

The *agonic* lines are, however, of more special interest, and of these there are three, viz. the *American agonic line*, the *European agonic line*, and the *Siberian oval*. Starting at the north magnetic pole, the American agonic line passes across Canada, the eastern United States, South America, to the south geographical pole. A curve continues this from the south geographical to the south magnetic pole, at every point of which the compass needle also lies in the geographical meridian, but its north pole points south instead of north, so that the curve is an isogonal of 180°. The European agonic follows an unexpected course, and again we have a part between the north geographical and north magnetic poles where the needle lies in the geographical meridian, but with its north pole pointing south, so that the curve is an isogonal of 180°. The third agonic line is an oval of which Japan is now the centre. At places between the American and European agonic lines the declination is *west*, inside the European it is *east*, inside the oval it is *west*, and between the oval and the American agonic line it is *east*.

Fig. 252 is a map showing the *isoclinals* and the *aclinic line* or *magnetic equator*. The magnetic equator lies north of the geographical equator in Asia and Africa, and south of it in America; it crosses the geographical at a point in the Atlantic and at another point in the Pacific Ocean. Starting at the magnetic equator, where the dip is zero, *i.e.* the dip needle lies horizontal, and travelling north along a meridian the dip increases, the north pole pointing downwards, and is 90° at the north magnetic pole; similarly, travelling south from the magnetic equator along a meridian the dip increases, the south pole pointing downwards, and is 90° at south magnetic pole.

Maps are also drawn showing the various *isodynamic lines* (for H, V, and T). The *isodynamic lines* joining places having equal values of the *total force* indicate that the latter reaches its minimum value in equatorial regions, and its maximum values in two regions in the northern hemisphere and in two regions in the southern hemisphere: these latter four regions are called **magnetic foci.**

The student must distinguish between the isogonals and Duperrey's lines; the latter indicate the magnetic meridans, are not nearly so irregular as the isogonals, and converge, not to four, but to two points only, viz. the magnetic poles.

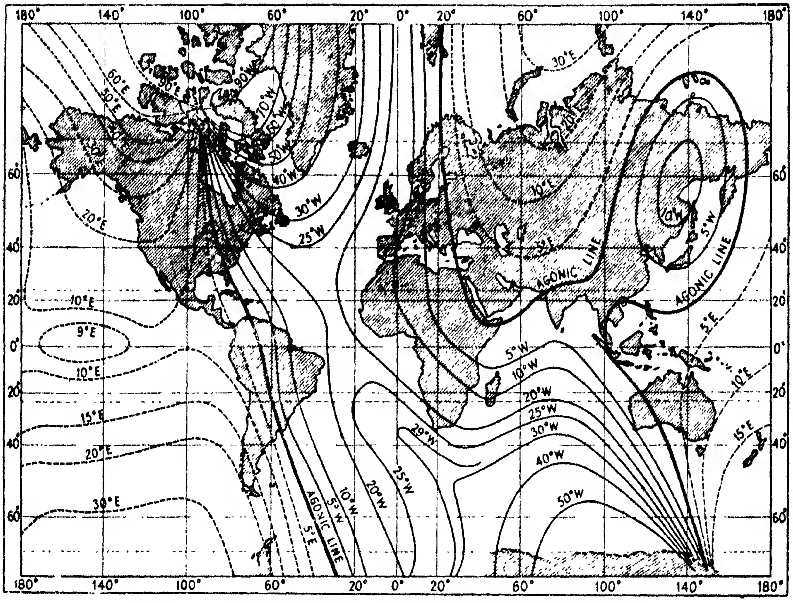


FIG. 251. Map showing Isogonals and the Agonic Lines.

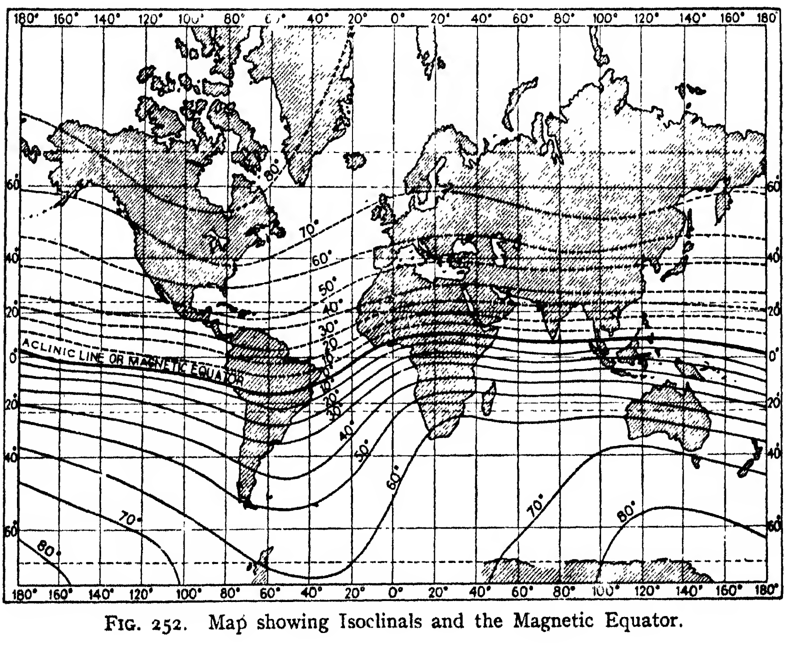


FIG. 252. Map showing Isoclinals and the Magnetic Equator.

6. Theory of the Earth's Magnetism

Although the earth's magnetism has been very carefully studied very little is known as to its actual origin—all theories at present are really inadequate.

An attractive theory is, of course, to attribute the earth's magnetism to magnetic material (largely ferromagnetic) in the earth itself, and Gauss in 1839 worked out an elaborate theory coming to the conclusion that this was the fundamental cause. If this were the sole cause, however, then to fit in with observed facts the deeper layers and centre of the earth would certainly have to contain a very much greater proportion of ferromagnetic material than is known to exist in the outer surface layers: but an objection to all this is that the temperature of the central regions of the earth is much above that at which ferromagnetic material loses its ferromagnetism.

In 1849 Grover contended that the earth's magnetism was due to the influence of electric currents circulating in the earth and around it, these being primarily due to the action of the sun and modified by the earth's movements, and by an extension of Gauss's work, Schuster in 1870 came to the conclusion that *at least* the daily variations in the elements were undoubtedly due to some cause outside the earth, "probably to electric currents in our atmosphere": further, not only the daily changes but the coincidence of the eleven-year cycle of magnetic changes with the eleven-year sun-spot cycle indicates a connexion between the earth's magnetism and changes on the sun. The suggestion then that the earth's magnetic field is due to currents of electricity in the upper ionised layers of air (which are ionised by radiation from the sun) must also be considered. A difficulty which arises, however, is that it is not easy to see how currents of sufficient strength could be produced to fully account for all the facts: moreover the strength of the earth's field *decreases rapidly as we travel upwards* from its surface.

Another contributing cause which has been suggested is that of currents set up in the earth itself by its own rotation: that this is not impossible is probably indicated by the fact that twenty years ago certain unusual changes in the rate of rotation of the earth were accompanied by unusual changes in the magnetic elements.

As stated above, our knowledge on the subject of the causes of the earth's magnetism is thus far from complete, although we are on a little surer ground in dealing with such matters as the daily and annual variations. Probably several factors contribute—

magnetic material in the earth, electric currents in the earth, electric currents in the upper atmosphere, electrons emitted by the sun, possibly action of moon, etc.

One further point may be noted which will be of interest to the mathematically minded. The magnetic field at the surface of the earth is, in general and neglecting local irregularities, somewhat similar to what it would be if the earth's magnetism were due to a very small magnet with its mid-point at the centre of the earth, its north pole pointing to the earth's south magnetic pole, and its axis making an angle of about 17° with the earth's axis. From the theory of a small magnet given in Art. 10, page 71, it is possible on this supposition to deduce expressions for the value of the intensity at various points on the earth's surface, and also certain facts about dip and declination at various places. As an example we may take the following:—

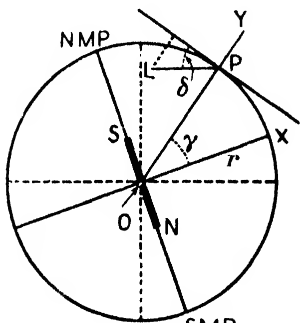


FIG. 253.

For the point P on the earth's surface (Fig. 253) the angle γ is the *magnetic* latitude for X is a point on the equatorial line of the magnet NS. Further, δ is the dip at P for it is the angle between the horizontal and the direction of the dip needle, viz. PL. Now the angle PON (viz. $90° + \gamma$) corresponds to α in Art. 10, page 71, and the angle YPL (viz. $90 + \delta$) corresponds to θ. It was shown that $\tan \alpha = 2 \tan \theta$: hence $\tan (90 + \gamma) = 2 \tan (90 + \delta)$ or $\cot \gamma = 2 \cot \delta$ or $\tan \delta = 2 \tan \gamma$, i.e. *the tangent of the angle of dip is twice the tangent of the magnetic latitude.*

Again applying the result (13) of Art. 10, page 72, to Fig. 253, it is easy to deduce that if T_a be the total intensity of the earth's field at the magnetic equator (say at X), the total intensity T at P is given by:—

$$T = T_a\sqrt{1 + 3 \sin^2 \gamma},$$

where, of course, $T_a = M/r^3$ the moment of NS (*i.e.* of the earth on this supposition) being M.

B—ATMOSPHERIC ELECTRICITY

7. Some Facts about the Electric Discharge in Air

We will first consider a few *observed experimental facts* in connexion with the electric discharge between two conductors separated by air at or near ordinary atmospheric pressure and charged to different potentials—say the discharge between the positive and

negative poles of a Wimshurst electrical machine: *explanation* of the fundamental facts will be dealt with presently.

When the gradient of potential in the insulating medium (air) is raised sufficiently high by the approach of the positive and negative poles of the machine to each other, the medium breaks down under the stress, and the accumulated energy is liberated by **disruptive discharge** between the poles. The appearance presented may be either that of a single or branched line of light (straight, curved, or zig-zag) from one pole to the other, when it is called a **spark,** or that of a brush-like glow, with branches diverging from a stem springing from one of the conductors, when it is called a **brush.**

FIG. 254*a*.

The discharge takes the spark or brush form according as the quantity of electricity to be discharged is large or small. When the quantity is large and the distance small the spark is short, straight, and intense. As the distance increases the spark line ceases to be straight, and takes a branching form similar to that shown in Fig. 254*a*. It should be noted that in a spark of this kind the tips of the branches point from the positive to the negative pole. With a limited quantity of electricity the discharge takes the *brush* form. A bright luminous brush of a light violet colour branches from the positive terminal in the way shown in Fig. 254*b*, whilst the negative is covered with a luminous glow. When brush discharge takes place between two terminals, such as the poles of a Wimshurst machine, which are supplied with electricity at a uniform rate, the difference of potential between the terminals remains practically constant during the discharge. In the case of a spark under the same conditions, the difference of potential rises to a maximum at the instant of discharge and at once falls to zero to rise again to a maximum for the next spark. The shape of the opposing charged conductors has also some effect on the kind of discharge obtained: rounded surfaces tend to produce sparks and pointed ones brushes.

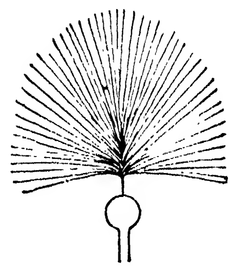


FIG. 254*b*.

If the two conductors be joined to a *very powerful source of electricity* a fresh discharge may follow before the effect of the preceding one has disappeared. In such cases instead of a series

of sparks each following its own zig-zag path, the discharge forms a continuous band of glowing gas known as an **arc**: moreover the heat developed is so very great that the hot gas rises and in doing so results in the arc assuming the bow shape indicated in Fig. 255. This property is made use of in some practical appliances employed to lengthen and finally break any arc which may be started by lightning between overhead electric power transmission lines.

The distance which separates the two conductors when the discharge takes place is called their **striking distance,** and for a given pair of conductors similarly placed it is found to be very nearly proportional to their potential difference: it depends, however, also upon the shape of the conductors. Thus for a pair of spheres the striking distance is about one inch per 100,000 volts, while for a point facing a flat plate it is much greater, viz. about one inch per 20,000 volts; and for a pair of points greater still. It should be noted also that "striking distance" refers to the *starting* of the discharge and not to its *maintenance*. For example, suppose we have two spheres which by means of a machine are kept at a potential difference of 10,000 volts; then in order to start a spark between them they must be brought to within $\frac{1}{10}$ inch (the striking distance), but after the spark has once started they may be gradually separated to a much greater distance and it will still continue.

FIG. 255.

A somewhat similar fact is also seen in the *gas-discharge lamps* now on the market. At present we are mainly concerned with the electric discharge—spark and brush—in air at ordinary pressure, whereas in these lamps the gas medium between the conductors or electrodes is *rarefied*. Electric discharge in gases at low gas pressure is a complex and important phenomena which is dealt with in detail in Chapter XIX., but it is interesting to note here that in these gas discharge lamps it takes a much bigger P.D. to start or "strike" the discharge than it does to keep it going once it is started. Thus a 10 ft. neon tube (neon sign) needs a "striking voltage" of about 3200 volts, but, once started, 1500 volts will keep it going.

It has been shown that the relation between spark length and potential difference for sparks more than 2 mm. long is given by:—

$$V = a + bl,$$

where V is the potential difference, l the spark length, and a and b

constants. Baille's results for air gave $a = 4{\cdot}997$ and $b = 99{\cdot}593$, with l in cm. and V in electrostatic units (1 e.s. unit = 300 volts).

The colour of the discharge depends on the medium (air or other gas, etc.) between the conducting charged surfaces, on the nature of any foreign particles in the medium, and on the material of the conducting surfaces, for particles of the latter are volatilised.

If the gaseous medium between the terminals is in a rarefied condition, however, *i.e.* gas pressure low, the medium mainly decides the colour of the glow discharge: thus in a neon gas discharge tube the colour is a rich red-orange, with mercury vapour a distinct blue would be obtained, with mercury vapour and neon a bluish-white, and with various mixtures various other shades of colour.

When a disruptive discharge of any form is examined by means of a rapidly rotating mirror it is found to be of an *oscillatory* character. Thus the image of a short straight spark seen in a revolving mirror is a number of short straight parallel lines of light separated by narrow intervals corresponding to the period of oscillation of the discharge.

When the discharge takes place *through a conductor* it is accompanied by all the effects produced by an electric current. Heat is developed, a magnetic field is set up, liquid may be decomposed. When the conductor is of bad conducting material violent mechanical effects are often produced. When the discharge takes place through the human body, the physiological effects which accompany the *shock* depend on the *energy* of the discharge, that is, upon the quantity discharged as well as upon the difference of potential, and also upon the time rate of discharge. The larger the quantity, and potential difference, and the more sudden the discharge, the greater the shock.

A Digression.—There is one point in connexion with the electric " shock " which may be mentioned here as a matter of interest, for it is often puzzling to the beginner. The student will often hear that a *very high frequency current*—even at high P.D.—will not cause a " shock," that it is possible to pass one of several amperes—sufficient to light many glow lamps—through the body without harm. This is true. A similar C.C. or ordinary A.C. would cause a " shock "—perhaps death.

An electric " shock " is produced as follows:—The cells of which the body is made up contain liquid, mainly common salt solution, which is dissociated or split up into +ve and −ve ions, viz. positive sodium and negative chlorine ions (page 12). When a P.D. is applied, the positive and negative ions set off towards opposite walls of the cells causing a change in the concentration of the liquid. This change stimulates the nerves (this is the " shock ") and causes chemical changes which result in death if the changes are very marked,

i.e. if the electric discharge through the body is a heavy one. Now in the case of a *very high frequency* current the " flows " first one way, then the other, *follow each other so rapidly* that they cancel each others effect: the ions have not time to get moving off to any extent in accordance with the first " flow " before the current reverses causing movement in the opposite direction. Hence there is no " shock." High frequency treatment is often used in hospitals: the " cure " is mainly due to the local heating effect.

8. Electrical Potential at a Point in the Air

It has been found by direct measurement that the potential at a point in the open air is always different from the potential of the earth, and usually higher than it. One method of measuring this potential is by means of a type of *water dropper*. (Read again page 157.) In this case a metal cistern, fitted with a tap having a very fine nozzle, is filled with water, carefully insulated, and fixed in position so that the end of the nozzle is at the point in the air at which the potential is to be measured. Water is allowed to drop rapidly from the nozzle, and owing to the inductive action on the drops as they are detached from the nozzle, the cistern gradually becomes charged up to the potential at the point of the nozzle. If the cistern is connected to an absolute electrometer, or to one pair of quadrants of a quadrant electrometer, the potential it attains when equilibrium is set up can be directly measured, and this gives the potential of the air at the point selected.

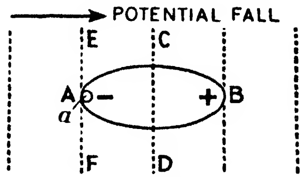


FIG. 256.

Perhaps the following will make the principle of action of the water-dropper clearer in this case. For simplicity in the diagram (Fig. 256) suppose the potential of the air decreases in the direction left to right and that an insulated conductor AB is placed as shown. A −ve charge is induced at A, a +ve at B, and the whole of AB acquires a uniform positive potential, say that of the equipotential surface CD. Now suppose a small portion (*a*) of the conductor at A drops off carrying its small −ve charge with it: the potential of AB will rise a little above the value of CD. Suppose that in some way the conductor AB grows again to replace the piece that has fallen off, and that then this new piece drops off again carrying its small −ve charge with it: the potential of AB rises a little more. It is clear that if this " growing on " and " falling off " be repeated, the potential of AB will rise until it is equal to the potential of the air at A, *i.e.* to that of the equipotential EF. Similarly the potential of all the water in the cistern (and the cistern) rises (or falls) until it is at the same potential as the air at the nozzle (which corresponds to the end A).

A more modern method is to employ a wire tipped with radium or other radio-active substance; this ionises the air, and the charge on the end of the wire is neutralised by the oppositely charged ions.

The potential at a point in the air is, in fine weather, always positive, and increases with height above the ground: the rate of increase with height is very variable. During wet and changeable weather the potential may be negative, and is again always very variable in value. The electrification of the air also varies, even under settled conditions of weather, with the season and the hour of the day: it seems to be a maximum at the times of greatest variation of temperature, and a minimum during the hours of constant temperature.

9. Lightning, Thunder, Aurora Borealis or Northern Lights

An early theory attributed atmospheric electricity to the processes of evaporation, vegetation, and combustion. This theory supposed that the water vapour arising from water on the surface of the earth carries a positive charge with it, leaving the water and the earth negatively charged; the processes of vegetation and combustion are also supposed to carry into the air a positive electrification. Experiments indicate, however, that these operations are insufficient to account for all the observed phenomena, and even the work of various experimenters is inconsistent.

It has been known for some time that the *splashing of liquids* results in electrical separation. Thus, if water falls on a metal plate the air around becomes negatively electrified, the spray positively electrified; but salt water produces an opposite result. Further, it has been shown that when pure water is broken into drops by means of an air jet, electrical separation ensues, the water exhibiting a positive charge: such experiments led to the suggestion that water in the atmosphere may be broken up by air currents, etc., and the electrification partly accounted for in this way.

We have seen that although normally air is an insulator and that air atoms cling to their electrons, there are conditions under which electrons can be, and are, forced out, such atoms losing electrons in this way becoming therefore positive ions. As a matter of fact, most gases exist as molecules composed of two or more atoms, and to be more exact, *a positive gas ion is a molecule* which has lost electrons from its atoms, but this is a detail which is not important from our present point of view. This forcing out of electrons and the resulting production of positive ions—*ionisation* as we have already named it—can be brought about by heat, by

ultra-violet rays from the sun, by electrons emitted by the sun, by emanations from radio-active material, by X-rays, by cosmic rays which reach the atmosphere from remote space, etc. It can be definitely taken, then, that the air is to a certain extent ionised, although so far as the lower regions of the atmosphere are concerned the ionisation is not sufficient to so improve the conductivity as to interfere with our ordinary electrostatic experiments.

Further, it must be remembered that the air molecules (and any ions and electrons present) are in a constant state of movement (thermal agitation) which is greater the higher the temperature, and in this movement constant collisions are being made with other molecules. Moreover it is clear that if the movement of the detached electrons and ions in ionised air could be sufficiently increased—say by a P.D. between different parts of the air which would accelerate the electrons in one direction and the positive ions in the other direction—the electrons (and ions) may be so accelerated that in collision with other atoms they would cause more electrons to be expelled and more positive ions to be produced, and so on: thus the ionisation would be cumulative and the conductivity of the air much increased.

Now there is of course water vapour in the upper atmosphere. It is well known that the process of condensation necessitates a nucleus, *e.g.* a dust particle, etc., and it is easy to understand that ions and electrons will act as nuclei. Thus drops of water formed by the condensation of vapour on, say a positive ion of air in the upper regions, will be positively charged, and when several drops combine a charged cloud will be produced, the potential of which may reach a high positive value. This positively charged cloud acts inductively on the earth, causing it to have a negative charge under the cloud. Further, the big potential difference between cloud and earth and the big electric force between the two causes more electrons to be torn out of air atoms, *i.e.* produces more ionisation of the air in between. This cumulative ionisation really means that the air between cloud and earth ceases to be insulating, for the positive ions tend to move down to the negative earth to neutralise the negative there, and the electrons (free or "loaded"—p. 131), tend to move up to the positive cloud. Finally the conducting power of the air may be so improved by this ionisation, and the P.D. between cloud and earth may be so big, that the P.D. is able to drive electricity across the gap, and a discharge of electricity takes place between cloud and earth neutralising both of them. This rush of electricity is our well-known *lightning*.

An electrically charged cloud is thus probably made up of a very large number of isolated charged drops of water, and is probably charged *throughout its mass* and not on its surface only. As the cloud grows by the drops uniting to form larger drops, the potential of each must evidently rise. For, if eight small drops unite to form one larger one, the charge on this drop will be eight times that of one of the smaller drops, but its radius will be only twice as great, and, therefore, the potential of each drop will be four times as great and the surface density twice as great as before coalescence. The potential of a heavy cloud made up of comparatively large droplets may in this way rise to a very high value.

Lightning, then, is a " disruptive discharge " of electricity on a large scale between a charged cloud and the earth (Fig. 257) or, it may be, between two clouds charged to widely different potentials. Now, what about the *light* and *heat* when lightning occurs? We " see " the flash, and we know that lightning often sets fire to trees, hay-stacks, etc., which it strikes. When positive ions and electrons join up again in an ionised gas the energy which was necessary to separate them when the gas was ionised reappears as heat, whilst some is radiated away, and the frequency of some of this radiation falls within the range limits we call " light." It is the joining up of these in the *path of the discharge* which causes the light of the flash. The familiar *colour* of a lightning flash depends on the kind of atoms in the air (and water vapour), viz. hydrogen, ozygen, and nitrogen. If the atmosphere consisted of other gases lightning would have a different colour: if the air, for example, were mainly neon, lightning would be a magnificent red-orange. This " heat " and " light " when electrons are captured by ions on a large scale is dealt with again later.

Latest research seems to indicate that the process of a flash is as follows:—At first a little tongue of light stretches earthwards about 50 yards from the cloud. The light then fades out. It then reappears and stretches about another 50 yards until the ground is reached. As soon as this reaches the ground the main stroke begins: a brilliant flame sweeps up from the ground to the cloud retracing the previous path. This stroke lasts about 50×10^{-6} sec.

Lightning itself is usually of the *forked* variety shown in Fig. 257. *Sheet* lightning is probably due, to a great extent, to reflections in the clouds of discharges between clouds at too great a distance for thunder to be heard. *Globe* lightning consists of " balls of fire " which move slowly along and then sometimes burst with a loud explosion: the explanation is not known, and it is rare.

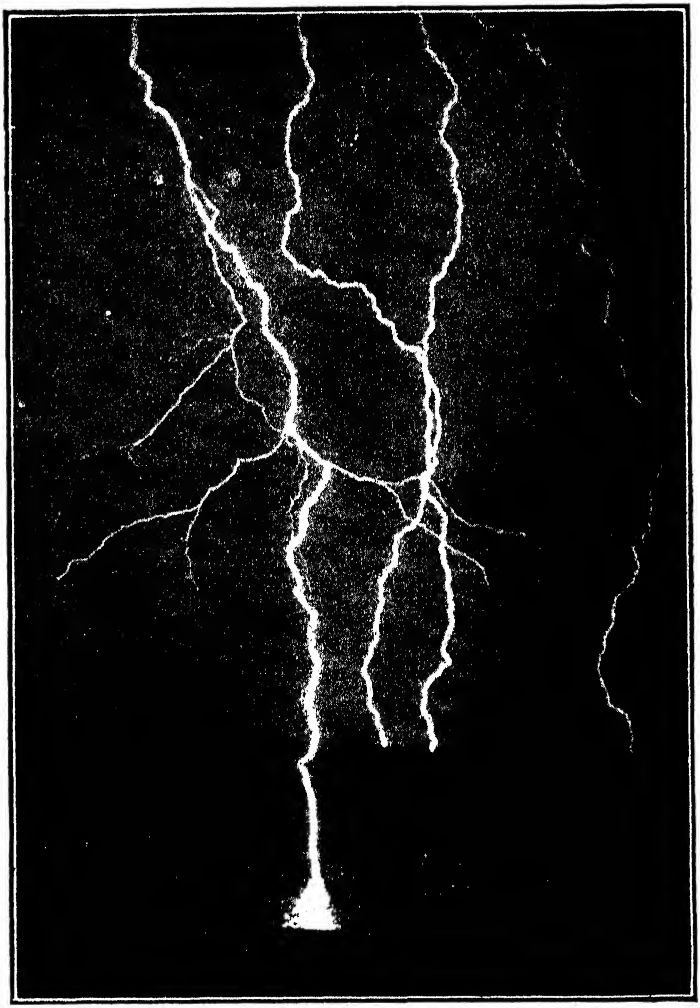


FIG. 257. LIGHTNING.

Photograph taken at Sydney Observatory. But for the storm the night was pitch dark; the faint picture of the district round about was due to light diffused by the flashes. The part of the foremost flash (on the water) seen in the picture was 1540 feet long: the whole length of the flash was not photographed.

The sound—*thunder*—is caused by the sudden closing in of the air in the path of the lightning after the tremendous heat and expansion due to ions and electrons joining up again. If the path is short and straight the *thunder clap* is heard: if it is zig-zag we have the *thunder rattle.* The *rumbling* or rolling is due mainly to echoes among the clouds.

Lightning conductors consist of iron or copper rod or flat strip running from the top to the bottom of the building to be protected. At its upper end it is sharp pointed, and its lower end is well earthed. If a lightning flash occurs in the vicinity of the building it tends to strike the highest point—the lightning conductor—and thus finds an easy path to earth without damage to the building. But the lightning conductor may *prevent* the flash altogether. If the

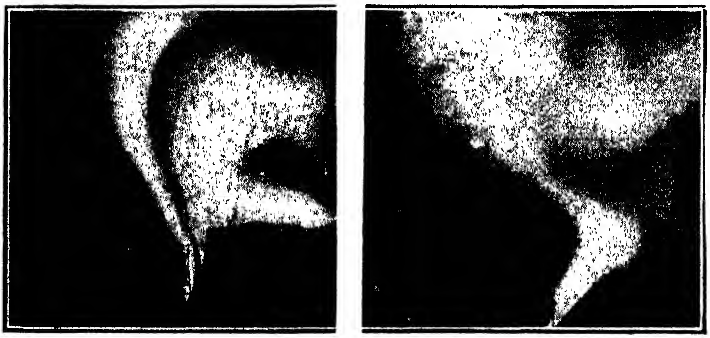


FIG. 258. AURORA BOREALIS OR NORTHERN LIGHTS.
(British Polar Year Expedition, Fort Rae, North-West Canada.)

thunder cloud is positive the "density" of the negative induced at the top points of the lightning conductor will be great: thus, as explained under "action of points" (Chapter V.), a stream of electrons and negative ions of air proceeds from the points neutralising the positive cloud above and preventing the flash altogether. There really should be several such lightning conductors to a building: the general principle is to *surround* the building with a network of wires and to give the lightning many easy paths to earth.

Yet another excellent example of light emitted by recombination of ions is the *aurora* (Northern Lights). This is a bluish—often curtain-like—glow hanging in the sky in high latitudes (Fig. 258). It is due to the joining-up again of ions and electrons produced in the upper atmosphere by electrons from the sun, the electrons being deflected towards the poles by the magnetic field of the earth.

CHAPTER IX

PRODUCTION OF AN ELECTRIC CURRENT. GENERAL EFFECTS OF A CURRENT

TO obtain a continual flow of electricity from one point to another some conducting path must be put between them, and means must be devised for keeping the two points at different potentials, in which case, assuming the path is the usual wire, electrons will continue to "flow" (pages 17-21) in the direction lower to higher potential, each electron having a definite small (negative) charge. We refer to this as a "current" in the wire, the number of electrons passing any section in one second determining the "current strength," but we only know it is there by certain effects which it produces. (We often speak of the current as flowing from higher to lower potential—the conventional direction.)

One method of producing and maintaining this P.D. is by chemical action as, for example, in the various types of voltaic cells. This was briefly referred to in Chapter I. (page 20), and it will be an advantage to consider here in more detail the action of the *simple cell* there mentioned, for it will assist the reader to understand much that follows. Other methods of producing a P.D. to obtain a continued current are dealt with in subsequent chapters: on the "commercial scale" they are produced by dynamos (or D.C. generators) and alternators (A.C.) which usually involve conductors *moving in magnetic fields*.

1. Theory of the Action of a Simple Voltaic Cell

It will be remembered that when the *insulator* vulcanite is rubbed (to secure close *contact*) with fur the net result is that *electrons* pass the junction from fur to vulcanite so that the vulcanite is negative and the fur positive, and a P.D. is set up between them.

The simple cell consists of plates of zinc and copper in a dilute solution of sulphuric acid. The following experiments will illustrate some facts about the action.

(1) Place a plate of common zinc (Zn) in dilute sulphuric acid (H_2SO_4); a violent action ensues, zinc is dissolved from the plate, zinc sulphate ($ZnSO_4$) is formed, hydrogen gas (H_2) is evolved, and the solution becomes hot. The chemical action is expressed by the equation $Zn + H_2SO_4 = ZnSO_4 + H_2$.

(2) Amalgamate the zinc (*i.e.* coat with mercury) and replace: no action is *observed*. Insert a plate of copper; no action is *observed*.

(3) Place the amalgamated zinc and the copper side by side in the acid but not touching each other; still no action is *observed*.

(4) Join the two plates by a wire (Fig. 259) and it will be found that (*a*) zinc dissolves into the solution and zinc sulphate is formed; (*b*) hydrogen gas appears *at the copper plate* (note this); and (*c*) a current flows in the circuit as can be proved by bringing a compass needle near it: from the direction of deflection it will be seen that the *conventional* direction of the current in the wire is from the copper to the zinc, whilst in the cell it is from zinc to copper. There is also heat produced in the wire and in the liquid.

(5) Disconnect the wire and connect the plates to a voltmeter: it will register a P.D. between them of about 1 volt. Replace the voltmeter by an ammeter reading up to about 5 amperes: it will register a current of about 1 or 2 amperes at first, but this will decrease and particularly so as more and more hydrogen collects on the copper plate: if the hydrogen bubbles be brushed off the copper, the current almost regains its former value, but decreases again as further bubbles collect on the plate.

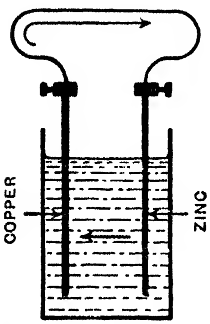


FIG. 259. The *conventional* current direction in the wire is indicated.

As can be shown by a quadrant electrometer, the copper is at a higher potential than the zinc and is called the *high potential plate*, the portion of it outside being called the *positive pole*: the zinc is the *low potential plate*, the outside portion the *negative pole*. In the outside circuit the current was said, in the early days, to flow from copper to zinc—*high to low potential*—whilst inside it was said to go from zinc to copper, the energy of the chemical action taking the current, as it were, from *low to high potential* inside. The chemical action was often likened, in fact, to that of a pump lifting water from a lower to a higher level, from which position the water would run down again, doing work in virtue of the energy the pump had conferred upon it. In the cell the consumption of the zinc provides the energy which keeps the current flowing.

The pump analogy *is very faulty*, however, unless further facts are included, for amongst other defects it gives the impression, which is not correct, that there is a rise in potential all the way through the liquid from the low potential zinc to the high potential copper, and that the "pumping action" consists in moving the electricity *up* this potential slope or potential gradient all the way from zinc to copper. Below is a brief account of the essential facts

of the modern theory, but further details of some of the principles involved are given in subsequent Chapters.

Consider first a zinc plate in a dilute solution of zinc sulphate. So far as the plate is concerned, we know that zinc is electro-positive (page 12), and its outer electrons not very firmly held in the atom: in fact they wander out of the atom, leaving positive zinc ions. So far as the solution is concerned we know that there are positive zinc ions (Zn) and negative sulphions (SO_4) for the solution is an *electrolyte* and some at least of its molecules are dissociated or split up into these oppositely charged ions (pages 12, 21, 130): note that metallic ions such as zinc, copper, potassium, etc., and hydrogen are the *positive* ions.

Now in this case (metal in electrolyte) there is a tendency for *positive* Zn *ions* (note this) of the metal to cross the boundary to the solution, and at the same time there is a tendency for the zinc ions in the solution to cross the boundary in the other direction to the plate (this diffusion across boundaries is a well-known fact in many branches of science). In the case of a metal like zinc the tendency for the positive Zn ions of the metal to pass to the solution is greater than the tendency of the Zn ions of the solution to pass to the metal, and *more do pass into the solution than out of it.*

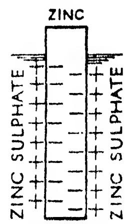


FIG. 260.

But when these extra positive ions pass from the metal they leave it negatively charged. The final result is therefore that the metal becomes at a lower potential than the solution. At the boundary we have a "contact potential difference," the zinc being at a lower potential than the solution: this takes place in a *very narrow film* at the boundary so that we have a kind of molecular or *very thin film* condenser at the boundary, the zinc being the negative and the solution the positive coating of the condenser. The maximum P.D. is soon reached, for the zinc becomes so much negative and the other side so positive that the P.D. opposes any further transfer across the boundary. Modern work gives the potential of the zinc as about 0·76 volt *below* the potential of the solution side of the very narrow film. (Results of latest work on these P.D. values are given in Chapter XIII.) This may be indicated as in Fig. 260, the signs + and − showing that the zinc is below the liquid in potential.

Now take a copper plate in a solution of copper sulphate ($CuSO_4$), an electrolyte which is dissociated into positive copper ions and negative sulphions. Here again there is a tendency for positive

copper ions to pass across from the plate to the solution and also from the solution to the plate. But in this case the tendency to pass from solution to plate is greater than the tendency to pass the other way, and *more positive copper ions do pass to the plate.* This means that the plate acquires a higher potential than the solution at the boundary (Fig. 261). The potential of the copper is about 0·34 volt *above* the potential of the solution side of the very narrow film.

These boundary P.D.'s are established practically instantaneously when the plates are placed in the solutions, but the actual number of ions which is transferred to attain the P.D. is not great: with zinc in zinc sulphate the zinc passing to the solution to set up the P.D. is not more than $5{\cdot}5 \times 10^{-8}$ grm. per sq. cm. of the plate surface. The P.D. set up at a metal-electrolyte junction in the manner indicated is usually referred to as the **electrode potential** of the metal with respect to the electrolyte.

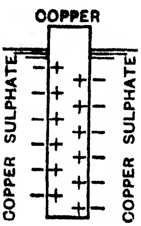


FIG. 261.

As a matter of fact, as will be seen in Chapter XIII., when a metal is placed in a solution of the same metal the metallic ions in the solution exert a "pressure" known as **osmotic pressure** tending to drive metallic ions upon the metal, and the ions in the metal also exert a pressure known as **solution pressure** tending to drive metallic ions into solution. In the case of zinc in zinc sulphate the osmotic pressure of the positive zinc ions in solution is less than the solution pressure of the zinc: the result is that more *positive* zinc ions pass *from the zinc plate* and an electric double layer is formed, the zinc plate being negative and the solution side positive. This continues until the greater of the two, *i.e.* the solution pressure, is balanced by the smaller osmotic pressure *plus* the electric double layer. In the case of copper in copper sulphate the osmotic pressure of the positive copper ions in solution is greater than the solution pressure of the copper, so that more copper ions are driven on the copper and the electric double layer is of opposite sign, *i.e.* the copper plate is positive and the solution side negative. If in any case of a metal and a solution the osmotic pressure were equal to the solution pressure no double layer would be formed and no P.D. would exist at the boundary of metal and solution.

It may be noted here that the greater the concentration of the solution the greater is the osmotic pressure. In the case of the zinc plate in zinc sulphate, the stronger the solution the more positive ions will be driven to the plate and the less, therefore, will be the *negative* potential of the plate,

i.e. the electrode potential will be less. On the other hand, in the case of copper in copper sulphate, the stronger the solution the greater will be the *positive* potential of the copper plate, *i.e.* the electrode potential will be greater.

Now similar results happen if the zinc and copper plates are put in dilute sulphuric acid (which is dissociated into positive hydrogen ions and negative sulphions), although the positive ions in the solution, viz. hydrogen, are not the same kind as those of the metals. Suppose a plate of zinc and a plate of copper to be in the acid (this is our " simple cell "). At the zinc the tendency of positive zinc ions to pass to the solution is greater than the tendency of positive hydrogen ions to pass to the zinc, and more do, so that the zinc is again at a lower potential than the solution side of the very narrow film: latest research gives the zinc as about ·62 volt

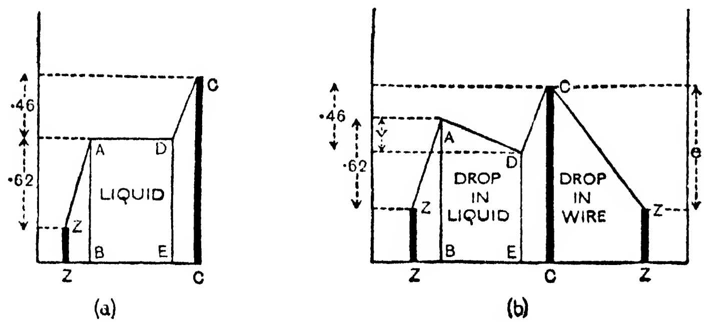


FIG. 262.

below the solution. At the copper the tendency for the positive hydrogen ions to pass across to the copper is greater than the tendency of copper ions to pass the other way, so that the copper is again at a higher potential than the solution side of the boundary film: the copper is about ·46 volts *above* the solution.

Fig. 262 (*a*) shows roughly the case so far, vertical distances denoting potentials and AB and DE the outer surfaces of the films (much exaggerated in width) at the zinc and copper. There is a " jump-up " of potential from the zinc Z to AB of ·62 volt. The acid, being a conductor and no current flowing, has the same potential throughout (shown by the level line AD). Then there is a " jump-up " of potential from DE to the copper C of ·46 volt. The P.D. between the zinc and copper is (·62 + ·46) = 1·08 volt, and this P.D. at the terminals of the cell when it is on " open circuit," *i.e.* when its poles are not connected so that it is not

giving a current, measures what is termed the **electro-motive-force** (E.M.F.) of the cell.

Now join the poles by a wire. Electrons "flow" in the wire from the zinc to the copper, and (as they are negative) this raises the potential of the zinc and lowers the potential of the copper. So far as the potential slopes in the films are concerned, however, the "actions" come on again, so that the "jump-up" in the thin film at the zinc is kept practically equal to ·62 (Fig. 262 (*b*)), and that at the copper equal to ·46 (neglecting the influence of any hydrogen gas bubbles collecting there—see below). But the liquid is no longer of uniform potential for "current" is flowing. There is a fall of potential from AB to DE, *i.e.* in the direction of the conventional current, and negative SO_4 ions move *up* this slope from low to high, *i.e.* from DE to AB or in the direction copper to zinc, whilst positive H ions move *down* the slope from high to low, *i.e.* from AB to DE or in the direction zinc to copper. If V volts be the drop in potential from AB to DE then V is what the technical man refers to when he speaks of "the volts lost in the cell." If e volts be the terminal P.D., *i.e.* the fall in volts in the wire, then from the figure it follows that:—

$$\cdot62 - V + \cdot46 - e = 0; \quad \therefore \ \cdot62 + \cdot46 = e + V;$$

$$\text{But } \cdot62 + \cdot46 = 1\cdot08 = \text{E.M.F.} = E, \text{ say}; \ \therefore \ E = e + V;$$

$$\textit{i.e.}\ \text{E.M.F.} = \text{Terminal P.D.} + \text{“Volts used” in the cell.}$$

Thus in the outside circuit the current consists of a drift of electrons in the direction zinc to copper, whilst inside the cell *negative* SO_4 ions move towards the zinc and *positive* H ions towards the copper. When this current is flowing zinc dissolves from the zinc plate combining with SO_4 and forming zinc sulphate ($ZnSO_4$), hydrogen gas is evolved at the copper, and heat is produced in the wire and in the cell. The actions may be represented as follows:—

The sulphuric acid at the outset and before current passes is dissociated into positive H ions and negative SO_4 ions thus:—

$$H_2SO_4 \rightleftharpoons 2H^+ + SO_4^{--}$$

the + and − signs denoting positive and negative unit charges: a unit charge here means a charge equal to an electron charge, say e. Notice that a hydrogen ion has a single positive charge (*i.e.* a hydrogen *atom* H loses an electron and becomes a hydrogen *ion* with a unit positive charge H^+), and the SO_4 group *two* negative charges: but SO_4 joins with 2H so that the opposite charges balance in the molecule H_2SO_4.

Now suppose the plates joined so that current can flow and for simplicity consider one dissociated molecule of acid. (*a*) The negative SO_4 passes along to the zinc with its *negative* charge 2*e* (*i.e.* SO_4^{--}). (*b*) Zinc dissolves in the acid as positively charged zinc ions, *i.e.* a zinc atom parts with two electrons (*negative* charge 2*e*), becomes a positive zinc ion (Zn^{++}), and joining with the SO_4^{--} forms zinc sulphate, $ZnSO_4$. (*c*) A *negative* charge 2*e* "flows" along the wire (electronic current) in the direction zinc to copper. (*d*) The $2H^+$ passes along to the copper with the *positive* charge 2*e*: this is neutralised by the *negative* electron charge 2*e* from the copper thus resulting in two ordinary hydrogen *atoms* H which join up to form a molecule of hydrogen H_2 and gaseous hydrogen is evolved at the copper, $2H^+ + 2e$ (negative) $= 2H = H_2$ (Fig. 262(*c*)).

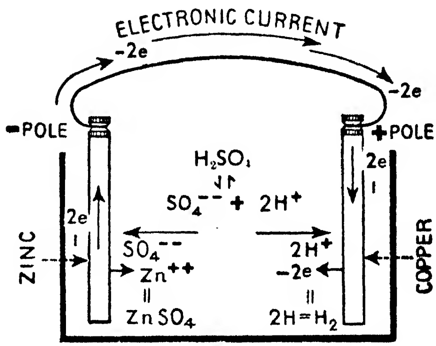


FIG. 262(*c*).

Note that when the cell is on open circuit the *potential difference between the terminals* is $\cdot 62 + \cdot 46$, *i.e.* is equal to the E.M.F. (E) of the cell. (Fig. 262(*a*)). When the cell gives a current the *potential difference between the terminals* is *e* which is less than E: it is in fact equal to $E - V$ where V is the drop in volts in the liquid from AB to DE (Fig. 262(*b*)).

Electricity is really not "made" or "generated" in a cell, although we sometimes speak of them as generators. Electrical charges are always present in the materials forming the cell, and atoms of the materials lose or gain electrons and become ions when "chemical changes" occur.

When zinc is merely dissolved in sulphuric acid the energy of the chemical action *goes direct to heat*. When it is dissolved in a cell the poles of which are joined by a simple conductor, the chemical energy is converted into electrical energy, *i.e.* current flows, but *the energy finally goes to heat*, partly in the conductor and partly in the cell itself. Should the external circuit contain, say, a motor, a certain amount of the energy is utilised in driving the motor, *i.e.* in mechanical work, *but the balance again appears as heat* in the circuit.

The simple cell has two main defects known as **local action** and **polarisation**. Common zinc contains impurities, such as iron, lead, arsenic, etc.; these, together with the zinc, being in contact with the acid give rise to a number of local currents all over the surface

of the plate, one result being that some zinc is uselessly consumed. This, termed "local action," is prevented by *amalgamating* the plate, *i.e.* coating its surface with mercury. The latter dissolves the zinc, forming a uniformly soft amalgam, which covers up the impurities; as the zinc is consumed the impurities fall to the bottom of the cell.

In the experiments on page 262 it was found that hydrogen bubbles *appeared at the copper plate* when the poles of the cell were connected, and we have now seen how this comes about. It was also found that as the hydrogen on the copper increased the current became less, but it became greater again when the hydrogen bubbles were removed from the plate. This deposition of hydrogen on the copper is one important cause of what is termed "polarisation." It reduces the current in two ways: (1) the gas has a large resistance, (2) the surface character of the positive plate is, of course, altered and the result is that a back E.M.F. is set up opposing the E.M.F. of the cell: thus the resultant E.M.F. of the cell is less and so is the current. The various forms of primary cells—Daniell's, Grove's, Bunsen's, the Bichromate, etc. (Chapter XIII.) were mainly devices to eliminate polarisation.

Polarisation in a cell is not due entirely to the formation of hydrogen bubbles on the high potential plate as is so often assumed, but is really the resultant of several effects. For example, certain changes in concentration of the solution (or solutions) take place when a cell gives a current, and the effect of these may be to definitely reduce the resultant of the P.D.'s in the cell which determines its E.M.F. To quote one case only, when current passes zinc is dissolved from the zinc plate so that there is a local concentration of zinc sulphate which lessens the electrode potential at the zinc (see above). These effects are referred to as **concentration polarisation.** In actual practice some polarisation has already set in before hydrogen gas collects on the high potential plate.

2. Other Cases of Potential Differences at Junctions

(1) It will be clear that, say, two zinc plates in sulphuric acid will not function as a "cell" for the potential rise v_1 from one zinc to AB (Fig. 262) will be cancelled by an equal fall from CD to the other zinc, *i.e.* the E.M.F. $= v_1 - v_1 =$ zero. If, however, an electrolyte consists of two parts, one more concentrated than the other, and a plate of the same material be put in each part, then the *electrode potentials* will not be equal; there will be a resultant P.D. between the plates and if they are joined outside current will flow

in the circuit. Such an arrangement is called a **concentration cell**: the E.M.F. is only small, but the cell is important in "theory" in dealing with what is known as the "thermodynamics" of the voltaic cell (see later).

Thus imagine a concentrated solution and a dilute solution of copper sulphate "separated" by a thin porous diaphragm, and that a copper plate is in each solution. Now the osmotic pressure which tends to drive positive copper ions from the solution to the plate is greater for a concentrated solution than for a dilute solution, so that more positive copper ions pass from the strong solution to its plate than pass to the other plate from the weak solution. Thus the copper plate in the strong solution will be at a higher potential than the other plate—in other words the electrode potential in the strong $CuSO_4$ compartment will be greater than that in the weak compartment—and, on joining the plates by a wire, current (conventional) will flow in it from the copper plate in the concentrated solution to the copper plate in the dilute solution. Copper will be deposited on the higher potential plate from the concentrated solution and copper will be dissolved from the other plate into the dilute solution (see Art. 4), and this will tend to continue until the solutions are equal in strength.

Of course *electrons* really "flow" *in the wire* in the direction *from* the copper in the weak solution to the other (low to high potential). The electrode in the weak solution thus loses electrons and the positive copper ions *pass into solution*. The electrode at the other end of the wire, *i.e.* in the strong solution, gains electrons, and positive copper ions there are neutralised and *deposited from the solution*.

Similar results occur with two zinc plates in concentrated and dilute solutions of, say, zinc chloride. More *positive* zinc ions are driven from the strong solution to the plate than from the weak solution to its plate, so that the zinc in the strong solution is at the higher potential, *i.e.* is *less* negative. As before, current (conventional) flows in the wire from the zinc in the concentrated solution to the other zinc: zinc is deposited from the strong solution on the plate

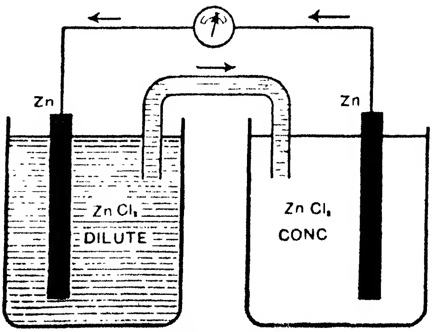


FIG. 263.

and zinc passes from the plate to the weak solution so that the strong solution gets weaker and the weaker gets stronger as above.

There is, of course, *no resultant chemical action* as in the simple cell (for actions are equal and opposite in the two compartments): the energy of the current may be said to be derived from the difference of the osmotic pressures of the two solutions. Fig. 263 shows a simple form of concentration cell: the two solutions communicate by means of the bent tube B.

(2) Whenever two different *metals* A and B are placed in contact there is a tendency for some *electrons* (note this) to diffuse across the junction from A into B and also from B into A. If the concentration of "free" electrons in A is greater than the concentration of free electrons in B, the number of electrons passing from A to B will be greater than the number passing from B to A. If, in addition, the electrons of A are less firmly held by their attracting atomic nuclei than those of B, the transfer from A to B will be increased, whilst if A's electrons are more firmly held than B's electrons, the transfer from A to B will be diminished. If on the whole the transfer of electrons from A to B is greater than that in the opposite direction it is clear that B will acquire a negative potential, *i.e.* a potential difference will be established at the junction, the metal B being at a lower potential than A (Fig. 264 (*a*)). The action continues, of course, until the potential difference established does not permit any further transfer. These P.D.'s produced at a junction of two metals are spoken of as **contact potentials.** The phenomenon is thus somewhat similar to that of the junction between a metal and an electrolyte dealt with above and in Art. 1, but in those cases *positive ions* diffused across the boundary whereas in the contact of two metals only *electrons* diffuse across.

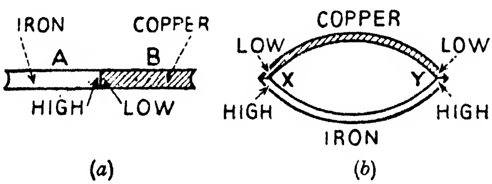


FIG. 264.

As an example, if a bar of copper and a bar of iron be put in contact, say end to end, the diffusion of electrons at the boundary in the direction iron to copper is greater than that from copper to iron, so that a P.D. is established at the junction, the copper being at the lower potential. If two pieces of the *same* metal are put in

contact the diffusion of electrons is the same in both directions so that no P.D. is set up. These potential differences at the junctions of metals are very small.

To obtain a continued *current* a "complete circuit" is, of course, necessary. Consider a circuit of copper and iron as in Fig. 264 (*b*). There is a P.D. at junction X, the iron being at a higher potential than the copper, but there is an equal P.D. at Y, and as these are in opposite directions in the closed circuit they cancel each other's effect so that no current flows. If *one* junction is heated, however, the diffusions across that junction alter. With some pairs of metals the diffusions in opposite directions across the hot junction become more nearly equal, so that the P.D. at that junction is less, whilst with other pairs the diffusions in opposite directions become more unequal and the P.D. at the junction greater. In both cases the P.D.'s at X and Y will no longer balance and electrons will flow round the circuit, *i.e.* a current will be produced. In the case of copper and iron the current (conventional direction) is from copper to iron through the hot junction and iron to copper through the cold (Fig. 264(*c*)). Currents produced in this way are called **thermo-electric currents** and have many applications in practice. The subject is fully dealt with in Chapter XIV.

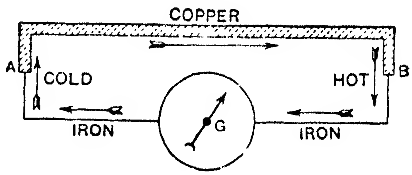


FIG. 264(*c*).

Note that in the simple cell there will be a contact P.D. where the outside copper wire is joined to the zinc plate and this should be taken into account in considering the E.M.F. of the cell: but it is small and negligible compared with the other junction P.D.'s.

(3) Contact differences of potential are also set up between two different electrolytes and also between a strong and a weak solution of the same electrolyte, but in these cases the "transfers" across the boundary which cause the P.D.'s are complicated, for in the case of two solutions in contact *both positive and negative ions* diffuse across the boundary in both directions, and the "mobility" of the ions also comes into the problem.

A very simple case, and the only one we need consider at this stage, is a contact of two solutions of the same electrolyte which differ in concentration, say a strong and a weak solution of

hydrochloric acid (HCl), in which the former is, say, three times as concentrated as the latter. This electrolyte is dissociated into +ve hydrogen ions and −ve chlorine ions, each ion having a unit charge, viz. $HCl \rightleftharpoons H^+ + Cl^-$. At the junction the number of positive hydrogen ions passing from the strong to the weak solution is nearly three times as great as the number passing in the opposite direction, and this alone would tend to make the weak solution at the higher potential at the boundary. But the number of negative chlorine ions passing from the strong to the weak is also about three times as great as the number passing in the opposite direction, and this alone would tend to make the weak solution at an equal negative potential. The two actions together would therefore seem to cancel each other's effects. But hydrogen ions are lighter than chlorine ions and *they travel across more rapidly* so that on the whole the weak solution acquires a higher potential at the junction. Similar results take place at the contacts of different kinds of electrolytes. It will be noted that these P.D.'s depend on concentration and velocity of ions (" mobility of ions " it is usually called).

Note that in the concentration cell of Fig. 263 there will be a small P.D. at the junction of the two solutions in addition to those at the metal-electrolyte junctions: the same applies to other cells if different electrolytes are in contact or if changes in concentration take place.

3. Magnetic Effects of a Current

The presence of a current in a circuit is only known by certain effects which it produces. The three main current effects which are used for the detection—and also the measurement—of a current are: (1) the *magnetic* effect, (2) the *chemical* effect, (3) the *heating* effect. In this chapter only a brief reference will be made to a few points in connexion with these which must be known before proceeding further.

So far as the magnetic effects are concerned, most of the facts necessary for our present purpose were given in Chapter II. (pages 25, 40, 43). As stated, it was soon discovered that *when a current is started in a wire, a magnetic field is immediately produced in the space round about the wire.* The strength of this field depends on the strength of the current, and is measured in the way already indicated, *i.e.* by the force on unit pole or by the number of magnetic lines of force per sq. cm. or by the magnetic potential gradient: the direction of the field depends on the direction of the current.

Several cases of magnetic fields due to currents were given on pages 40-42. Thus taking the current in a straight vertical wire,

Fig. 46 (*a*) and (*c*) show the direction of the field when the (conventional) current is flowing down the wire and (*b*) and (*d*) when the current is upwards: in the first case the field direction is counter-clockwise and in the second case clockwise. Again, place a compass needle on the table, and hold a wire *above* and parallel to it, as shown in Fig. 265. Pass a current through the wire (*a*) from south to north, (*b*) from north to south, and note in which direction the north pole of the needle is deflected: taking the conventional current again it will be found that in (*a*) the north pole of the needle is deflected towards the *west*, and in (*b*) towards the *east*. Hold the wire *below* the needle and again pass a current (*c*) from south to north, (*d*) from north to south; in (*c*) the north pole of the needle is deflected towards the *east*, and in (*d*) towards the *west*.

The results of the above enable us to find the direction of the current from that of the field or vice versa: they can be obtained from the following rules, assuming the conventional current direction.

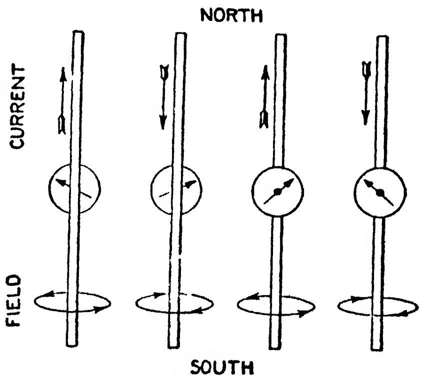


FIG. 265.

(*a*) Ampère's Rule. — *Imagine a man swimming in the circuit in the direction of the current and with his face towards the needle: the north pole of the needle will be deflected towards his left hand.*

(*b*) Corkscrew Rule (Maxwell).—*Imagine an ordinary right-handed screw to be along the wire, and to be twisted so as to move in the direction of the current: the direction in which the thumb rotates is the direction in which the north pole tends to move round the wire.*

Yet another rule—*right-hand rule*—for the current in a straight wire was given on page 40. Of course, all these rules really amount to the same, so that the student need only learn one of them.

Again, Fig. 53 gives the magnetic field in the case of a current in a circular coil of wire. The direction of the field is given by the rules quoted above, but it is well to note (as already mentioned) that, *looking at a face of the coil* if the (conventional) current is *clockwise* the direction of the field inside the coil is *away from* the

observer, *i.e.* the magnetic lines *enter* the clockwise current face just as they *enter* the *south* pole of a magnet: in fact if the coil were suspended in the earth's magnetic field it would turn so that this clockwise current face was towards the south. If the (conventional) current at the coil face looked at is *counter-clockwise* the field inside is *towards* the observer: the magnetic lines *emerge* from the face in question as they do from the *north* pole of a magnet, and this face would turn towards the north if the coil were suspended (see also Fig. 16).

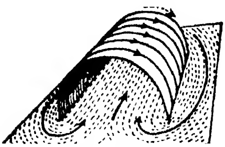


FIG. 266.

The magnetic field of a solenoid carrying a current was depicted in Fig. 52, and it can be shown by iron filings as indicated in Fig. 266. Note that the solenoid resembles a bar magnet, the counter-clockwise (conventional) current end corresponding to the north pole of the magnet; inside the magnetic lines run from S to N (Fig. 266) and outside from N to S (see also Fig. 52). The four rules given on page 43 for determining the polarity of the bar of iron magnetised by a current in the solenoid also serve to decide the "polarity" of the empty solenoid when the current is passing: the "end rule" (Fig. 267) is perhaps the most convenient.

The magnetic effects of a current are used in defining the electromagnetic unit current, in the working of many measuring instruments —galvanometers, ammeters, voltmeters, etc.—in dynamos, motors, and many other appliances. Further details are given in Chapter XI.

4. Chemical Effects of a Current

We have seen that certain liquids are decomposed when current passes through them. The process is termed *electrolysis*, the liquid is, of course, the *electrolyte*, and the containing vessel the *voltameter* or *coulometer*. The conducting plates by which the current enters and leaves the liquid are called the **electrodes**: that electrode which is joined to the positive pole of, say, the battery supplying the current, is called the **anode** (or positive electrode), and that which is joined to the negative pole of the battery is called the **cathode** (or negative electrode). The constituents of the liquid which travel to the electrodes are, of course, *ions*, that travelling to the cathode being called the *cation* and that travelling to the anode the *anion*.

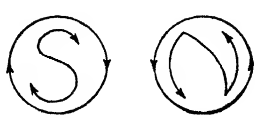


FIG. 267.

Only two practical examples of electrolysis (to which reference must be made in the next chapter) will be briefly considered at this stage: further details of these chemical effects are given in Chapter XIII.

A typical example is afforded by a solution of copper sulphate. Let Fig. 268 represent a voltameter containing a solution of copper sulphate, A and K two platinum plates—the electrodes—of which A is joined to the positive pole of the battery, and is the anode, whilst K is joined to the negative pole and is the cathode. On passing the current *metallic copper is deposited in a thin layer on the cathode*, whilst oxygen is liberated at the anode: the copper sulphate solution becomes weaker.

If copper electrodes be used in the above, then, as before, copper is deposited in a thin layer on the cathode, and in most cases in practice an equal amount is taken from the anode into solution forming copper sulphate; thus the loss in weight of the anode equals the gain in weight of the cathode, and the average strength of the solution remains the same. In practice the anode loss is not quite equal to the cathode gain, for the loss may include impurities which have become detached as the copper dissolved.

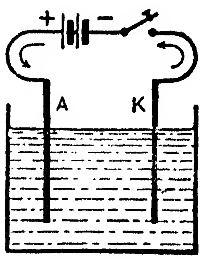


FIG. 268.

From what has been said in Arts. 1, 2, the general principle of action in the above will be readily understood. The solution is dissociated into positive copper ions and negative sulphions, thus:—

$$CuSO_4 \rightleftharpoons Cu^{++} + SO_4^{--}$$

each ion carrying two unit charges so that the opposite charges balance in the molecule $CuSO_4$.

First assume platinum electrodes. On joining up the battery (Fig. 268) the positive Cu ions move towards the negative electrode (cathode), and the negative sulphions SO_4, towards the positive electrode (anode). At the cathode the positive Cu ions "give up their charge," becoming neutral or ordinary Cu atoms and copper is deposited.* At the anode the negative sulphions, SO_4, "give up their charge" and become neutral. *But an uncharged group* SO_4

* Of course what happens is that electrons "flow" along the wire in the direction negative pole of battery to cathode (electronic current) so that the cathode gains electrons which neutralise the arriving positive copper ions: thus they become ordinary copper *atoms* and copper is deposited.

is incapable of a separate existence, and so it immediately reacts with water (H_2O) forming sulphuric acid and liberating oxygen which is given off. To avoid writing an *atom* of oxygen (O) in the equation and to write a *molecule,* viz. two atoms (O_2) instead, we assume two molecules of $CuSO_4$ and express the equations thus:—

$$\cancel{2\,(CuSO_4) = 2\,(Cu) + 2\,(SO_4)}$$
$$2\,(SO_4) + 2\,(H_2O) = 2\,(H_2SO_4) + O_2.$$

The result is, then, that copper is deposited on the cathode, oxygen is given off at the anode, and the solution becomes weaker (in the equations two molecules of copper sulphate have disappeared).

If copper electrodes be used, copper from the solution is deposited on the cathode as before. If the copper anode is such that it is readily " attacked " by the SO_4, copper is merely taken from it to form copper sulphate ($Cu + SO_4 = CuSO_4$) and the *average* concentration of the solution remains the same. If the anode is not in this condition, however, the action there may be rather complex: some SO_4 may attack the plate forming $CuSO_4$: some may react with water as above liberating oxygen: some oxygen may combine with the copper forming copper oxide ($2\,Cu + O_2 = 2\,CuO$) which forms a blackish coat on the plate, whilst some oxide may dissolve in sulphuric acid forming copper sulphate. The *main* result, however, is the formation of copper oxide and the blackening of the surface of the plate.

Repeat the above experiment with a solution of silver nitrate (*a*) with platinum electrodes, (*b*) with silver electrodes. Results similar to the above will be obtained: in both cases *silver will be deposited on the cathode.* The explanation is again readily understood. When dissolved in water the silver nitrate molecules ($AgNO_3$) are dissociated and we have silver ions, Ag, which are positive and the nitrions, NO_3, which are negative:—

$$AgNO_3 \rightleftharpoons Ag^+ + NO_3^-.$$

The positive silver ions travel to the negative cathode and the negative nitrions to the positive anode. With platinum electrodes. the silver *ions* at the cathode give up their charge, become ordinary silver *atoms,* and silver is deposited. The NO_3 ions at the anode give up their charge, *but an uncharged group,* NO_3, *cannot exist free,* so it combines with water forming nitric acid (HNO_3) and liberating oxygen. In writing down the equations we start with four molecules of silver nitrate (for the same reason as mentioned above) and express the actions thus:—

$$4\,AgNO_3 = 4\,Ag + 4\,NO_3$$
$$4\,NO_3 + 2\,H_2O = 4\,HNO_3 + O_2.$$

Note that the concentration of the silver nitrate gets less. With silver electrodes, silver is deposited on the cathode as before, but an equal amount is taken from the anode to form silver nitrate ($Ag + NO_3 = AgNO_3$), and the average concentration of the solution remains the same.

Similarly water (H_2O) can be decomposed into hydrogen and oxygen, the hydrogen appearing at the cathode and the oxygen at the anode; if a special voltameter be used so that the gases can be collected it will be found that the volume of hydrogen is double the volume of oxygen. But this case is rather complex and is explained on page 379.

The above experiments illustrate the method of electroplating; thus to silverplate an iron spoon it must be made to form the cathode in a silver solution, the anode being a silver plate. In practice special arrangements have to be made in order to get a coherent layer which will take a good polish.

Many devices for finding which is the positive pole of a generator—battery or dynamo—or for finding which is the positive and which the negative of a pair of supply mains, use the chemical effects of the current. Thus one type of "pole finding paper," as it is called, consists of paper soaked in a solution of potassium iodide and starch. If two wires are taken from, say, the generator and placed some distance apart on the paper, the potassium iodide will be decomposed, iodine appearing at the *anode*: here it reacts with the starch producing a blue coloration which therefore indicates the wire joined to the positive. To avoid the risk of short circuit and excessive current, a suitable resistance must be put into one of the "leads."

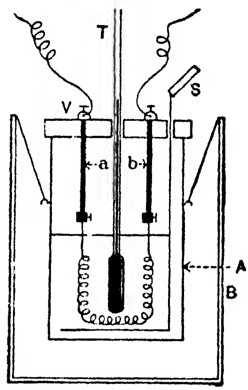


FIG. 269.

The chemical effects of a current are used in defining the practical unit of current strength (the ampere), in accumulators, copper refining, electroplating, production of alkalies, extraction of metals from their ores, etc.

5. Heating Effects of a Current

The fact that a conductor is heated by the passage of an electric current is well known owing to its application in electric lighting, heating, etc.; the following experiments are, however, instructive:—

A *calorimeter* suitable for the measurement of quantities of heat is shown in Fig. 269. A is a copper pot hung by threads inside a

larger copper pot B. The inner surface of B and the outer surface of A are smooth and polished to reduce loss of heat by radiation from A: in some calorimeters the space between A and B is packed with felt or other bad conductor of heat to reduce heat loss from A by conduction. A contains water and is fitted with a stirrer S, a thermometer T, and a vulcanite lid V through which pass two stout copper wires *a* and *b* provided with terminals as shown.

(*a*) Fit a spiral of german silver wire (insulated) to *a* and *b*, and join the coil in series with a battery, an ammeter, a rheostat, and a key. Note the temperature of the water. Then pass a current for 5 minutes, stirring the water and keeping the current constant by means of the rheostat. Note the current and the rise in temperature. Let the water cool to its original temperature, and then repeat with a different current flowing for 5 minutes. It will be found that:—

$$\frac{\text{Heat in Case 1}}{\text{Heat in Case 2}} = \frac{\text{Temp. rise in Case 1}}{\text{Temp. rise in Case 2}} = \frac{(\text{Curr. in Case 1})^2}{(\text{Curr. in Case 2})^2}$$

or *the heat produced is proportional to the square of the current*: doubling a current produces four times the heat in the same time.

(*b*) Arrange two calorimeters in series with a battery, and rheostat. The calorimeters should be alike, and contain equal quantities of water, but the spirals should be of different lengths, so that one A has a resistance R_1 and the other B a resistance R_2. The values R_1 and R_2 should either be *known* or be *determined* by *experiment* as explained later: if they are cut from the same wire, however, the resistances may be taken to be proportional to the lengths. Pass a current for 5 minutes and get the rises in temperature (say θ_1° C. and θ_2° C.). It will be found that $\theta_1/\theta_2 = R_1/R_2$, *i.e.*

$$\frac{\text{Heat in A}}{\text{Heat in B}} = \frac{\text{Resistance of A}}{\text{Resistance of B}}$$

or *the heat produced is proportional to the resistance.*

(*c*) From two or three experiments it will be easy to deduce also that *the heat produced by a current is proportional to the time the current flows.*

Combining the above results, if a current I units flows through a resistance R units for *t* seconds and H units is the heat produced, we can write:—

$$\text{H is proportional to } I^2Rt; \quad \therefore\ H = kI^2Rt$$

where *k* is a constant depending on the various units employed.

The practical unit of current strength is called an *ampere,* and the practical unit of resistance an *ohm*: these are defined in the next chapter. The unit of heat is called a *calorie* which is usually defined as the heat required to raise the temperature of one gramme of pure water one degree Centigrade: strictly the calorie should now be defined as the heat required to raise 1 grm. of water from

14·5° C. to 15·5° C. for after various proposals this definition has been accepted *internationally*. (To be even more exact the 14·5° and 15·5° should refer to what is known as the international gas thermometer scale.) Now in the above expression for the heat produced by a current, if the heat be expressed in calories, the current strength in amperes, the resistance of the conductor in ohms, and the time the current flows in seconds, k has the value ·24 and we have:—

$$\text{Heat in calories} = \cdot 24 I^2 R t.$$

The whole subject of the heating effects of currents is, however, fully dealt with in Chapter XII.

CHAPTER X

SOME UNITS AND SIMPLE THEORY IN CURRENT ELECTRICITY

A CURRENT in a wire is a flow of electrons in it, and the greater the number of electrons (*i.e.* the greater the quantity of electricity) passing any section of the wire in one second the greater is said to be the **current strength.** The analogy which follows will make the general idea clearer.

Consider a pipe AB through which water is steadily flowing in the direction A to B. Some idea of the strength of this water current may be conveyed by a statement of the quantity of water entering A, leaving B, or passing any section of the pipe in a definite time, say one second; in short, *the strength of the current may conveniently be defined as the rate of flow of water through the pipe.* The total quantity flowing past any section in a given time is then obtained by multiplying the current strength by the time in seconds. Further, when the pipe is quite full, the quantity entering A per second must be equal to the quantity leaving B or passing any section of the pipe in that time, however uneven the bore may be; *that is, the current strength is the same at all parts of the pipe.*

These elementary ideas have their electrical analogies. *Current strength* is defined as *the quantity of electricity passing any section of the conductor per second,* i.e. *as the rate of flow of electricity in the circuit, and the total quantity which passes in a stated period is given by the product of the current strength and the time in seconds*; further, *the current strength is the same at all parts of a simple conducting circuit* however the parts differ in resistance, but, as we have seen, there is a fall of potential in the conventional direction of the current (*i.e.* a rise in the direction of the actual electronic current).

1. Current Strength. The Electromagnetic Unit. The Ampere

(1) It seems natural to suggest that the unit of current strength be *that current in which an electrostatic unit quantity of electricity passes any section of the conductor in one second.* This is the " C.G.S. electrostatic unit of current strength," but it is much too small for use with most of the work in the present branch of our subject. The strength of a current is measured, in fact, in terms of the intensity of the magnetic field produced at a given distance from the conductor carrying it. Consider a wire bent into a circle of radius r arbitrary

units, and carrying a current of strength I arbitrary units; investigation shows that the intensity of the magnetic field at the centre of the circle (or the force on unit pole) is (*a*) directly proportional to the current I, (*b*) directly proportional to the length, $2\pi r$, of the circular conducting path, and (*c*) inversely proportional to the square of the radius, r. These statements are dealt with in detail in Chapter XI. We can therefore write for the force F on unit pole at the centre of the coil—

$$\text{F is proportional to } \frac{2\pi r \mathrm{I}}{r^2} \text{ or } \mathrm{F} \propto \frac{2\pi \mathrm{I}}{r}; \quad \therefore \mathrm{F} = k\frac{2\pi \mathrm{I}}{r}$$

where k is a factor depending on the units we employ.

Just as in the case of unit pole and unit e.s. quantity, it will be convenient to so choose the unit of current strength that (measuring r in cm. and F in dynes) k becomes unity, in which case:—

$$\mathrm{F} = \frac{2\pi \mathrm{I}}{r} \text{ dynes} \quad \ldots\ldots\ldots \quad (1)$$

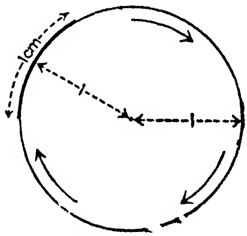


FIG. 270.

Suppose the strength of the current in these units is unity and the radius r is 1 cm., then F is 2π dynes. From this we get one definition of unit current, and it is called the **absolute or C.G.S. electromagnetic unit of current strength**: thus *the electromagnetic unit current is that current which, flowing in a single circular coil of one cm. radius, exerts a force of 2π dynes on a unit pole at the centre.*

As the coil is 1 cm. radius, its circumference is 2π cm., so that if we take the force on unit pole due to 1 cm. of the circumference (Fig. 270) it will be 1 dyne. Thus we get the usual definition as given below. This e.m. unit is rather large, and for practical work we take *one-tenth* of it, which is called an **ampere** (after Ampère). Clearly, as the ampere is $\frac{1}{10}$ e.m. unit, it will exert a force of $\frac{1}{10}$ dyne on the unit pole in the case above. Summarising, we have:—

The C.G.S. electromagnetic unit of current strength is that current which, flowing in a wire 1 cm. long bent into an arc of 1 cm. radius exerts a force of 1 dyne on a unit pole at the centre.

The practical unit is the ampere: the (true) ampere is that current which, flowing in a wire 1 cm. long bent into an arc of 1 cm. radius, exerts a force of 0·1 dyne on a unit pole at the centre.

The ampere as defined above ($\frac{1}{10}$ e.m. unit) is known as the **true ampere** for a reason which will be seen presently.

$$1 \text{ electromagnetic current unit} = (3 \times 10^{10}) \text{ e.s. current units.}$$
$$1 \text{ (true) ampere} = \tfrac{1}{10} \text{ e.m. unit} = (3 \times 10^{9}) \text{ e.s. units.}$$

(2) Another definition of the e.m. unit of current strength was referred to on page 68. A current flowing in a closed circuit produces a magnetic field at a point P, the intensity of which is proportional to the current strength, say i units. Imagine the current circuit (assumed for simplicity to be *in air*) replaced by a uniform magnetic shell, the boundary of which coincides with the current circuit: it produces a magnetic field at P, the intensity of which is proportional to the strength of the shell (page 83). Now suppose the strength of the shell be altered until the field it produces at P is equal to the field at P due to the current, and let ϕ be the strength of this " equivalent shell ": then the e.m. unit of current strength has been so chosen that i in these units, and ϕ are *numerically the same*, i.e. $i = \phi$, for, as stated, the unit is defined thus:— **the C.G.S. electromagnetic unit of current strength is that current which flowing in any closed circuit has magnetic effects the same as those of a uniform magnetic shell of unit strength whose boundary coincides with the current circuit** (*the medium is air—strictly vacuo*). The (true) ampere is, of course, $\frac{1}{10}$ of this.

If the medium be changed to one of permeability μ the potential (and field) due to the shell becomes $1/\mu$ of what it was in air (Chapter III.). More generally, then, we can say:—

$$\left.\begin{matrix}\text{Field due to}\\ \text{current}\end{matrix}\right\} \propto i: \qquad \left.\begin{matrix}\text{Field due to shell}\\ \text{of same contour}\end{matrix}\right\} \propto \frac{\phi}{\mu},$$

and if the fields are equal, *i.e.* if the shell is the " equivalent shell ":—

$$\frac{\phi}{\mu} \propto i; \quad \therefore \frac{\phi}{\mu} = ki,$$

where k is a factor depending on the units employed. Now the e.m. unit of current is so chosen that k is unity and therefore $\phi/\mu = i$: thus $\phi = \mu i$, so that *a current circuit is equivalent to a uniform magnetic shell with the same boundary if the strength of the shell* $= \mu i$, where i is the current in e.m. units. If the medium be air (strictly vacuo), $\mu = 1$ and $\phi = i$: hence i is unity if ϕ is unity, and the definition of the e.m. unit current follows as given above.

(3) The Standards Committee of the Board of Trade used the " chemical effects " of a current in its definition of the practical unit of current strength—the ampere. Consider a silver nitrate voltameter with silver electrodes (page 276): on passing a current, silver is deposited on the cathode and experiment proved that:—

(*a*) The amount of silver deposited was proportional to the current strength. (*b*) The amount deposited was proportional to the time the current flowed. (*c*) If a current of $\frac{1}{10}$ e.m. unit, *i.e.* 1 ampere, was passed, ·001118 grm. of silver was deposited every second.

Consequently we get the following definition: **the ampere is that steady current which, flowing through a solution of nitrate of silver in water, deposits silver on the cathode at the rate of ·001118 grm. per sec.** The ampere defined in this way is called the **international ampere.** It was intended to be an exact practical realisation of the true ampere of $\frac{1}{10}$ e.m. unit, but it is just a very little less (about ·025 per cent.): we neglect the slight difference and take the two to be identical, as they were intended to be.

The number ·001118 (which gives the "grm. deposited per ampere per sec.") measures what is termed the **electro-chemical equivalent** of silver: thus *the electro-chemical equivalent of a substance is the mass of it which would be liberated in electrolysis by a steady current of one ampere in one second* (see Art. 2). If a steady current of one ampere flows through a solution of copper sulphate (page 275) ·0003293 grm. of copper is deposited on the cathode every second: ·0003293 is the electro-chemical equivalent of copper.

The above is the usual "wording" of the ampere definition: the exact *explanation* of the deposition of the silver is given in Chapter XIII.

If a current I amperes flows through, say, a solution of silver nitrate for t seconds, and w grm. is the increase in weight of the cathode, then $w = \cdot 001118 \times I \times t$; or writing z for the electro-chemical equivalent:—

$$I = \frac{w}{z \times t} \text{ amperes (strictly international)} \quad \ldots\ldots\ldots\ldots (2)$$

2. Quantity. The Electromagnetic Unit. The Coulomb

Since Quantity = Current strength × Time in seconds, unit current flowing for unit time will result in the transfer of unit quantity: hence:—

The C.G.S. electromagnetic unit quantity is the quantity conveyed by the electromagnetic unit current in one second.

The practical unit is the coulomb (after Coulomb). **The (true) coulomb is the quantity conveyed by a current of one (true) ampere in one second.**

The "C.G.S. electrostatic quantity unit" was defined on page 159: it is clear that it is the quantity conveyed by the electrostatic unit current in one second.

1 electromagnetic quantity unity = 3×10^{10} e.s. units.

1 true coulomb = $\frac{1}{10}$ e.m. unit = 3×10^{9} e.s. units.

Further, we have the **international coulomb** which can be defined as *the quantity conveyed by a current of one international ampere in one second*, but the usual definition is as follows:—Since one international ampere in one second deposits ·001118 grm. of silver from a solution of silver nitrate, and one ampere flowing for one second means the passage of one coulomb, **the international coulomb is that quantity of electricity which liberates ·001118 gramme of silver from a solution of silver nitrate.** Thus one-fifth of an (international) ampere flowing for five seconds would also mean that one (international) coulomb had passed, and ·001118 grm. of silver would be deposited: clearly, also, *the electro-chemical equivalent of a substance is the mass of the substance liberated by one coulomb of electricity.* Like the international ampere, the international coulomb is really a little less than the true coulomb of $\frac{1}{10}$ e.m. unit, but in practice we can regard them as identical.

Another practical quantity unit termed the "ampere-hour" is employed. *An ampere-hour is the quantity of electricity conveyed by a steady current of one ampere flowing for one hour.* But one ampere in one hour will transfer 60×60, *i.e.* 3600 coulombs; hence 1 ampere-hour = 3600 coulombs.

Since a current I amperes flowing for t seconds means the transfer of $\mathrm{I}t$ coulombs, then from (2) above we have in the experiment referred to:—

$$\mathrm{I} = \frac{w}{zt}; \quad \therefore \mathrm{I}t = \frac{w}{z}, \text{ i.e. } \mathrm{Q} = \frac{w}{z} \text{ coulombs.}$$

3. Potential Difference. The Electromagnetic Unit. The Volt

We have seen (page 162) that a charge moving from one point to another at a different electrical potential would always involve *work being done, i.e.* it would mean *some change in energy*: work was done against the repulsion of A (Fig. 176) in moving a positive charge from the lower potential C to the higher potential B, and work was done by the electrical force if the positive charge moved from B to C. We also saw that the P.D. between two points in e.s. units was represented numerically by the work done, in ergs, when *an electrostatic unit charge* moved from one point to the other, so that if the work done or energy change was one erg the P.D. between the two points was one e.s. unit.

Similar reasoning applies to current electricity, and the unit we get is called the **absolute or C.G.S. electromagnetic unit of potential difference.** As an illustration, consider two points A and B in a simple wire through which a (conventional) current is flowing in

the direction A to B, and in which, therefore, A is at the higher potential. The wire AB is heated, and by the law of conservation of energy this heat must have been produced at the expense of an equal amount of energy which has disappeared from the electric circuit. The moving electricity is endowed with energy and some of this has been changed into heat energy in the part AB of the wire.

Imagine, now that the electromagnetic unit quantity passes from A to B and that W ergs is the energy which, in this case, is *subtracted from* the electric circuit and appears as heat: the P.D. between the two points is W electromagnetic units of potential. Thus *the P.D. in e.m. units between two points is represented numerically by the work done (or energy transformed) in ergs when the e.m. unit quantity passes between the two points.* If the work be one erg the P.D. is one e.m. unit (*not* 1 erg). This unit is very much smaller than the e.s. unit of potential.

If the unit quantity (+ve) had been forced from low to high potential work would have been done on it and energy *added to* the electric circuit: this is referred to again later.

The e.m. unit of P.D. is too small for practical purposes, and for such work we use the **volt** (named after Volta) which is 10^8 e.m. units. Clearly, if the P.D. is one volt the work done when the e.m. unit quantity passes will be 10^8 ergs. Now a coulomb is $\frac{1}{10}$ e.m. unit quantity, so that the work done when a coulomb passes will be $\frac{1}{10}$ of 10^8, *i.e.* 10^7 ergs, or one *joule*, and this fact is used in defining the volt. Summarising then:—

The P.D. between two points is one C.G.S. electromagnetic unit if the work done or energy transformed is one erg when the electromagnetic unit quantity passes.

The practical unit is the volt: the P.D. between two points is one (true) volt if the work done or energy transformed is one joule when one (true) coulomb of electricity passes.

1 electromagnetic potential unit $= 1/(3 \times 10^{10})$ e.s. unit.
1 true volt $= 10^8$ e.m. units $= 1/300$ e.s. unit.

Just as we have the "international ampere" and the "international coulomb" so we have the **international volt**: it is just a little larger than the true volt of 10^8 e.m. units (in practice we regard them as identical) and is defined on page 292.

In practical work the P.D. between two points is sometimes called the ***pressure*** (*not* an ideal name for it), and sometimes the ***voltage*** between the two points.

4. Potential Difference (P.D.) and Electromotive Force (E.M.F.)

Another term in constant use is *electromotive force* (E.M.F.)—referred to on pages 266, 267—which is also measured, in practice, in volts. We will first illustrate, in an elementary way, its use *in practice*, taking a cell or battery as our source of E.M.F.

(1) Consider a battery *on open circuit, i.e.* with the outside circuit disconnected. Imagine an electrostatic voltmeter (no continuous current through it) joined to its terminals, and that the reading is 50 volts; this measures the total "voltage" this type of battery can develop, and this total voltage *given numerically by the P.D. at its terminals on open circuit* is the E.M.F. of the battery. If an outside circuit be switched on the voltmeter reading will fall by an amount depending on circumstances. Suppose the reading is now 47 volts: this measures the terminal P.D. under present conditions or the "volts used" in getting the current through the *external* circuit, the other three volts being the P.D. used in getting the current through the *internal* circuit, *i.e.* through the battery itself.

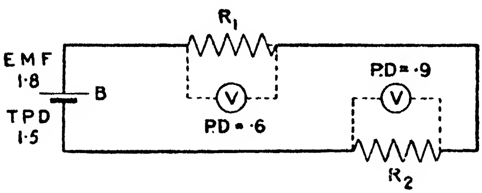


FIG. 271.

Consider as a further example Fig. 271. The *E.M.F.* of the cell is 1·8 volts. The voltmeter V across the resistance R_1 reads ·6 volt which is the *P.D.* at the ends of R_1. Similarly ·9 volt is the *P.D.* at the ends of R_2, and the balance, viz. ·3 volt, is the *P.D.* through the materials of the cell itself (we are neglecting the connecting wires). The E.M.F. (1·8) is numerically equal to the sum of the P.D.s. The terminal P.D. in Fig. 271 is 1·5 volts: it would be 1·8 volts if the cell were on open circuit.

The preceding shows what for practical purposes may be regarded as the distinction between P.D. and E.M.F. *The E.M.F. of, say, a battery is the total voltage it is capable of developing, and is numerically equal to the P.D. at its terminals on open circuit*; when the current flows the terminal P.D. may be anything according to circumstances, *but it is always less than the E.M.F.* If E volts be the E.M.F., e volts the terminal P.D. when a current flows, and V be the volts used in the battery:—

$$E = e + V,$$

that is, the E.M.F. is numerically equal to the sum of the P.D. outside and the P.D. through the materials of the battery itself.

Now we have seen that the P.D. in volts between two points in a conductor is represented numerically by the work done (or energy transformed) in joules when one coulomb of electricity passes from one point to the other. Similarly the **E.M.F. of a battery (or dynamo) in volts is represented numerically by the number of joules of work done (or energy transformed) when one coulomb of electricity flows " completely round the circuit."**

(2) In order to maintain a current in a circuit *energy must be supplied by some source to the circuit.* Thus if the circuit contains a copper wire say, the flowing of the current means the heating of the wire and this demands a continuous supply of energy. Several sources of energy can be used for the maintenance of the current, the most important being as we have seen the voltaic cell and the dynamo. In the cell the chemical action supplies the necessary energy; the products of the chemical action in the cell possess less chemical energy than the materials from which they are formed and the balance equals the energy of the current. In the dynamo a steam engine or other form of engine provides it. Whenever energy of another form is changed into energy of a current we say that an E.M.F. is present in the circuit and the E.M.F. is measured by the energy supplied by the source in one second when it is maintaining unit current in the circuit. Thus **the E.M.F. of a battery in volts is measured by the joules of energy it supplies in one second when it is maintaining a current of one ampere in the circuit,** *i.e.* it is measured by the joules of energy supplied per coulomb output. In the case of our simple battery circuit this energy supplied per coulomb output is equal to the energy transformed—in this case dissipated as heat—partly outside and partly inside as one coulomb passes round.

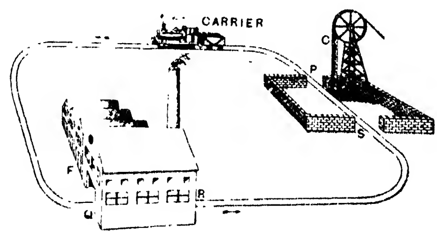


FIG. 272.

Perhaps the following analogy will further help the student, and although far from perfect it is free from some of the defects of the pump and head of water analogies often given.

Suppose C (Fig. 272) is a colliery, F a factory, and that a continuous line of wagons travels round the track conveying coal from C to F: suppose the

wagons are exactly alike, and that by " shoot " arrangements each is given 12 cwt. of coal at the colliery. We might call the wagons " carriers ": the 12 cwt. of coal given to each carrier represents a definite amount of " energy." Now consider one wagon (carrier) leaving with its 12 cwt. of coal (energy). Some of its coal (energy) will be used in going along the track to F, and then the bulk of the remaining coal (energy) will be delivered there, where it will be used to produce heat, light, set machinery in motion, etc. But the wagon must not deliver all its coal at F, for some coal (energy) will be required to get it back to the colliery gate, and then a little more will be wanted to get it along the colliery yard. We will assume that by the time it gets completely round, the wagon (carrier) has parted with all its coal (energy): it gets a further supply of 12 cwt. and the performance is repeated.

Fig. 273 shows the corresponding electrical case. The battery corresponds to C, the connecting wires to the rails, the lamps to F, and the inside of the battery to the colliery yard: the carriers are " coulombs," and the energy is " joules." The carriers (coulombs) are endowed with energy (joules) by the battery (as a result of the " chemical " action in the battery): they give up most of their energy (joules) to the lamps, but some energy is given up in going along PQ, along RS, and through the battery. In going completely round, the carriers (coulombs) part with all their energy (joules), but they get a fresh supply and continue the journey.

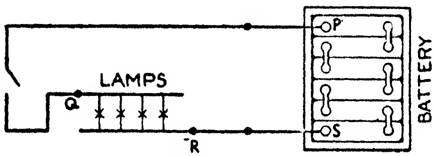


FIG. 273.

Now if the above battery *endows* each coulomb with 12 joules of energy, then a total of 12 joules of work will be done, *i.e.* 12 joules of energy will be *delivered to* the circuit when a coulomb flows *completely round* the circuit, and the " E.M.F. " of the battery is 12 volts. If each coulomb *parts with* ·2 joule of energy in going along PQ, then ·2 volt is the " P.D." between the points P and Q. Similarly, if each coulomb *parts with* 11 joules to the lamps, ·2 joule to RS, and ·6 joule in going through the battery, then 11 volts, ·2 volt, and ·6 volt are the " potential differences " on these various parts. The E.M.F. is equal to the sum of all the various P.D.s ($\cdot 2 + 11 + \cdot 2 + \cdot 6 = 12$). The terminal potential difference when the current is flowing is ($\cdot 2 + 11 + \cdot 2$) $= 11 \cdot 4$ volts: it would be 12 volts if the battery were on " open circuit."

(3) Referring now to the simple cell (Fig. 262, page 265), we saw that immediately the plates were placed in the solution an **electrode potential** (say e_1) was developed at the surface of contact of the zinc and the solution, and another (say e_2) was developed at the contact surface of the copper and the solution, and that the

algebraic sum of these two (in this case $(e_1 + e_2)$) measured the *electromotive force* (E.M.F.) of the cell (say E). In actual practice there may be other potential differences at surfaces of contact in a cell. In some cells more than one kind of solution is in use and there may be a slight P.D. at the contact surface of the two solutions (page 271): a solution may be of different concentrations thereby setting up a contact P.D. at the separating surfaces (page 271): a *copper* terminal may be attached to the *zinc* and there will be a contact P.D. at the junction (page 270): moreover some of these P.D.s may act in one direction and some in the other, and all should be taken into account. **The algebraic sum of the differences of potential occurring at all the surfaces of contact of different materials intervening (in the cell) between the terminals measures the E.M.F. of the cell** and it is numerically equal to the joules of energy the cell provides per coulomb of electricity output.

From Fig. 262 the facts previously mentioned can readily be seen. In (*a*) the cell is on open circuit, and here *the P.D. between the terminals is* $(e_1 + e_2)$, i.e. *is equal to the E.M.F.* (E) as stated.

In (*b*) the cell is shown with its poles joined by a wire and current flowing. The electrode potentials at the zinc-solution surface (e_1) and at the copper-solution surface (e_2) are the same as before, so that E, the E.M.F., viz. $(e_1 + e_2)$, is as before. As current is flowing there is a fall of potential (say e) in the outside wire in the direction of the (conventional) current, and there is a fall (say V) through the solution. From the figure

$$e_1 - V + e_2 - e = 0, \text{ i.e. } e_1 + e_2 = e + V; \therefore E = e + V,$$

as previously stated. The P.D. at the terminals is now e, which is less than the E.M.F. (E). Further, the E.M.F. is numerically equal to the sum of the P.D. on the outside circuit (e) and the P.D. through the solution (V). Finally, from the definition of P.D. (using practical units), e (volts) is numerically equal to the joules of energy delivered to the outside circuit, and V (volts) is equal to the joules delivered to the inside circuit, when one coulomb passes: hence, as stated, *the E.M.F.* (E) *in volts is numerically equal to the work done in joules when one coulomb passes completely round the circuit.*

By the action of the cell each coulomb is *raised* in potential to the extent $(e_1 + e_2)$ volts: hence here energy is *supplied—added* to the circuit (Art. 3)—and (since joules = volts × coulombs—Art. 13), the cell therefore supplies $(e_1 + e_2)$, *i.e.* E joules of energy for each coulomb output.

(4) When a current flows through a copper sulphate voltameter fitted with copper electrodes, copper is removed from one plate A and copper is deposited on the other plate B. If the current be sent in the opposite direction for the same time an equal amount of copper is removed from B and copper is deposited on A: one current undoes the work of the other. An operation of this kind

is said to be **reversible.** Most chemical operations depending on the flow of a current are reversible.

When a current flows through a conductor heat is produced. If an equal current be sent for the same time in the opposite direction the conductor is heated equally. An operation of this kind is said to be **irreversible.***

Now the E.M.F. of a cell is really measured by the energy it provides *in reversible processes* when unit quantity passes. If the energy transformation between two points in a circuit is completely *irreversible* we speak of a P.D. between the two points.†

Thus E.M.F. has a definite direction (which determines the direction of the current) and is reversible: if the current be reversed by applying a bigger E.M.F. in the opposite direction then energy is *absorbed from* the circuit at the smaller E.M.F.—not *supplied to* it.

A full explanation of the difference between P.D. and E.M.F. from this point of view is, however, beyond the scope of this book: for further details see *Advanced Textbook of Electricity and Magnetism.*

Example.—*If* 19,800 *joules of work be done in the outside circuit of a battery, and* 1800 *joules in the inside circuit* (i.e. *in the battery itself*) *due to a steady current of* 1 *ampere flowing for half an hour, find* (*a*) *the E.M.F. of the battery,* (*b*) *the terminal potential difference* (*T.P.D.*) *when the current is flowing,* (*c*) *the volts "lost" in the battery.*

$$\text{No. of coulombs} = 1 \times (\tfrac{1}{2} \times 60 \times 60) = 1800.$$

Work done when 1800 coulombs pass *completely round* the circuit =

$$19800 + 1800 = 21600 \text{ joules;}$$

$\therefore$ Work done when 1 coulomb passes *completely round* $= \frac{21600}{1800} = 12$ joules;

$$\therefore \textit{E.M.F. of battery} = 12 \textit{ volts} = E \quad \ldots\ldots\ldots\ldots\ldots (a)$$

Again, work done by 1800 coulombs in outside circuit = 19,800 joules;

$$\therefore \text{Work done by 1 coulomb in outside circuit} = \frac{19800}{1800} = 11 \text{ joules;}$$

$$\therefore \textit{Terminal P.D.} = 11 \textit{ volts} = e \quad \ldots\ldots\ldots\ldots\ldots (b)$$

Finally, work done by 1800 coulombs in inside circuit = 1800 joules;

$$\therefore \text{Work done by 1 coulomb in inside circuit} = 1 \text{ joule;}$$

$$\therefore \textit{Volts used or "lost" in battery} = 1 \textit{ volt} = V \quad \ldots\ldots\ldots (c)$$

$$\text{Note } E = e + V.$$

* Certain chemical operations *can be* irreversible in their nature, and heating effects *can exist* which are reversible (see Chapter XIV.).

† If partly irreversible and partly reversible we also speak of the P.D. between the two points.

5. Resistance. The Electromagnetic Unit. The Ohm

The resistance of a body may be defined in a general way as *that property of it which opposes the flow of electricity*, but precise definitions are as follows.

Consider first a simple wire, through which a current is passing, and in which, therefore, we have the *irreversible* process, the production of heat. Experiment (and theory) shows that the heat, in energy units, is proportional to the square of the current and the time, and it depends also on the material, dimensions, etc., of the conductor. In symbols—

$$\text{Heat in energy units} \propto I^2t = RI^2t,$$

where R is a factor depending on the material, etc., and known as its **resistance** (see page 279). If I and t be each unity, then the heat *in energy units* is numerically equal to R: hence *the resistance of a wire (temperature uniform) is represented numerically by the heat, in energy units, developed in one second when unit current passes*; or, more general—*the resistance of any conductor is represented numerically by the heat, in energy units, developed in one second (by irreversible processes) when unit current passes.*

If I and t and the heat energy be each unity, R is unity; hence—

The resistance of a conductor is one C.G.S. electromagnetic unit if the heat in energy units produced per second (by irreversible processes) when the electromagnetic unit current passes is one erg.

The practical unit is the ohm. The resistance of a conductor is one (true) ohm if the heat in energy units produced per second (by irreversible processes) when one (true) ampere passes is one joule.

Since a current of 1/10 absolute unit produces in the ohm (named after Ohm) 10^7 ergs of heat per second, if R be the value of the ohm in absolute units, $10^7 = R\,(1/10)^2$ or $R = 10^9$, *i.e.*

$$1 \text{ (true) ohm} = 10^9 \text{ electromagnetic units.}$$

Again consider a simple wire with a steady P.D. between two points A and B; a certain steady current will be flowing. If the P.D. be altered the strength of the current will also be changed, but experimentally it can be proved that *if the temperature of the wire be kept constant the ratio of the P.D. to the current is constant, i.e.* if V denote P.D.:—

$$\frac{\text{Potential difference}}{\text{Current}} = \text{a constant or } \frac{V}{I} = R,$$

where R denotes the constant, and this is numerically the same as the R above, *i.e.* it is the *resistance* between the points A and B. If V and I be each unity R is unity: hence:—

A conductor has a resistance of one C.G.S. electromagnetic unit if a P.D. of one electromagnetic unit applied to its ends causes a current of one electromagnetic unit to flow through it.

The **practical unit is the ohm. A conductor has a resistance of one (true) ohm if a P.D. of one (true) volt applied to its ends causes a current of one (true) ampere to flow through it.**

$$1 \text{ (true) ohm} = \frac{1 \text{ volt}}{1 \text{ ampere}} = \frac{10^8 \text{ e.m. units}}{1/10 \text{ e.m. unit}} = 10^9 \text{ e.m. units.}$$

Just as we have the international ampere, coulomb, and volt, so we have the **international ohm**: it was intended to be a practical statement of the true ohm of 10^9 e.m. units as defined above, but it is just a little greater, although, in practice, we take them as identical. It was defined in this way:—Various experiments were performed to find the exact dimensions of a column of mercury which would have a resistance of one true ohm or 10^9 e.m. units: the results gave the following definition:—**the (international) ohm is the resistance of a column of mercury 106·3 cm. long, 1 sq. mm. in cross-section** (mass 14·4521 grm.) **at the temperature of melting ice (0° C.).**

Conductance is the reciprocal of resistance; thus a wire of resistance R has a conductance of 1/R. The practical unit is the **mho,** which is the conductance of a material of resistance one ohm.

We can now define the *international volt* referred to on page 285. **The international volt is that P.D. which will cause a current of one international ampere to flow through a resistance of one international ohm.** This is clear because $V/I = R$; $\therefore V = IR$, and if I and R be each unity, V is unity. As stated, it is just a little bigger than the true volt of 10^8 e.m. units.

A word of warning, particularly to examination candidates, about "defining" the various units may be given. In one of our definitions of the *true ohm* we used the relation $V/I = R$, *i.e.* we defined one *true* unit (R) in terms of the other two *true* units (V and I): this was correct for V and I had been *separately* defined, V by "work," and I by "force on unit pole." Again, in the definition of the *international volt* we used the relation $V = IR$ defining one *international* unit (V) in terms of the other two *international* units (I and R): this also was correct for I and R had been *separately* defined, I by "electrolysis," and R by the "mercury column." Unless care be taken,

however, this method of defining units from the relation $R = V/I$, *e.g.* defining *every* unit in terms of the other two (which is sometimes done) will mean taking the "path of least resistance" and "talking round in a circle."

6. Summary of True, International, and Legal Practical Units

As a further help, the true and international units of current strength, resistance, and P.D. are summarised below, and to these are added the *legal units* as they are called, for the student will come across them in his reading.

(1) *Current Strength.*—The **true ampere** is $\frac{1}{10}$ of the C.G.S. e.m. unit. It is that current which, flowing in a wire 1 cm. long bent into an arc of 1 cm. radius, exerts a force of 0·1 dyne on a unit pole at the centre. The **international ampere** is that steady current which, flowing through a solution of silver nitrate, deposits ·001118 grm. of silver on the cathode in one second. The **legal ampere** is the current which gives a certain reading on a standard instrument called an ampere balance (Chapter XI.) kept at the National Physical Laboratory: the balance was standardised by the deposition of silver.

(2) *Resistance.*—The **true ohm** is 10^9 C.G.S. e.m. units. One definition is that it is the resistance a wire must have in order that a P.D. of one *true* volt (of 10^8 e.m. units) may cause a current of one *true* ampere (of $\frac{1}{10}$ e.m. unit) to flow through it. The **international ohm** is the resistance of a column of mercury 106·3 cm. long, mass 14·4521 grm. (cross-section 1 sq. mm.) at 0° C. The **legal ohm** is the resistance at 16·4° C. between the terminals of a "standard ohm" kept at the National Physical Laboratory: this standard was, however, checked by the mercury column.

(3) *Potential Difference.*—The **true volt** is 10^8 C.G.S. e.m. units. It is the P.D. between two points if the energy transformed be one joule when one *true* coulomb ($\frac{1}{10}$ e.m. unit of quantity) passes. The **international volt** is the P.D. which, when applied to a wire of resistance one *international* ohm, will cause a current of one *international* ampere to flow, and it may be taken as 1/1·0183 of the E.M.F. of a Weston cell at 20° C. (page 394). The **legal volt** is $\frac{1}{100}$ of the P.D. which produces a certain deflection on a standard electrostatic voltmeter kept at the National Physical Laboratory: this is intended to be the same as the international volt just as in the other "legal" cases above.

7. Ohm's Law

In order that a current may flow between two points in a conductor there must be a P.D. between them. The relationship between these two quantities was expressed by **Ohm's Law** which states that **if the temperature of a conductor be kept constant the current is proportional to the potential difference**: in algebraic form—

$$I \propto V \text{ or } I = kV; \quad \therefore \frac{V}{I} = \frac{1}{k} = \text{a constant} = R.$$

Clearly, the law can also be expressed thus:—**When the temperature of a conductor is kept constant the ratio of the steady direct P.D. between two points (no generator of E.M.F. being between them) to the resulting steady direct current is constant.**

The constant R is, as we have seen, a measure of the *resistance*, and k of the *conductance* (1/R), but it should be noted that the term "resistance" does not enter into the statement of the law as originally given. However, from the relation:—

$$\frac{\text{Potential difference}}{\text{Current}} = \text{a constant} = \text{Resistance},$$

we get three equations which are of great importance, viz.—

$$\frac{V}{I} = R; \quad \therefore V = IR; \quad \therefore I = \frac{V}{R} \ldots\ldots\ldots\ldots(3)$$

With regard to units, if V in, say, the last expression be in volts and R in ohms, then I will be in amperes; whilst if V and R be in electromagnetic units I will be in electromagnetic units, and so with the other expressions.

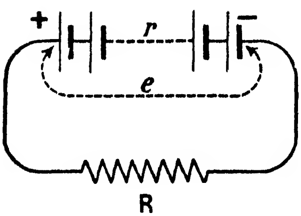


FIG. 274.

We can apply the above to a complete circuit. Let E denote the E.M.F. in a circuit, I the total current, and R the *total* resistance. The energy *provided by reversible processes* when unit quantity passes is E, and therefore it is EQ or EIt where Q is the quantity transferred by the current I flowing for time t. The energy represented by the *irreversible heat production* is I^2Rt: hence:—

$$EIt = I^2Rt;$$

$$\therefore I = \frac{E}{R}, \text{ i.e. Total current} = \frac{\text{E.M.F.}}{\text{Total resistance}} \ldots\ldots(4)$$

the E.M.F. being, of course, the *resultant* E.M.F. in the circuit, *i.e.* the algebraic sum of direct E.M.F.s and any reverse or back E.M.F.s.

Note now the simple circuit shown in Fig. 274. If E volts be the E.M.F. of the battery, e volts the terminal P.D., *i.e.* the P.D. between the ends of the outside resistance R ohms, and V the volts employed in getting the current through the *internal* circuit (*i.e.* through the battery) of resistance r ohms, then for the current I we have:—

$$I = \frac{\text{E.M.F.}}{\text{Total resistance}} = \frac{E}{r + R} \dots\dots\dots\dots\dots\dots \quad (5)$$

$$I = \frac{\text{Terminal P.D.}}{\text{External resistance}} = \frac{e}{R} \dots\dots\dots\dots\dots\dots \quad (6)$$

$$I = \frac{\text{Internal fall of potential}}{\text{Internal resistance}} = \frac{V}{r} = \frac{E - e}{r} \dots\dots \quad (7)$$

the current being the same at all parts since it is a series circuit.

Again from (5) and (6) we have that:—

$$\frac{E}{r + R} = \frac{e}{R}; \quad \therefore e = E\left(1 - \frac{r}{R + r}\right)$$

and it follows that when a current is flowing the terminal P.D. (e) is always less than the E.M.F. (E). If the outside resistance R be *infinite, i.e.* the cell on open circuit, then $r/(R + r)$ is zero and $e = E$: the E.M.F. is thus numerically equal to the terminal P.D. on open circuit.

Example.—*A battery has an E.M.F. of 2 volts When its poles are joined by a resistance the current is found to be ⅔ ampere, and the terminal P.D. 1·8 volts. Find the resistance of the battery and the outside resistance.*

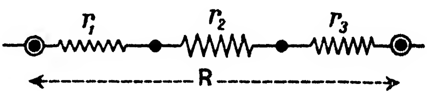


FIG. 275.

1·8 volts is the P.D. between the ends of the outside resistance and the current outside is ⅔ ampere. From R = V/I we therefore have:—

$$\text{Resistance outside} = \frac{\text{Volts outside}}{\text{Current outside}} = 1{\cdot}8/\tfrac{2}{3} = 2{\cdot}7 \text{ ohms.}$$

Since 2 volts is the E.M.F. of the battery, and 1·8 volts the terminal P.D., the volts employed in getting the current of ⅔ ampere through the battery itself is $(2 - 1{\cdot}8) = {\cdot}2$ volt, and we have:—

$$\text{Resistance of battery} = \frac{\text{Volts in battery}}{\text{Current in battery}} = {\cdot}2/\tfrac{2}{3} = {\cdot}3 \text{ ohm.}$$

8. Joining Resistances in "Series" and in "Parallel"

(*a*) If several resistances r_1, r_2, r_3 be joined *in series* (*i.e.* end to end) as in Fig. 275, the total resistance R is clearly their *sum, i.e.* $R = r_1 + r_2 + r_3$.

(*b*) Joining resistances *in parallel* means joining them, for example, like the three resistances r_1, r_2, and r_3 in Fig. 276(*a*). Here

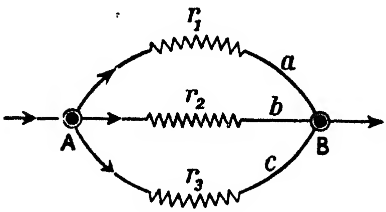


FIG. 276(*a*).

if V be the P.D. between the points A and B, we have for the currents in the three conductors:—

Current in $r_1 = V/r_1$; Current in $r_2 = V/r_2$; Current in $r_3 = V/r_3$;

$\therefore$ Total current

$$= \frac{V}{r_1} + \frac{V}{r_2} + \frac{V}{r_3}.$$

If R = joint resistance of r_1, r_2, r_3, Total current = V/R;

$$\therefore \frac{V}{R} = \frac{V}{r_1} + \frac{V}{r_2} + \frac{V}{r_3}, \text{ i.e. } \frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} \quad \ldots\ldots (8)$$

Thus if three resistances of 2, 4, and 20 ohms be in parallel, we have $1/R = 1/2 + 1/4 + 1/20 = 16/20$ and therefore $R = 1\frac{1}{4}$ ohms. Note that *when resistances are in parallel, the joint resistance is less than the resistance of the smallest of them.*

If the three resistances are equal, say each equal to r, then (8) becomes $1/R = 3/r$; $\therefore R = \frac{1}{3}r$, and thus *the joint resistance of n equal resistances in parallel is $1/n$ of the resistance of one of them.*

In the case of two resistances in parallel it is convenient to remember another expression for the joint resistance—

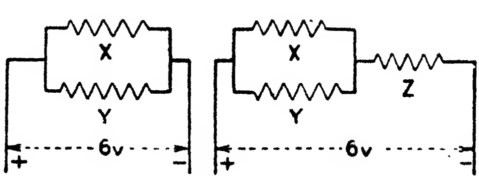
$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{r_1 + r_2}{r_1 r_2}; \quad \therefore R = \frac{r_1 r_2}{r_1 + r_2} \quad \ldots\ldots\ldots (9)$$

$$\text{i.e. Joint resistance} = R = \frac{\text{Product of resistances}}{\text{Sum of resistances}}$$

Whilst the current is the same at all parts of a simple (series) circuit however the parts differ in resistance, in a parallel arrangement *the current divides inversely as the resistances*; thus, taking the wires a and b of Fig. 276(*a*):—

$$\frac{\text{Current in } a}{\text{Current in } b} = \frac{V}{r_1} \div \frac{V}{r_2} = \frac{r_2}{r_1} = \frac{\text{Resistance of } b}{\text{Resistance of } a} \quad \ldots (10)$$

Examples.—(1) *A resistance* X *of* 4 *ohms is in parallel with a resistance* Y. *A.P.D. of* 6 *volts is applied and the total current is* 2·5 *amperes. Find* (*a*) *the current in* X *and in* Y, (*b*) *the resistance of* Y, (*c*) *what*



(*b*) FIG. 276. (*c*)

resistance Z must be put in series with the XY combination to cut the total current down to 1·5 amperes? (See Figs. 276 (*b*), (*c*).)

$$\text{Curr. in X} = \frac{\text{P.D. on X}}{\text{Res. of X}} = \frac{6}{4} = 1{\cdot}5\ \textit{amps.};\ \therefore\ \text{Curr. in Y} = 2{\cdot}5 - 1{\cdot}5 = 1\ \textit{amp}.$$

$$\text{Resistance of Y} = \frac{\text{P.D. on Y}}{\text{Current in Y}} = \frac{6}{1} = 6\ \textit{ohms}.$$

$$\text{Resistance of X and Y in parallel} = \frac{4 \times 6}{4 + 6} = 2{\cdot}4 \text{ ohms.}$$

If x = resistance of Z the total resistance = $(2{\cdot}4 + x)$ ohms.

$$\text{Total current} = \frac{\text{Total P.D.}}{\text{Total Res.}},\ \textit{i.e.}\ 1{\cdot}5 = \frac{6}{2{\cdot}4 + x};$$

$$\therefore\ 3{\cdot}6 + 1{\cdot}5x = 6 \text{ or } 1{\cdot}5x = 6 - 3{\cdot}6 = 2{\cdot}4;\quad \therefore\ x = 1{\cdot}6\ \textit{ohms}.$$

Answers.—(*a*) 1·5 amps.: 1 amp., (*b*) 6 ohms, (*c*) 1·6 ohms.

(2) *Three wires,* A, B, *and* C, *of* 2, 4, *and* 6 *ohms resistance respectively, are arranged in parallel, and the total current passing is* 22 *amperes. Find the joint resistance and the current in each wire.*

If R = joint resistance:—

$$\frac{1}{R} = \frac{1}{2} + \frac{1}{4} + \frac{1}{6} = \frac{6 + 3 + 2}{12} = \frac{11}{12};\quad \therefore\ R = \frac{12}{11} = 1{\cdot}09 \text{ ohms.}$$

Current in A = $\frac{6}{11}$ of 22 amperes = 12 amperes
,, ,, B = $\frac{3}{11}$ of 22 ,, = 6 ,,
,, ,, C = $\frac{2}{11}$ of 22 ,, = 4 ,,

Note that the rules for "joint resistance of resistances in series and in parallel" are the other way about to those for "joint capacitance of condensers in series and parallel" (page 212).

9. More Complicated Divided Circuits: Kirchhoff's Laws

The joint resistance, and the currents in the various branches of a divided circuit can be found by the above method, provided that there are no cross connexions as are indicated in Fig. 278. In this and other complex arrangements applications of *Kirchhoff's Laws* enable the solutions to be obtained.

Kirchhoff's two laws are as follows:—(1) *In any network of wires carrying currents the algebraic sum of the currents meeting at any point is zero.* (2) *In any closed path (or mesh) in a network the sum of the E.M.F.s acting in that path is equal to the sum of the products of the resistances of, and currents in, the various parts of the path.*

The first law presents no difficulty; thus, to take a simple case (Fig. 277), it is clear that $I = I_1 + I_2 + I_3$, and therefore

$$I - I_1 - I_2 - I_3 = 0.$$

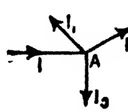


FIG. 277.

Current flowing *to* the point A is given the positive sign, and current *from* A the negative sign in writing down the sum.

The simplest illustration of the second law consists of a battery of E.M.F., say, E volts and resistance r ohms joined to an outside resistance R ohms. If I be the current we have $I = E/(R + r)$; $\therefore E = IR + Ir$. If there were two cells working in opposition in the circuit their E.M.F.s being E_1 and E_2 and resistances r_1 and r_2, and if I be the current, then $I = (E_1 - E_2)/(R + r_1 + r_2)$; $\therefore E_1 - E_2 = IR + Ir_1 + Ir_2$.

The above are simple illustrations, but the full meaning and

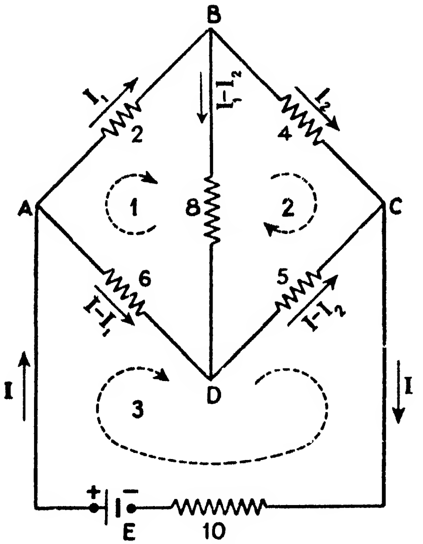


FIG. 278.

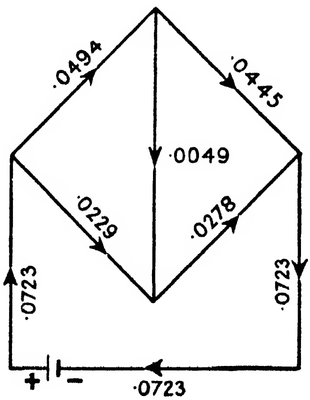


FIG. 279.

utility of the law will only be realised if we show its application to a definite (and important) network of conductors. In Fig. 278 five resistances AB, BC, CD, DA, and BD of 2, 4, 5, 6, and 8 ohms respectively are joined as indicated and the points A and C are connected to a cell of E.M.F. 1 volt, the total resistance of the path AEC being 10 ohms: *the problem is to find the currents in the various branches and the joint resistance of the network.*

As in any algebraic problem we begin by giving our unknowns suitable symbols. Let I denote the current from the cell. At A it divides, a part I_1 going along AB and the balance $(I - I_1)$ along AD. At B the current I_1 *either* divides, part going *down* BD and

part along BC, *or* I_1 is joined by current coming *up* DB: we will assume the former, calling I_2 the part along BC, and therefore the balance down BD will be $(I_1 - I_2)$. Should our choice of direction be the wrong one, our answer will merely come out with a minus sign showing that the current goes the other way. At D the currents in AD and BD join giving a current $(I - I_1 + I_1 - I_2) = (I - I_2)$ along DC and at C this joins with the current I_2 in BC giving a current I in the bottom branch back to the battery.

Now suppose we start at A, travel round mesh 1 *clockwise* say, and write down the products of current and resistance for the three sides of the mesh. If we travel *with* the current we mark the product +ve, and if *against* the current −ve: thus we get for mesh 1:—

$$I_1 \times 2 + (I_1 - I_2)\,8 - (I - I_1)\,6 = 0,$$

for there is no E.M.F. in this mesh. Similarly for mesh 2:—

$$I_2 \times 4 - (I - I_2)\,5 - (I_1 - I_2)\,8 = 0,$$

and for mesh 3:—

$$I \times 10 + (I - I_1)\,6 + (I - I_2)\,5 = 1.$$

Collecting the terms in each of these equations we get:—

$$-3I + 8I_1 - 4I_2 = 0 \quad \ldots\ldots\ldots\ldots (a)$$

$$-5I - 8I_1 + 17I_2 = 0 \quad \ldots\ldots\ldots\ldots (b)$$

$$21I - 6I_1 - 5I_2 = 1 \quad \ldots\ldots\ldots\ldots (c)$$

and on solving these the currents I, I_1, and I_2 are determined. Adding (*a*) and (*b*) we get:—

$$-8I + 13I_2 = 0 \quad \ldots\ldots\ldots\ldots (d)$$

Multiplying (*a*) by 3 and (*c*) by 4 and adding gives:—

$$75I - 32I_2 = 4 \quad \ldots\ldots\ldots\ldots (e)$$

and on solving (*d*) and (*e*) we get $I = \cdot0723$.

Substituting this value for I in (*d*) gives $I_2 = \cdot0445$.

Substituting the values of I and I_2 in (*a*) gives $I_1 = \cdot0494$.

Thus the currents in the various branches are (1) total current in CEA $= I = \cdot0723$ amp.; (2) from A to B $= I_1 = \cdot0494$ amp.; (3) from A to D $= I - I_1 = \cdot0229$ amp.; (4) from B to D $= I_1 - I_2 = \cdot0049$ amp.; (5) from B to C $= I_2 = \cdot0445$ amp.; (6) from D to C $= I - I_2 = \cdot0278$ amp. The current distribution is shown in Fig. 279.

The joint resistance of the network ABCD is found thus:—If R = this joint resistance, the *total* resistance is (10 + R) ohms. The E.M.F. is 1 volt. Now total current = E.M.F./Total resistance: hence $\cdot0723 = 1/(10 + R)$ from which $R = 3\cdot83$ ohms.

The second law might be expressed in a slightly different form, viz. *the sum of all the changes in potential round any closed circuit or mesh is zero.* Thus consider a simple circuit (Fig. 280) of two cells in opposition, X of E.M.F. E_1 and Y of E.M.F. E_2: let R be the outside resistance, I the current, and assume the resistance of the

cells is negligible. Starting at any point A and travelling round clockwise back to A there is a potential *fall* of IR in the wire, another potential *fall* of E_2 in going through Y, and a potential *rise* E_1 in going through X. As we are back to the *same potential point* A, the total potential falls must equal the total rises, *i.e.* $IR + E_2 = E_1$ or $E_1 - E_2 - IR = 0$, *i.e.* the sum of the potential changes round the mesh is zero. This agrees with the second law as initially stated, viz. $IR = E_1 - E_2$. (We would have arrived at the same conclusion if we had travelled round counter-clockwise: there would have been a fall E_1, a rise E_2, and a rise IR, and $IR + E_2 = E_1$ or $E_1 - E_2 - IR = 0$ as before.)

10. Laws of Resistance

(1) *The resistance of a conductor is directly proportional to its length.*

(2) *The resistance of a conductor is inversely proportional to its area of cross-section*; thus if one wire has double the area of cross-section of another of the same material, and equal lengths be taken, the thick one will have half the resistance of the thin one: the thick one practically corresponds to two of the other in parallel.

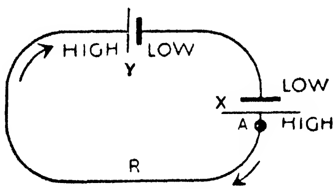


FIG. 280.

(3) *The resistance of a substance depends on its material*; thus a piece of platinum has about 6 times the resistance of a piece of copper of the same dimensions.

(4) *The resistance of a substance depends on its temperature*: this is dealt with in Art. 11.

(5) *The resistance of a substance depends on its molecular condition, density, purity, hardness, etc.* A decrease in the density of copper results in increased resistance. Wires subjected to mechanical strain increase in resistance. In general the resistance of an alloy is much greater than that of the substances forming it. The resistance of a rod of *bismuth* is considerably affected by a magnetic field, especially if the latter be transverse to the rod; its resistance increases and this effect is utilised for the measurement of magnetic fields. *Selenium* decreases in resistance when exposed to light: this property of selenium is used for turning current automatically into the lights of floating buoys when darkness sets in, and into train lights on entering tunnels.

The first three laws given above may be expressed thus:—

$$R = S\frac{l}{a} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

where R is the resistance of the conductor, l its length, a its cross-sectional area, and S a factor depending on the material and known as its **specific resistance or resistivity**. If l be one centimetre, and a one square centimetre, R is equal to S; hence **the resistivity of any material is the resistance of a piece of it one centimetre in length and one square centimetre in cross-section,** e.g. *the resistance of a cube of one centimetre side* (the "cm. cube"): frequently S is expressed in terms of the "inch cube." Note particularly it is "cm. cube" or "inch cube," not cubic centimetre or cubic inch: a cubic inch of copper may be almost any length.

In using the formula care must be taken with the units. Thus if S be given in microhms per cm. cube, then l must be in cm., a must be in sq. cm., and R will be in microhms. If S be given in ohms per inch cube, then l must be in inches, a must be in sq. inches, and R will be in ohms.

Note that since 1 inch = 2·54 cm., an "inch cube" has 2·54 times the length and $(2{\cdot}54)^2$ times the cross-section of a "cm. cube." Hence:—

$$\text{S per inch cube} = \frac{2{\cdot}54}{(2{\cdot}54)^2}, \text{ i.e. } \frac{1}{2{\cdot}54} \text{ times S per cm. cube.}$$

Again, the thickness of a wire is often settled by a number termed its "gauge"; thus 20 B.W.G. (Birmingham Wire Gauge) refers to a wire ·35 inch in diameter; 16 S.W.G. (Standard Wire Gauge) is a wire of ·064 inch diameter. The larger the "gauge" number the smaller the diameter. (French Gauge an exception.)

The conductors of submarine and electric light cables are formed of stranded wire on account of the greater pliability and less liability to complete fracture than in the case of solid cores. Thus 7/16 cable (S.W.G.) refers to one whose conductor is seven stranded wires, each of 16 gauge (now often expressed as 7/·064″).

Examples.—(1) *In a certain plant the dynamo is 300 yards away from the house. The total resistance of the cable is $\frac{2}{7}$ ohm. What is the cross-section of the cable, the resistivity of the copper being ·66 microhm per inch cube?*

$$R = S\frac{l}{a}; \quad \therefore a = S\frac{l}{R}.$$

$$S = {\cdot}66 \text{ microhm} = \frac{{\cdot}66}{1000000} \text{ ohm per inch cube; } R = \frac{2}{7} \text{ ohm}$$

$$l = 600 \text{ yards (go and return)} = (600 \times 36) \text{ inches;}$$

$$\therefore a = \frac{{\cdot}66 \times 600 \times 36 \times 7}{10^6 \times 2} = {\cdot}05 \text{ sq. inch.}$$

(2) *Compare the resistances of two copper wires, one of which is 4 metres long and weighs 7·5 grm., the other being 5 metres long and weighing 4·2 grm.*

Let R_1, l_1, and a_1 apply with their usual meanings to the first wire, and R_2 l_2, and a_2 to the second wire.

$$\frac{R_1}{R_2} = \frac{Sl_1/a_1}{Sl_2/a_2} = \frac{l_1}{l_2} \times \frac{a_2}{a_1}.$$

Now if W be the mass of a wire and D the density (mass of unit volume) the volume of the wire is W/D. But the volume is $a \times l$. Hence $al = W/D$, *i.e.* $a = W/Dl$. Substituting in the above:—

$$\frac{R_1}{R_2} = \frac{l_1}{l_2} \times \frac{W_2/Dl_2}{W_1/Dl_1} = \frac{l_1 \times l_1 \times W_2}{l_2 \times l_2 \times W_1} = \frac{4 \times 4 \times 4{\cdot}2}{5 \times 5 \times 7{\cdot}5} = {\cdot}35.$$

Sometimes the "mil" and "circular mil" are given in problems. A *mil* is $\frac{1}{1000}$ inch, and a *circular mil* is the area of a circle of diameter one mil. In such problems, S is often given in *ohms per mil-foot* (one foot of wire of diameter one mil), in which case l must be in feet, a in circular mils, and R will be in ohms. The area of a circle in circular mils is given by the square of its diameter in mils. An example will make the method of working quite clear.

Example.—*A copper wire is 2 miles long and ·08 inch diameter. Find its resistance if the specific resistance of copper is 10·4 ohms per mil-foot.*

Diameter in mils $= {\cdot}08 \times 1000 = 80$; $\therefore$ Area $a = 80^2$ circular mils.

Length $l = 2$ miles $= 2 \times 1760 \times 3 = 10560$ feet.

S = 10·4 ohms per mil-foot;

$$\therefore\ R = S\frac{l}{a} = \frac{10{\cdot}4 \times 10560}{80 \times 80} = 17{\cdot}16 \text{ ohms}.$$

11. The Effect of Heat on Resistance

Metals increase in resistance when their temperature is raised and decrease in resistance when cooled. The *process* of conduction in metals was explained on page 129, and from that, this effect of heat on resistance might be anticipated. As the temperature is raised the thermal agitation means increased vibration of the ions constituting the "space lattice," so that the roads or avenues between the ions (in which the electrons forming the current move) are *less* clearly defined; thus the electrons are more impeded in their motion by collisions, and the resistance of the metal increases. Conversely, lowering the temperature makes the avenues *more* clearly defined, the electrons are less impeded in their motion, and the resistance of the metal is reduced.

Consider a piece of copper of resistance 1 ohm at 0° C. If its temperature were raised to 1° C. it would increase in resistance by

about 0·004 ohm: if raised to 5° C. it would increase by (·004 × 5) ohm. If its resistance were 10 ohms at 0° C. and its temperature were raised to 5° C. its *increase* in resistance would be (10 × ·004 × 5) ohm. so that its total resistance at 5° C. would be given by

$$\text{Res. at } 5^\circ \text{ C.} = 10 + (10 \times \cdot 004 \times 5),$$

and thus if R_t = resistance at t° C., and R_o = resistance at 0° C.

$$R_t = R_o + (R_o \times \cdot 004 \times t); \quad \therefore\ R_t = R_o(1 + at) \quad ..(12)$$

where we are writing a for ·004. This a is called the **temperature coefficient of resistance**: thus **the temperature coefficient is numerically equal to the increase in unit resistance for unit rise in temperature** (strictly 0° C. to 1° C.). Put another way, the temperature coefficient of the resistance of a material is the ratio:—

$$\frac{\text{Increase in resistance for } 1^\circ \text{ C. rise in temperature}}{\text{Original resistance (strictly at } 0^\circ \text{ C.)}}.$$

For many pure metals the resistivity is almost proportional to the absolute temperature, *i.e.* the coefficient is about 1/273.

The temperature coefficient is frequently expressed as a percentage, *e.g.* for copper, ·4 *per cent. per degree Centigrade*. Further, instead of taking the resistance at 0° C. as the standard, we use the ordinary room temperature. Hence a copper wire whose resistance at ordinary air temperature is, say, 20 ohms, would increase in resistance by about ·4 per cent. of 20 ohms (*i.e.* by ·08 ohm) for every 1° C. its temperature was raised.

A more exact expression for R_t than the one above is $R_t = R_o(1 + at + \beta t^2)$ where β is a very small constant for the substance considered.

Lowering the temperature of the metal lessens the resistance. It was surmised quite early that at the absolute zero of temperature (− 273·7° C.) the resistance of a metal would be nil. Complicated refrigerating plant is necessary for experiments at very low temperature and further research is necessary, but there is no doubt that a few substances when cooled sufficiently do seem to lose their resistance altogether (they are then called **supra-conducting**). If a ring of lead, for example, be sufficiently cooled and a current be started in it (by a method explained later) electrons can move so easily in it that the current will keep on flowing sometimes for several days without any external arrangement (battery, etc.) to keep it up. When such metals lose their resistance they do so *suddenly* (at temperatures a few degrees above the absolute zero), showing that there is some *sudden* change in the electron make-up of the atoms of the material, but what *exactly* takes place is not yet fully known. We refer to the matter again later.

Alloys *increase* in resistance when heated *but not to the extent that pure metals do.* The variation in the case of *manganin* is so small that it can almost be neglected. *Platinoid* has a temperature coefficient about $\frac{1}{18}$ that of the pure metals, *German silver* about $\frac{1}{6}$, and *resista* about $\frac{1}{4}$. The high resistance and small temperature coefficients of these alloys lead to their use in the construction of standard resistances for testing purposes.

Carbon, electrolytes, and insulators all *decrease* in resistance when heated, *i.e.* they have negative temperature coefficients.

The curves in Fig. 281 show the variation of resistance with temperature for a few substances.

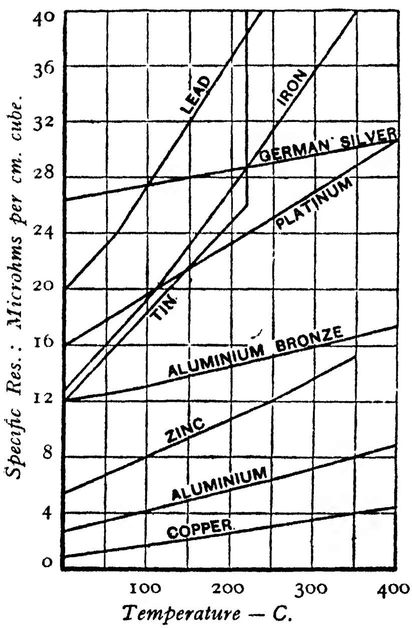


FIG. 281.

12. Insulation Resistance of a Cable

In dealing with the "dielectric" or "insulation" resistance of a cable it must be remembered that the *thickness* of the insulating covering corresponds to the "length" in Art. 10 (for the current leaks from the central conducting core *through the insulating material to the outside* and to earth), and that a long cable gives a *greater sectional area* of insulating material, and therefore has *less* insulation resistance, than a short one. In fact, whilst the *resistance of the conductor* of a cable is, of course, proportional to the length, the *insulation resistance* is *inversely* proportional to the length of the cable, as will be proved presently. Taking two cables of lengths l_1 and l_2, insulated with the same material and having equal internal and equal external diameters, and calling their *insulation resistances* R_1 and R_2:—

$$\frac{R_1}{R_2} = \frac{l_2}{l_1}, \; i.e. \; \frac{\text{Insulation Res. of cable A}}{\text{Insulation Res. of cable B}} = \frac{\text{Length of B}}{\text{Length of A}},$$

i.e. the insulation resistances are *inversely* as the lengths. Thus, if the insulation resistance of a cable is 400 megohms per mile, half a mile will have an insulation resistance of 800 megohms, and two miles an insulation resistance of 200 megohms.

An expression for the insulation resistance of a cable may be found thus:—Let l be the length of the cable, S the resistivity *of the dielectric or insulating covering*, and r_2 and r_1 the external and internal radii of the insulation (Fig. 282); let C be a layer of insulation of infinitely small thickness dr and radius r. For the resistance of this layer, say σ, we have

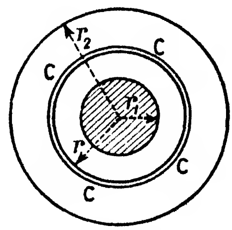


FIG. 282.

$$\sigma = \mathrm{S}\frac{dr}{2\pi rl},$$

for dr corresponds to the length and $2\pi rl$ to the cross-sectional area of our formula of Art. 10. For the total insulation resistance R we therefore have:—

$$\mathrm{R} = \frac{\mathrm{S}}{2\pi l}\int_{r_1}^{r_2}\frac{1}{r}\,dr = \frac{\mathrm{S}}{2\pi l}\Big[\log_e r\Big]_{r_1}^{r_2}$$

$$\therefore\ \mathrm{R} = \frac{\mathrm{S}}{2\pi l}\log_e\frac{r_2}{r_1} \ \textit{or}\ \mathrm{R} = \cdot 366 \times \frac{\mathrm{S}}{l}\log_{10}\frac{r_2}{r_1} \quad \ldots\ldots(13)$$

Taking two cables of lengths l_1 and l_2 insulated with the same material and having the same values for r_1 and r_2 it follows that $\mathrm{R}_1/\mathrm{R}_2 = l_2/l_1$, *i.e.* the insulation resistances are inversely as the lengths as stated above. Further, if two cables be joined end to end (Fig. 283) the conductor resistances are in series but the insulation resistances are *in parallel*: thus if R_1 and R_2 are the separate insulation resistances and R the joint resistances $\mathrm{R} = \mathrm{R}_1\mathrm{R}_2/(\mathrm{R}_1 + \mathrm{R}_2)$.

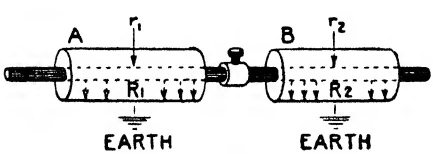


FIG. 283.

13. Electrical Energy. Units of Electrical Energy

In settling their quarter's account with the "Electricity Co." many people say that they are paying for "current," for "electricity," but this is wrong. "Electricity" comes into the house and also goes out: we do not "consume" any of it. The current brings

" electrical energy " into the house and then goes on its way. We pay for that " electrical energy " it has delivered.

It has been explained that whenever a P.D. exists between two points in a circuit an *energy transformation* occurs between them. When a current flows along a copper wire, energy is taken from the electric circuit appearing as heat in the wire; when a current flows through the filament of a lamp, energy is taken from the electric circuit, part appearing as heat and part as light. Passing a current through a motor gives a transformation into heat and mechanical work. In all cases the electrical energy which disappears is equivalent to the new forms into which it changes.

(*a*) In Art. 3 we saw that if the P.D. between two points is one e.m. unit and one e.m. unit quantity of electricity passes the work done or energy transformed is one erg: this is taken as the ***absolute unit*** of work or energy so that from an electrical point of view **an erg is the work done (or energy transformed) between two points in a circuit when the P.D. between the points is one e.m. unit and the e.m. unit quantity passes.** If the P.D. be V e.m. units and one e.m. unit quantity passes the work done will be V ergs; if Q e.m. units pass the work will be VQ ergs, and if the Q e.m. units be transferred by a current I e.m. units in t seconds Q is equal to It and the energy transformation is VIt ergs; hence

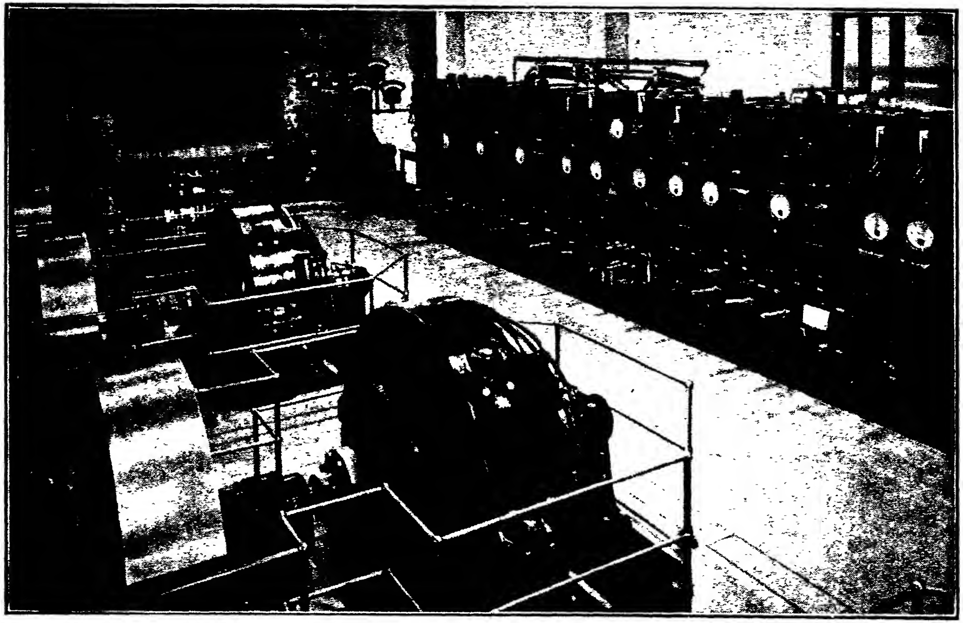
$$\text{Energy in ergs} = VIt = I^2Rt = \frac{V^2}{R}t \quad \ldots\ldots\ldots(14)$$

where V, I, and R are in *electromagnetic units* and t is *in seconds.*

(*b*) If the P.D. be one volt (10^8 e.m. units) and one coulomb (1/10 e.m. unit) passes the energy transformation is clearly 10^7 ergs; this is taken as the *practical unit* of electrical energy and is called a joule; hence **a joule is the work done (or energy transformed) between two points of a circuit when the P.D. between the points is one volt and one coulomb of electricity passes.** If the P.D. be V volts and one coulomb passes, the work done is V joules; if Q coulombs pass, the work is VQ joules; if the Q coulombs be transferred by a current of I amperes flowing for t seconds, Q is equal to It and the energy transformation is measured by VIt joules; hence—

$$\text{Energy in joules} = VIt = I^2Rt = \frac{V^2}{R}t \quad \ldots\ldots\ldots(15)$$

where V, I, and R are in *volts, amperes,* and *ohms* respectively, and t is *in seconds.*



THE B.B.C. NORTH REGIONAL TRANSMITTING STATION.

Part of the Power House showing the main switchboard and three of the four C.C. generators.

(*c*) By (15) above, if the P.D. be one volt, the current one ampere, and the time *one hour, i.e.* 3600 seconds, the energy transformation will be 3600 joules. This is another energy unit called a **watt-hour**; hence **a watt-hour is the work done (or energy transformed) between two points in a circuit when the P.D. between the points is one volt and one ampere flows continuously for one hour.** We might also say that *a watt-hour is the work done between two points in a circuit when the product of the P.D. in volts, the current in amperes, and the time in hours is equal to unity.* From the definition it follows that if T be the time *in hours*—

$$\text{Watt-hours} = VIT = \text{Volts} \times \text{Amperes} \times \text{Hours} \ldots\ldots (16)$$
$$= I^2RT = (\text{Amperes})^2 \times \text{Ohms} \times \text{Hours} \ldots (17)$$
$$= \frac{V^2}{R}T = \frac{(\text{Volts})^2 \times \text{Hours}}{\text{Ohms}} \ldots\ldots\ldots\ldots\ldots (17)$$

(*d*) A still larger unit of energy has been chosen as the legal one by the Board of Trade; it is called a **kilowatt-hour** (kWh.) or **Board of Trade unit** (B.O.T. unit) or sometimes (but rarely) the "kelvin," and is equal to 1000 watt-hours. Clearly, **a B.O.T. unit is the work done (or energy transformed) between two points in a circuit when the P.D. is one volt and one ampere flows continuously for 1000 hours.** We might also say that *a B.O.T. unit is the work done between two points in a circuit when the product of the P.D. in volts, the current in amperes, and the time in hours is equal to* 1000. From the definitions it follows that to get the B.O.T. units or kWh. of energy we simply work out the watt-hours as above, and divide by 1000, *i.e.*

$$\text{No. of B.O.T. units or kWh.} = \frac{\text{No. of watt-hours}}{1000} \ldots (19)$$

1 joule $= 10^7$ ergs $= \cdot 7375$ ft.-pounds.

1 watt-hour $=$ 3600 joules $=$ 2654 ft.-pounds nearly.

1 B.O.T. unit $=$ 1000 watt-hours $=$ 3,600,000 joules $=$ 2,654,000 ft.-pounds.

It is the number of Board of Trade Units of electrical energy supplied to us (and which has been converted into heat and light, etc.) that we pay for in our quarter's account.

Various systems of charging for electrical energy are in use. In the *flat rate system* the consumer is charged a uniform price (say 4d.) for every B.O.T. unit he "consumes." Usually the tariff is reduced to about one-third for energy used for cooking, heating, driving machinery, etc.: this encourages the use of energy during the day when the load on the station is lighter, and some generators would, otherwise, be idle.

In the *rateable value system* the consumer pays a definite amount per year according to the rateable value of the house in which he lives (say 10 per cent. of the rateable value) and then, in addition, he pays a *small* price for every B.O.T. unit he consumes (say 1d. per unit).

Now the expenses of a generating station may be looked upon as consisting of two distinct parts: (1) *Readiness to serve* or *standing costs, i.e.* expenses incurred by the company in placing itself in a position to meet the *maximum* demand on the station at any moment: this includes interest on the money invested and devoted to the purchase of machines, cable, etc., rent, rates, management expenses, etc. (2) *Running costs* which vary with the output and include, for example, most of the coal costs, the oil, wages (partly), etc. Clearly, a consumer's payment must contribute towards both standing and running costs, and, logically, his payment towards standing costs should depend on *his* maximum demand at any time for plant must be ready at any moment to supply his greatest demand: hence we have the *maximum demand system* of charging.

Thus suppose the company find that £5 *per year per kilowatt* (see Art. 14) of the station output will meet the standing costs, and that other costs can be met by charging 1d. per unit for energy consumed. A consumer, A, who uses 1000 *B.O.T. units* in one year and whose maximum demand, *i.e.* greatest *power* consumption at any time was 1 kW., would be charged as follows:—

$$\left.\begin{array}{ll}\text{Max. demand charge (1 kW.)} & = £5\ 0\ 0 \\ \text{Consumption charge } (1000 \times 1)\text{d.} & = £4\ 3\ 4\end{array}\right\} \text{Total } £9\ 3\ 4.$$

Another consumer, B, using the same amount of energy during the year (1000 B.O.T. units), but whose maximum demand at any time was only ½kW., would be charged thus:—

$$\left.\begin{array}{ll}\text{Max. demand charge } (\tfrac{1}{2}\text{ kW.}) & = £2\ 10\ 0 \\ \text{Consumption charge } (1000 \times 1)\text{d.} & = £4\ \ 3\ 4\end{array}\right\} \text{Total } £6\ 13\ 4.$$

There are modifications of this method of charging.

14. Electrical Power. Units of Electrical Power

"Power" refers to "rate of doing work," and the power of any machine is measured by the *work it can do per second.*

(*a*) The absolute unit of power is *one erg of work per second.* Putting t equal to unity in (14) we get the three corresponding expressions for the power in the circuit in absolute units, viz.—

$$\text{Power} = VI = I^2R = V^2/R \text{ ergs per second} \quad \ldots .(20)$$

where V, I, and R are in e.m. units: *thus if the P.D. is* 1 *e.m. unit and a current of* 1 *e.m. unit is flowing, the power in the circuit is one absolute unit,* viz. 1 erg. per second.

(*b*) It has been shown that the work done in t seconds when a current of I amperes flows under a P.D. of V volts is V It joules. If V and I be each unity, the work per second is one joule; this is adopted as a unit of electrical power and is called a **watt**; hence a

watt is the power in a circuit when the P.D. is one volt and one ampere is passing. We can also say that *a watt is the power in a circuit when work is being done at the rate of one joule or* 10^7 *ergs or* ·7375 *foot-pound per second.* Putting t equal to unity in (15) we obtain the expressions for the power in watts in a circuit, viz.:

$$\text{Watts} = VI = \text{Volts} \times \text{Amperes} \quad \ldots\ldots\ldots\ldots (21)$$

$$= I^2R = (\text{Amperes})^2 \times \text{Ohms} \quad \ldots\ldots\ldots (22)$$

$$= \frac{V^2}{R} = \frac{(\text{Volts})^2}{(\text{Ohms})} \quad \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

One watt $= \frac{1}{746}$ horse-power: 1 H.P. = 746 watts.

(*c*) The **kilowatt** (kW.) is another unit of power, and is equal to 1000 watts; hence **a kilowatt is the power in the circuit when the P.D. is one volt and 1000 amperes is flowing or when the P.D. is 1000 volts and one ampere is flowing.** We can also say that *a kilowatt is the power in the circuit when the product of the P.D. in volts and the current in amperes is equal to* 1000. Clearly:—

$$\text{No. of kilowatts (kW.)} = \frac{\text{No. of watts}}{1000} \quad \ldots\ldots\ldots (24)$$

One kilowatt (kW.) $= \frac{1000}{746} = 1{\cdot}34$ H.P.: 1 H.P. = ·746 kW.

It follows from the definitions above and in Art. 13 that a watt-hour is " the work done in one hour when the power is one watt ": similarly a kilowatt-hour is " the work done in one hour when the power is one kilowatt."

The power of a D.C. dynamo is generally given in kilowatts. Thus a machine capable of giving 45 amperes at 2000 volts has a power output of 2000 × 45 = 90,000 watts or 90 kW., and is spoken of as a 90 unit machine. This dynamo would be driven, say, by an engine, and if we neglect all losses in the dynamo, *i.e.* assume it is 100 per cent. efficient, so that its " power output " equals the " power input " or the power spent in driving it, the necessary horse-power of the engine would be 90,000/746 = 120·6 H.P. No dynamo, however, is 100 per cent. efficient (there are losses in it due to heating, hysteresis, friction, etc.), although the efficiency may be as high as 95 per cent. If we assume 90 per cent. efficiency, then for the necessary H.P. of the engine we have:—

$$\text{Effic. of dynamo} = \frac{\text{Power output}}{\text{Power input}} = \frac{90}{100}; \quad \therefore \frac{90000}{\text{Power input}} = \frac{9}{10};$$

$\therefore$ Power input = 100,000 watts = 100,000/746 = 134 H.P.,

so that the H.P. of the engine must be 134, whilst the output of the dynamo is 90 kW. or 120·6 H.P.

15. Worked Examples

The following examples illustrate important points and should be carefully studied.

(1) *Twelve equal wires each of resistance r are joined up to form a skeleton cube, and a current enters at one corner and leaves at the diagonally opposite corner. Find the joint resistance between these corners.* [Inter. B.Sc.

An examination of Fig. 284 will show that the total current (say $6x$ for convenience) divides at A into three equal parts each equal to $2x$. At B, E, and D each of these again divides into two equal parts each equal to x. At H, C, and F these latter unite in pairs, giving three currents each equal to $2x$ uniting at G.

If V denote the P.D. between A and G, then, taking any one path, say AEFG,

$$V = 2xr + xr + 2xr = 5xr.$$

But if R denote the joint resistance,

$$V = 6xR, \; i.e. \; 6xR = 5xr;$$

$$\therefore \; R = \frac{5}{6}r.$$

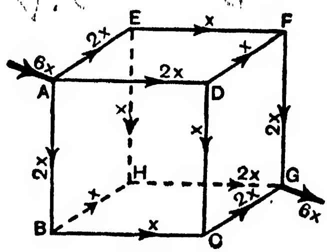


FIG. 284.

(2) *Twelve cells are arranged in series, the total internal resistance being 27 ohms. The outside resistance is 40 ohms and the terminal* P.D. 6 *volts. Find* (i) *the current,* (ii) *the fall of potential in each cell,* (iii) *the E.M.F. of the whole battery,* (iv) *the E.M.F. of each cell,* (v) *the terminal P.D. for each cell.*

When 12 equal cells are joined in series (+ve pole of one joined to −ve pole of the next) the E.M.F. is 12 times that of one cell and the internal resistance is also 12 times that of one cell (Chapter XIII.).

(i) Current $= \dfrac{\text{Terminal P.D.}}{\text{External resistance}} = \dfrac{6}{40} = \cdot 15$ ampere.

(ii) Fall of potential in battery = Current in battery × Res. of battery

$$= \cdot 15 \times 27 = 4\cdot 05 \text{ volts};$$

$$\therefore \text{ Fall of potential per cell} = \frac{4\cdot 05}{12} = \cdot 3375 \text{ volt.}$$

(iii) E.M.F. of battery = Terminal P.D. + Fall of potential in battery

$$= 6 + 4\cdot 05 = 10\cdot 05 \text{ volts.}$$

(iv) E.M.F. of each cell $= \dfrac{10\cdot 05}{12} = \cdot 8375$ volt.

(v) Terminal P.D. for each cell = E.M.F. − Fall of potential in cell

$$= \cdot 8375 - \cdot 3375 = \cdot 5 \text{ volt.}$$

Note also.—Fall of potential per cell = Current in cell × Res. of cell

$$= \cdot 15 \times \tfrac{27}{12} = \cdot 3375 \text{ volt.}$$

$$\text{E.M.F. of battery} = \text{Total current} \times \text{Total resistance}$$

$$= \cdot 15 \times (27 + 40) = 10\cdot 05 \text{ volts.}$$

$$\text{Terminal P.D. for each cell} = 6/12 = \cdot 5 \text{ volt.}$$

(3) *A dynamo supplies two hundred* 60 *watt lamps (the lamps are in parallel) at* 230 *volts. Find the current taken from the dynamo. If the lamps are alight for* 6 *hours find the cost of running them at* 5*d. per B.O.T. unit.*

$$\text{Watts} = \text{VI}; \quad \therefore \text{ Current (I) in each lamp} = \frac{\text{Watts}}{\text{V}} = \frac{60}{230} \text{ amperes;}$$

$$\therefore \text{ Total current from dynamo} = \frac{60}{230} \times 200 = 52\cdot 2 \text{ amperes.}$$

$$\text{B.O.T. units taken by each lamp} = \frac{\text{Watt-hours}}{1000} = \frac{60 \text{ (watts)} \times 6 \text{ (hours)}}{1000} = \cdot 36;$$

$$\therefore \text{ Total B.O.T. units absorbed} = \cdot 36 \times 200 = 72;$$

$$\therefore \text{ Cost at 5d. per unit} = 72 \times 5 \text{ pence} = £1 \text{ 10s.}$$

CHAPTER XI

MAGNETIC EFFECTS OF CURRENTS

WHENEVER electricity is set in motion magnetic forces are brought into existence. If two electrons, or protons, or negative ions, or positive ions, or two charged bodies in general are at rest there is a certain force—an electrostatic force—between them. If the two charges be set in motion, say along parallel paths, the electrostatic force between them becomes less owing to the setting up of magnetic force which partly cancels it. The amount by which the electrostatic force between the two decreases is greater the greater the velocity with which they move, but the electrostatic force would only disappear altogether if the velocity of the charged bodies could be made equal to the velocity of light (3×10^{10} cm. per sec.).

In practice, then, with moving charges electrostatic force is always present, although lessened, and the magnetic force is difficult to observe. When, however, we are dealing with the moving charges in a conductor—say the moving electrons which constitute a *current* in the wire—the electrostatic forces are cancelled by those of the positive ions through the avenues between which the electrons drift, and thus the magnetic forces show themselves and can be *readily* observed. All this is really a case of the general principle of **Relativity**, which not only shows that magnetic forces must accompany a current but also predicts their magnitudes.

In this chapter certain magnetic effects of currents are dealt with, and the intensities of the magnetic fields determined. As already indicated (page 68), there are two methods of approaching this subject, viz. the current element method and the magnetic potential or equivalent shell method: we shall deal first with the former method. The student should again read pages 40-42.

A—Current-Element Methods

1. Magnetic Field Due to a Linear Current

Before examining the magnetic field due to the current in a straight wire, consider the following simple experiments:—

(*a*) Fix a *long* wire vertically and pass a current *up* (conventional) the wire. Using the oscillation magnetometer, find the number of oscillations

(n_1) per minute at distance d_1 due magnetic east of the wire. Repeat at distance d_2 and let n_2 be the number per minute. Finally find n_0, the number per minute under the influence of the earth alone. On the east of the wire the earth's field and the field due to the current are in the same direction (see page 40): hence (Chapter IV.):—

$$n_1^2 - n_0^2 \propto \text{field due to the current at distance } d_1,$$
$$n_2^2 - n_0^2 \propto \text{field due to the current at distance } d_2,$$

and it will be found that $(n_1^2 - n_0^2) : (n_2^2 - n_0^2) = d_2 : d_1$, i.e. *the intensity of the field varies inversely as the distance.* This is known as Biot and Savart's experiment.

(*b*) Include an ammeter in the circuit, and also an adjustable resistance. With current I_1 passing, let n_1 be the number of oscillations per minute at a

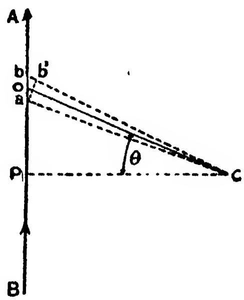


FIG. 285.

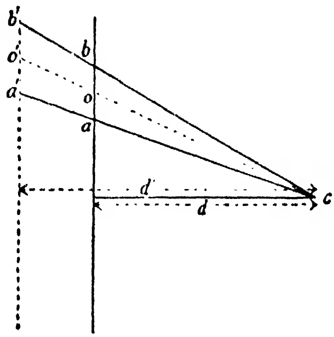


FIG. 286.

certain point, and with current I_2 let n_2 be the number per minute *at the same point*: then

$$n_1^2 - n_0^2 \propto \text{field at a certain distance when the current is } I_1,$$
$$n_2^2 - n_0^2 \propto \text{field at the same distance when the current is } I_2,$$

and it will be found that $(n_1^2 - n_0^2) : (n_2^2 - n_0^2) = I_1 : I_2$ (approx.), i.e. *the intensity of the field varies directly as the current strength.*

A rule for calculating the magnetic field was given by Laplace (known as Laplace's rule). The rule itself applies to each *small element* of the current circuit, but it can be extended to the complete circuit and the results verified by experiments. The rule embodies the following assumptions:—Let AB (Fig. 285) be a wire carrying a current: then considering the magnetic field at c due to a very small element ab it is assumed that the field is (1) Directly proportional to the length ab_1, that is, to the apparent length of ab as seen from c (ab_1 is drawn at right angles to co which joins c to o, the middle point of ab). (2) Inversely proportional to the square of the distance of the element from c, that is to oc^2. And to these may

be added, as we have seen, a third item, viz. that the field is (3) Directly proportional to the strength of the current in AB. Hence if H be the intensity of the field at c due to the element ab, and I be the strength of the current in *arbitrary* units, we have:—

$$H \propto \frac{I.ab_1}{(oc)^2} \text{ or } H = k\frac{I.ab \sin \alpha}{(oc)^2} \text{ or } H = k\frac{Iab \cos \theta}{(oc)^2},$$

where α = angle Poc, θ = angle Pco, and k = a constant depending on the unit of current chosen. As indicated in Chapter X., the e.m. unit current is really so chosen that *with* I *in these units*, $k = 1$: hence writing l for the *small* length ab, and r for the distance oc:—

$$H = \frac{Il \sin \alpha}{r^2} \quad \ldots\ldots\ldots\ldots (1)$$

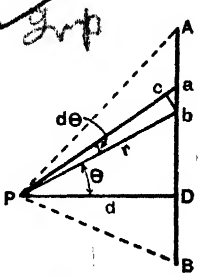


FIG. 287.

Note that the first of the Laplace rules given above could be expressed thus (since $ab_1 = ab \sin \alpha$): —" Directly proportional to the length of the element and to the sine of the angle between the direction of the element and the radius vector drawn to the point (c) from the element."

That the above fits in with Biot and Savart's experiment is seen thus:—Apply (1) to the two cases shown in Fig. 286, where d = the first distance of c from the wire, and d' the second distance, and we get:—

$$H = \frac{Iab \sin \alpha}{(oc)^2}; \quad H_1 = \frac{Ia'b' \sin \alpha}{(o'c)^2}; \quad \therefore \frac{H}{H_1} = \frac{ab}{a'b'}\cdot\frac{(o'c)^2}{(oc)^2}$$

$$\text{But } \frac{o'c}{oc} = \frac{a'b'}{ab}; \quad \therefore \frac{H}{H_1} = \frac{ab}{a'b'}\cdot\frac{(a'b')^2}{(ab)^2} = \frac{a'b'}{ab}$$

$$\text{And geometrically: } \frac{a'b'}{ab} = \frac{d'}{d}; \quad \therefore \frac{H}{H_1} = \frac{d'}{d},$$

i.e. the fields vary *inversely as the distance* from the wire (Biot and Savart).

We can now find an expression for the intensity of the magnetic field at a point due to the current in a straight (finite) wire. In Fig. 287 let BA be the wire carrying a current I e.m. units and P the point, d cm. from the conductor, at which the intensity is required; let $Pb = r$, the angle $DPb = \theta$, and the small angular increment $bPa = d\theta$; let the angle $DPA = \theta_2$ and the angle $DPB = \theta_1$. Clearly, $bc = rd\theta$;

$$\therefore \text{ Field at P due to } ba = \frac{I.bc}{r^2} = \frac{Ird\theta}{r^2} = \frac{I.d\theta}{r}.$$

$$\text{But } d/r = \cos \theta; \quad \therefore 1/r = \cos \theta/d;$$

$$\therefore \text{ Field at P due to } ba = I \cos \theta d\theta/d,$$

and the total field (H) at P due to the whole conductor BA is the integral of the above between the limits $\theta = -\theta_1$ and $\theta = \theta_2$; that is—

$$H = \frac{I}{d}\int_{-\theta_1}^{\theta_2} \cos\theta . d\theta = \frac{I}{d}\Big[\sin\theta\Big]_{-\theta_1}^{\theta_2} = \frac{I}{d}\Big[\sin\theta_2 - \sin(-\theta_1)\Big];$$

$$\therefore \text{ Magnetic field at P} = H = \frac{I}{d}(\sin\theta_1 + \sin\theta_2) \text{ gauss} \ldots (2)$$

and if the current I be in amperes this must be divided by 10.

In the case of an infinitely long conductor $\theta_1 = \theta_2 = \pi/2$, and therefore $\sin\theta_1 = \sin\theta_2 = 1$; hence:—

$$H = \frac{2I}{d} \text{ gauss or } H = \frac{2I}{10d} \text{ gauss} \ldots\ldots\ldots\ldots (3)$$

the second expression applying when the current I is in amperes.

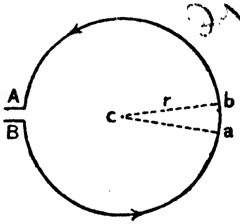


FIG. 288.

2. Work Done in Linking a Current

An important expression in electrical theory is that for the work done in moving a unit magnetic pole once round a current circuit. At this stage we will consider *only the special case of a long straight wire* carrying a current I e.m. units. The magnetic field at distance d cm. is $2I/d$ gauss, and therefore the force on a unit pole there is $2I/d$ dynes. Imagine a unit N pole carried once round the wire along the circular magnetic line of radius d in the direction, say against the magnetic force. Then:—

$$\text{Work done} = \text{Force} \times \text{Distance} = \frac{2I}{d} \times 2\pi d = 4\pi I \text{ ergs} \ldots (4)$$

Thus *the work done in linking the current once is* $4\pi I$ *ergs.* Moreover, energy being a scalar quantity, this is really independent of the path taken by the pole provided that it only goes round once and begins and ends at the same point.

Of course, if we can establish the expression $4\pi I$ ergs for the work done on unit pole in linking a linear current by some method *not* involving the assumption that the field is $2I/d$ (see Art. 8), then the result might be used to prove that $H = 2I/d$. Thus if H gauss be the field = force in dynes on unit pole, then:—

$$H \times 2\pi d = 4\pi I; \quad \therefore H = 2I/d \text{ gauss.}$$

3. Magnetic Field Due to a Circular Current

(1) FIELD AT THE CENTRE.—Consider the conductor AB to be looped into a circle as shown in Fig. 288, and let us determine the intensity of the magnetic field at c, the centre of the circle.

Considering the action of the element ab, the intensity of the field is $(\mathrm{I} \times ab)/r^2$, where I denotes the current in AB in e.m. units and r the radius of the circle in cm. Now, since each element of the conductor is similarly placed relatively to c, the intensity (H) of the field at c due to the conductor AB taken as a whole is $(\mathrm{I} \times \mathrm{AB})/r^2$, that is $(\mathrm{I} \times 2\pi r)/r^2$ or $2\pi\mathrm{I}/r$ (see again page 84). If the coil consists of n turns sufficiently thin to be regarded as coincident, this must be multiplied by n. Hence:—

$$\text{Field at the centre} = \mathrm{H} = \frac{2\pi n\mathrm{I}}{r}\ \text{gauss} = \frac{2\pi n\mathrm{I}}{10r}\ \text{gauss} \quad \ldots.(5)$$

the current I being in e.m. units in the first case and in amperes in the second case. The direction of the field at c is at right angles to the plane of the coil, and may be determined by Ampère's Rule; in Fig. 288 (conventional current direction *counter-clockwise*) it is outwards towards the reader.

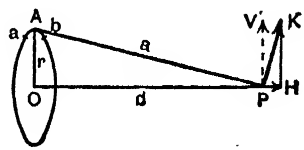


FIG. 289.

(2) FIELD AT ANY POINT ON THE AXIS.—The problem here is to find the field at *any* point P on the axis of the coil (Fig. 289). Again consider first the field due to the small element ab. This is clearly $(\mathrm{I} \times ab)/a^2$ in a direction at right angles to AP. Let PK represent this, and resolve it into two components, viz. PH *along* the axis, and PV perpendicular thereto. Only the component along the axis need be considered, for when the whole ring is taken into account the vertical components due to the various elements cancel each other. From the similar triangles HPK and OAP we have $\mathrm{PH}/\mathrm{PK} = r/a$ or $\mathrm{PH} = r\mathrm{PK}/a$: hence denoting the horizontal component PH by h:—

$$h = \frac{r}{a} \times \frac{\mathrm{I}.ab}{a^2} = \frac{r.\mathrm{I}.ab}{a^3},$$

and for the whole ring, the intensity H is obtained by summing the above for all the elements into which the ring is divided, *i.e.*

$$\mathrm{H} = \frac{r\mathrm{I}}{a^3}\Sigma ab = \frac{r\mathrm{I}}{a^3}2\pi r = \frac{2\pi r^2\mathrm{I}}{a^3}.$$

If the coil consists of n turns as in Case I above we have:—

$$\text{Field at P} = \text{H} = \frac{2\pi nr^2\text{I}}{a^3} = \frac{2\pi nr^2\text{I}}{(d^2 + r^2)^{\frac{3}{2}}} \text{ gauss} \quad \ldots\ldots\ldots(6)$$

the current I being in e.m. units: if I be in amperes this must be divided by 10. When $d = 0$ the above reduces to $\text{H} = 2\pi n\text{I}/r$, the result obtained in Case I. for the field at the centre of the coil.

Note on the "Moment of a Coil." The following point might be noted at this stage. Let a *small* coil of radius r carrying a current I e.m. units be fixed with its plane in the magnetic meridian: the field at P (Fig. 290) due to it is $2\pi r^2\text{I}/(d^2 + r^2)^{\frac{3}{2}} = 2\pi r^2\text{I}/d^3$ if r is small compared with d. If the coil be replaced by a small "end-on" magnet of moment M the field at P due to it is $2\text{M}/d^3$ (page 69). If this field be identical with the previous one, then $\text{M} = \pi r^2\text{I} = \text{AI}$, where A denotes the face area of the coil. Thus in the case considered *the circular current is equivalent to a magnet the moment of which is numerically equal to the current in e.m. units multiplied by the area of the coil face.* (If there are n turns, moment $= n\text{AI}$.)

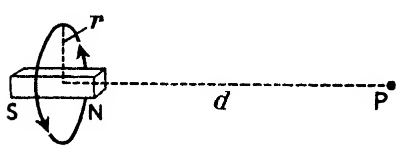


FIG. 290.

4. Magnetic Field on the Axis of a Solenoid

An extension of the results of Art. 3 shows that the field at the centre of a *long thin* closely wound solenoid is given by—

$$\text{H} = \frac{4\pi\text{SI}}{l} \text{ gauss} \quad or \quad \text{H} = \frac{4\pi\text{SI}}{10l} \text{ gauss} \quad \ldots\ldots\ldots\ldots(7)$$

the current in the first case being in absolute units and in the second case in amperes: S = total number of turns and l = length of solenoid in cm. The field is practically uniform except near the ends and is *half the above value at the ends.* Taking the second expression, since $4\pi/10 = 1{\cdot}257$ and S/l = turns per cm. $= n$, say,

$$\text{H} = 1{\cdot}257n\text{I} = 1{\cdot}257 \text{ (ampere turns per cm.)} \quad \ldots\ldots\ldots(8)$$

A more exact formula for the solenoid is as follows:—The field at P (Fig. 291 (a)) is really—

$$\text{H} = \frac{2\pi\text{SI}}{l}(\cos\theta_1 - \cos\theta_2) \quad \ldots\ldots\ldots\ldots\ldots(9)$$

If the solenoid be ***very long and thin*** and P be well removed from either end, practically $\theta_1 = 0$ and $\theta_2 = \pi$, so that $(\cos\theta_1 - \cos\theta_2) = 2$ and $H = 4\pi SI/l$ (as above). At the extreme end (say at C) of a long solenoid $/\theta_1 = \pi/2$ and $\cos\theta_1 = 0$, whilst θ_2 is practically π and $\cos\theta_2 = -1$, so that $H = 2\pi SI/l$, *i.e.* half the intensity at points well inside. Fig. 291 (*b*) gives approximately the variation of the field of a solenoid.

A simple proof of the expression (7) may be given if we assume the result deduced for one special case in Art. 2 to be of general application, viz. that *the work done in carrying unit pole once round a wire carrying a current* I *absolute units is* 4πI *ergs* (see Art 8). Let Fig. 292 represent a portion of the solenoid. Imagine unit pole moved along the closed path ABCDA: the work done is 4πI for *each turn* of wire. If there are n turns per unit length, the total turns in the part considered are nAB and the work done is $4\pi I \times n\overline{AB}$. The work in going along BC and DA is *nil*, since these paths are at right angles to the lines of force. The work in going along CD is negligibly small compared with that in going along AB, for the magnetic lines which are crowded into a small space inside (see Fig. 52) are spread out throughout the whole field outside, so that the force along CD, and therefore the work, is negligible. The work in going along AB is $H \times \overline{AB}$, where H is the intensity inside; hence $H \times AB = 4\pi I \times nAB$, so that $H = 4\pi nI$. But $n = S/l$, where S is the total turns on the solenoid, and l the length: hence—

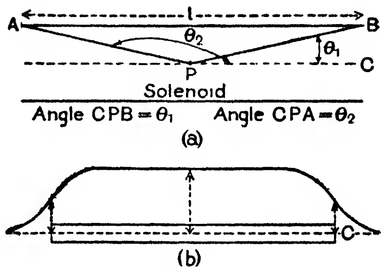


FIG. 291.

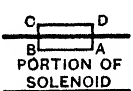


FIG. 292.

$$H = \frac{4\pi SI}{l} \text{ units} \quad or \quad H = \frac{4\pi SI}{10l} \text{ units.}$$

A simple application of the calculus, however, provides a proof of the formulae for the field of a solenoid:—

Consider the field at P (Fig. 293) due to a very thin slice of the solenoid of length dx. Let i be the "effective" current in unit length, so that $i.dx$ is the current in this thin slice dx. This constitutes a circular current, and we can apply the results of Art. 3 to find the field at P due to the slice: thus the field at P due to the slice is $2\pi r^2 idx/a^3$.

Now from Fig. 293, it is seen that $a d\theta/dx = \sin\theta$, so that $dx = a.d\theta/\sin\theta$: hence for the field at P due to the slice we have:—

$$\frac{2\pi r^2 i a d\theta}{a^3 \sin\theta} = 2\pi i.\frac{r^2}{a^2}.\frac{d\theta}{\sin\theta} = 2\pi i \sin\theta.d\theta$$

since $r/a = \sin\theta$. Thus if θ_1 and θ_2 be the values of θ for the ends of the solenoid, the total field H at P is clearly

$$H = 2\pi i\int_{\theta_1}^{\theta_2}\sin\theta.d\theta = 2\pi i\Big[\cos\theta\Big]_{\theta_2}^{\theta_1};$$

$$\therefore\ H = 2\pi i(\cos\theta_1 - \cos\theta_2).$$

If I be the current in the solenoid, S the total turns, and l the length, $i = SI/l$; hence:—

$$H = \frac{2\pi SI}{l}(\cos\theta_1 - \cos\theta_2)$$

which becomes $4\pi SI/l$ for an infinite solenoid as already indicated.

5. Current-Carrying Conductor Placed in a Magnetic Field

Just as a conductor carrying a current exerts a definite influence on a neighbouring magnet, so also does the magnet exert a reciprocal influence on the conductor. In fact the phenomena are connected in much the same way as the action and reaction of mechanical forces. When in many instruments a current coil surrounds a magnetic needle, the latter moves simply because the resistance offered to its motion at the point of support is small compared to that offered by the coil, which would usually be fixed in a wooden framework. If, however, the needle had been artificially restricted in its motion, and if also the coil had been suspended so that rotation of it could easily have been effected, then the coil would have moved and the needle remained at rest; for any couple action which is called into play tending to cause rotation of the needle in any one direction, will invariably and necessarily be accompanied by an equal couple action tending to cause rotation of the coil in the opposite direction.

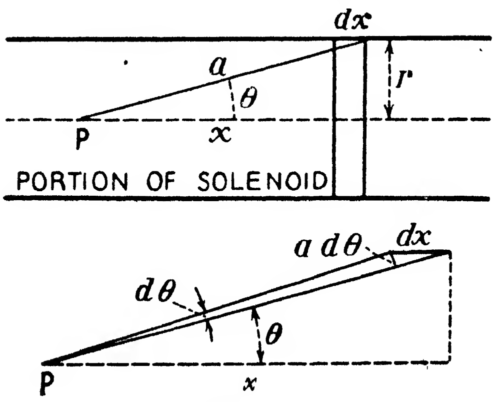


FIG. 293.

(1) DIRECTION IN WHICH THE CONDUCTOR MOVES.—Now, when a conductor is carrying a current there is a magnetic field round about it due to this current, and if the conductor be then put into *another* magnetic field due to a magnet (or another current), any force on the conductor tending to move it is, as explained, often regarded as due to the interaction of the two fields. By considering these two fields we can deduce how the conductor will move.

In Fig. 294, A is the cross-section of a straight wire *placed at right angles to the magnetic field* due to the poles N and S, and at right angles to the plane of the paper, *i.e.* the wire goes "into the paper" in the figure. The lines at B and C give the general direction of the magnetic lines of the field due to N and S. Imagine now that a (conventional) current passes *upwards* in the wire; the circle about the wire gives the direction of the magnetic lines of the field

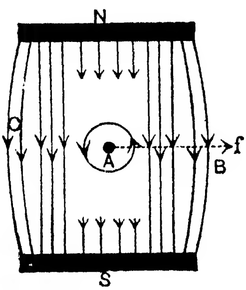


FIG. 294.

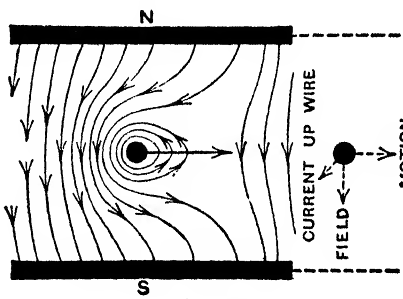


FIG. 295.

due to the current. It will be seen that on the side C the two sets of lines are in the same direction (thus the field is *strong* there) and therefore repel each other, whilst on the side B the two sets are in opposite directions and therefore more or less cancel each other (thus the field is *weak*). The result is that the wire is acted on by a force, indicated by *f*, towards the right, and the wire tends to move towards the right. The *actual* field between N and S is the resultant of the two fields above, and is indicated in Fig. 295.

See also Fig. 51. The current is going in the opposite direction, but the field is also in the opposite direction, and the wire again moves to the right: if *either* the current *or* the field had been reversed—but not both—the wire would have moved to the left.

Note that in the simple case considered the conductor and its current are *at right angles to the field* (this is often the case in practice), and that the direction in which the conductor moves is *at right angles*

to both current and field. From the figure it will be clear that the direction of motion of the wire is given by the following rule:—

Fleming's Left-Hand Rule.—*Hold the thumb and the first two fingers of the left hand mutually at right angles; place the forefinger in the direction of the lines of force of the field in which the conductor is situated (N to S), and turn the hand so that the middle finger points in the direction of the (conventional) current; the thumb will indicate the direction of motion.* (Fore finger—Force: mIddle finger—current I: thuMb—Motion.)

(2) MAGNITUDE OF THE FORCE ON THE CONDUCTOR.—An expression for the *magnitude* of the force on a current-carrying conductor when placed in a magnetic field may be deduced thus:—The field at P (Fig. 296) due to the element ab of the conductor is given by $\mathrm{I}.ab'/d^2$ or $\mathrm{I}.ab \sin \alpha/d^2$, where I is the current in e.m. units and d cm. is the distance Po (Art. 1). This, therefore, is the force in dynes which the current element would exert on a unit north pole at P: hence if a pole of strength m be put at P the force (F) on it due to the element will be:—

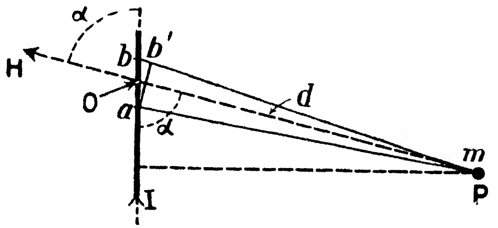


FIG. 296.

$$\mathrm{F} = m.\frac{\mathrm{I}.ab \sin \alpha}{d^2} = \mathrm{I}.l \sin \alpha.\frac{m}{d^2} \text{ dynes,}$$

where l is the length ab. Since action and reaction are equal and opposite, this is also the force which is experienced by the element l when the pole of strength m is put at P.

But a pole m at P produces a field H equal to $m/\mu d^2$ gauss at the place where the element is situated: thus for the force on the element l of conductor carrying a current I e.m. units in this field of H gauss we have:—

$$\text{Force} = \mathrm{F} = \mathrm{I}l \sin \alpha \frac{m}{d^2} = \mathrm{I}l \sin \alpha\mu \frac{m}{\mu d^2},$$

$$i.e. \text{ Force} = \mathrm{F} = \mu \mathrm{HI}l \sin \alpha = \mathrm{BI}l \sin \alpha \text{ dynes} \;\ldots\ldots(10)$$

where B = magnetic induction = μH. If the conductor is straight and the field uniform, then this result may be applied to a conductor of finite length. Note α = angle between l and direction of H.

In the case of a conductor of length l, placed *at right angles* to a uniform field of strength H, we have (since $\sin \alpha = 1$)—

$$\text{Force} = F = \mu H I l = B I l \text{ dynes} \qquad (11)$$

the current I being in e.m. units and l in cm. (Divide by 10 if I is in amperes.) In most cases in practice the conductor *is* at right angles to the field: moreover the medium is usually air for which $\mu = 1$ and $B = H$, so that the expression for the force on the conductor is often given as HIl dynes. The direction in which the conductor tends to move owing to this force acting on it is at right angles both to the field and to the current (left hand rule).

As an illustration, if a straight wire 20 cm. long carrying 15 amperes be placed at right angles to the earth's uniform field of strength ·18 unit the force tending to move the wire at right angles to itself and to the field (left-hand rule) is $(\cdot 18 \times 15 \times 20)/10$, *i.e.* 5·4 dynes: if the wire was lying east and west and the current going west to east the wire would move upwards.

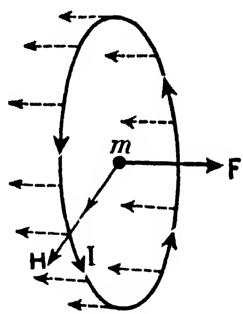


FIG. 297.

The force on a current-carrying conductor when placed in a magnetic field may be deduced in another way. Fig. 297 shows a coil radius r carrying a current I e.m. units. The force on unit north pole at the centre is $2\pi I/r$ dynes, and therefore on a north pole of strength m it is $2\pi I m/r$ dynes: this force F is in the direction indicated (towards the right). On dynamical grounds there is an equal opposite total force on the coil. From symmetry this is distributed all round the circle, so that the *force on unit length*, say, of the coil is

$$\frac{2\pi I m}{r \times 2\pi r} = I \frac{m}{r^2} \text{ dynes.}$$

But the radial field H at the coil due to the north pole m is $m/\mu r^2$. Hence for the force on unit length of the coil when in this field H we have:—

$$\text{Force } per\ cm. \text{ of coil} = I\frac{m}{r^2} = I\mu\frac{m}{\mu r^2} = \mu H I = B I \text{ dynes,}$$

and if the medium be air the *force per cm.* = HI dynes. The diagram shows that the force is at right angles to both H and I.

(3) WORK DONE WHEN A CURRENT-CARRYING CONDUCTOR MOVES IN A MAGNETIC FIELD.—At this stage we need only take the simple case of the conductor moving parallel to itself and at right angles to a uniform field (Fig. 298). In the figure AB of length l cm. carrying the current I e.m. units would move to the right (left-hand rule): let it move x cm. The force on AB is BIl dynes,

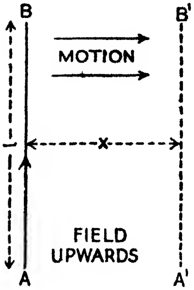


FIG. 298.

and the work done (force $\times$ distance) is therefore $BIlx$ ergs. But $l \times x$ is the area swept out, and Blx is therefore the *total* number of unit tubes of induction (N) cut during the displacement, for B is the number per unit area: hence:—

Work = Current (e.m. units) $\times$ Unit tubes of induction cut,

i.e. Work = IN ergs *or* Work = IN/10 ergs,

the current being in amperes in the second case. In an air medium the tubes of induction are also tubes of force.

6. Couple on a Current Coil in a Uniform Magnetic Field

Consider for simplicity a rectangular coil ABCD (Fig. 299) with its coil face making an angle θ with the direction of the uniform field H, the coil being capable of rotation about a vertical axis XX. The forces f on AD and BC are vertical, equal, and opposite, and can cause no motion of the coil (verify by Fleming left-hand rule). The forces F on AB and DC are, however, horizontal, and as the currents are in opposite directions the forces are in opposite directions (left-hand rule again): they therefore form a couple tending to rotate the coil. If l cm. be the lengths of AB and DC then the force F on each is μHIl dynes: hence:—

Couple on coil $= F \times Dk = \mu HIl \times AD \cos \theta$,

i.e. Couple on coil $= \mu HIA \cos \theta \ldots (12)$

where A is the face area of the coil $= l \times AD$. The current I is in e.m. units: if in amperes the result must be divided by 10. If the coil has n turns (12) becomes $\mu HInA \cos \theta$.

The couple becomes zero when $\theta = 90°$, for then $\cos \theta = 0$. Thus if the coil were suspended in the field it would tend to turn, so that its *face* was at right angles to the field (its axis therefore along the field), and its magnetic lines in the same direction as the lines of the field: or as it is often worded in practice, the coil turns so as to embrace as many magnetic lines as possible. These principles are applied in measuring

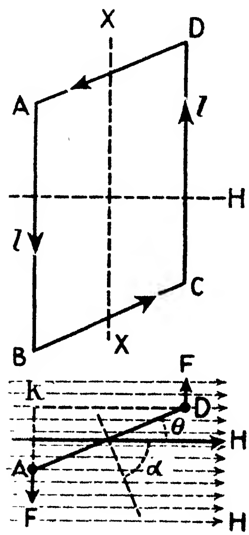


FIG. 299.

instruments of the moving coil type—galvanometers, ammeters, voltmeters, etc.

Note that θ is the angle between the *coil face* and the field: if we use the angle α between the *axis of the coil* and H, then $\alpha = (90^\circ - \theta)$, and the expression becomes μHIA sin α, or HIA sin α if the medium be air.

Again, if a magnet of moment M be suspended in a uniform field H with its magnetic axis making an angle α with the field, the couple on it is HM sin α (page 63). Thus, comparing with the last expression, we see that *the coil behaves like a magnet of magnetic moment* IA *with its magnetic axis perpendicular to the plane of the coil,* i.e. *along the coil's axis.* This fact was also deduced for a special case in Art. 3.

7. Force Between Current-Carrying Conductors

(1) DIRECTION IN WHICH THE CONDUCTORS MOVE. If two flat coils of wire be each suspended, so that they hang with their faces parallel to each other, *they attract each other if the currents go in the same direction, and repel each other if the currents go in opposite directions.* When a current flows in a coil it makes one " face " a north and the other face a south (page 274), so that this is just what would be expected. If the currents are in the same direction the faces of the coils which are opposite each other will have opposite " polarity," and there will be attraction: if the currents are in opposite directions these faces will have like polarity, and there will be repulsion.

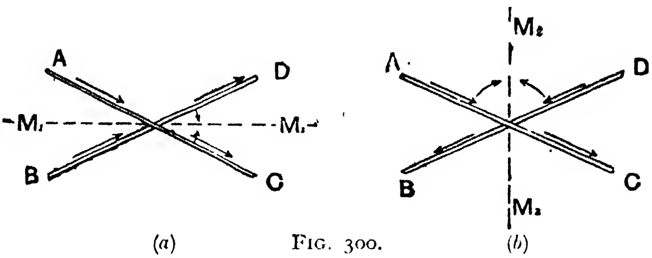


FIG. 300.

Ampère demonstrated in the case of straight circuits the following laws:—(1) *Two parallel currents attract or repel one another according as they flow in the same or in opposite directions.* (2) *Two non-parallel currents attract one another if both approach or both recede from the point of meeting of their directions, while they repel one another if one approaches and the other recedes from that point:* thus if AC and BD are two conductors carrying currents as indicated, then

the conductors will be urged to move, so that the currents in both of them tend to flow in the same direction, *i.e.* in Fig. 300 (*a*) they will be attracted to some mean position M_1M_1, and in Fig. 300 (*b*) to some mean direction M_2M_2. The first law was referred to on page 41, and Figs. 48, 49, 50, demonstrated the truth of it.

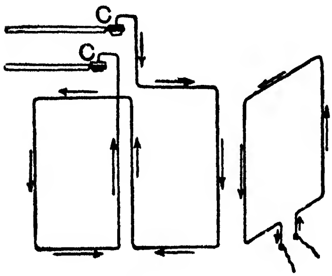


FIG. 301.

(*a*) Other experiments whereby Ampère's laws are established merely consist in having two circuits, one fixed and the other movable, placing them in various relative positions, and passing currents in sundry directions. One form is indicated in Fig. 301. The movable coil is bent in the form of two rectangles in such a way that the current flows through them in opposite directions (thus nullifying the effect of the earth on the apparatus) and the coil is suspended from the mercury cups C, C; on bringing the second current-carrying circuit into various positions the laws may be verified.

(*b*) Fig. 302 indicates another method. The tank contains water, and an insulated wire W passes vertically through it and carries a current, say downwards. The floating coil of Fig. 302 is placed in the tank and its movements noted. The coil turns round until it is " edge-on " to the vertical wire and so that the side of it where the current flows downwards is nearest the wire, and then it moves bodily towards the wire.

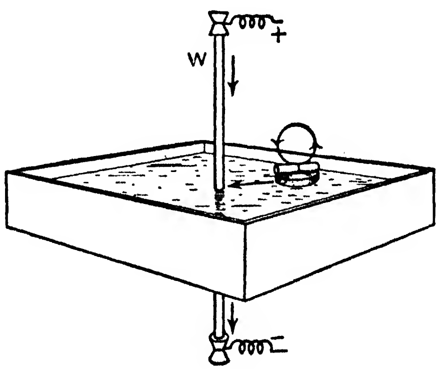


FIG. 302.

(*c*) It is interesting to examine the effect of a current in a wire on a stream of electrons, and for this what is known as a **cathode ray tube** can be used. This is described in detail in Chapter XXI., but briefly it consists of a bell-shaped glass vessel, exhausted, and fitted at the extreme narrow end with a filament cathode which is heated by a current. The thermal agitation together with a large P.D. which is maintained between the cathode and an anode (or anodes) situated in front of it cause electrons to be ejected from the cathode: these are accelerated towards the high potential anode and pass on through a hole as a narrow stream of electrons into the wide part of the tube (they are spoken of as

cathode rays). On fixing a wire parallel to the stream and passing a current (conventional) down the wire (*electronic* current therefore upwards) the beam is attracted towards the wire: if the current (conventional) in the wire is upwards the beam is repelled (Fig. 303).

(2) MAGNITUDE OF THE FORCES ON THE CONDUCTORS.—The values of the forces can easily be determined in a few simple cases:—

(*a*) *Two infinitely long parallel conductors.* Let AB and CD (Fig. 304) be the two conductors d cm. apart and carrying currents I_1 and I_2 e.m. units. The field at N due to the current in AB is $2I_1/d$ gauss. Now CD carrying the current I_2 is in this field, and for the force on it we have (Art. 5):—

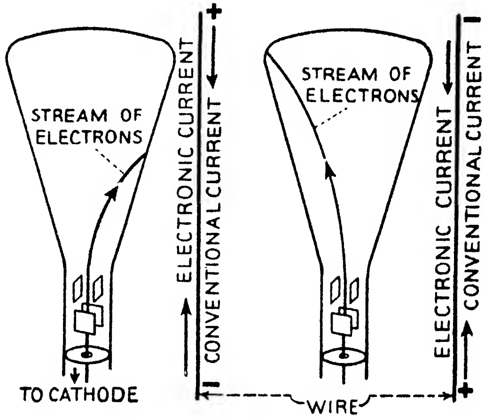


FIG. 303.

$$\text{Force per unit length of CD} = BI_2 = \mu HI_2 = \mu \frac{2I_1}{d} I_2,$$

$$i.e. \text{ Force per unit length} = \frac{2\mu I_1 I_2}{d} \text{ dynes} \ldots\ldots\ldots (13)$$

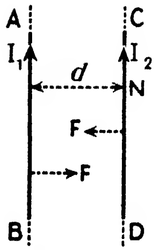


FIG. 304.

and similarly the force on unit length of AB is also $2\mu I_1 I_2/d$ dynes: if the medium be air, as is usual, μ of course is unity. Further, since the *product* of the two currents appears in the expression, if the currents I_1 and I_2 are in amperes divide by 100.

The direction of the force, say, on CD is given by the left-hand rule. Thus if the currents are in the *same direction* as in Fig. 304 the field H at N due to AB is at right angles to the plane of the paper and away from the reader (swimming rule applied to AB), and the left-hand rule shows that the force F on CD is *towards* AB, *i.e.* there is *attraction* between the wires: if the two currents are in opposite directions the force on CD is away from AB.

If $d = 1$ cm. and $I_1 = I_2 = 1$ e.m. unit, and the medium is air, then the force per cm. = 2 dynes. This gives another definition of the e.m. unit current, viz. **the e.m. unit current is that current**

which flowing in each of two long parallel wires one cm. apart in air (strictly vacuo) causes them to exert a force on each centimetre of each other of two dynes. The definition has the merit that it does not include the expression "unit magnet pole," and only involves effects which are always present when currents flow.

(*b*) *Two equal coaxial coils.* Consider two coils, A and B, each of radius r cm. arranged parallel and coaxially at d cm. apart, and carrying currents I_1 and I_2 e.m. units: assume an air medium for simplicity. The force on unit length of B due to A is $2I_1I_2/d$ dynes, and by symmetry the total force on B is the product of this and the circumference. Thus:—

$$\text{Force on B} = \frac{2I_1I_2}{d} \times 2\pi r = \frac{4\pi r I_1 I_2}{d} \text{ dynes} \quad \ldots\ldots\ldots(14)$$

and similarly for the force on A.

The full treatment of the forces between coils of different dimensions would take us beyond the scope of this book. (See *Advanced Textbook of Electricity and Magnetism.*)

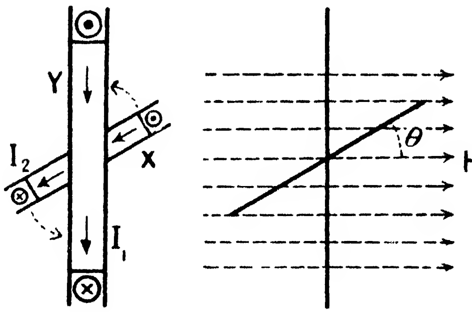


FIG. 305. X is drawn rather large for clearness.

(*c*) *Couple on a small coil at the centre of a larger coil.* It is important, however, to find the couple on a small coil X placed at the centre of a larger coil Y when current flows in both, for this has an important application in certain measuring instruments known as **dynamometer** instruments. Let I_1 be the current in Y (Fig. 305), and I_2 the current in X (e.m. units), r_1 the radius of Y, and r_2 the radius of X. The magnetic field H at the centre of Y is $2\pi I_1/r_1$, and we can assume this uniform over the space occupied by the small coil X (this field is, of course, along the axis of Y). The couple on X in this field is by Art. 6 equal to $HI_2A \cos\theta$ where A is the face area of X and θ the angle between the *face* of X and the field. Hence:—

$$\text{Couple on X} = \frac{2\pi I_1}{r_1} \times I_2 \times \pi r_2^2 \times \cos\theta = \frac{2\pi^2 r_2^2 I_1 I_2}{r_1} \cos\theta \quad \ldots(15)$$

which becomes zero when $\theta = 90°$. Thus X experiences a couple

tending to twist its plane into that of Y, so that the two fields become coincident: as in Art. 6 the small coil X tends to set its face at right angles to the field of Y and to embrace as many magnetic lines as possible.

B—MAGNETIC POTENTIAL OR EQUIVALENT SHELL METHODS

On pages 66-68, 83 it was shown that the magnetic effects of a current circuit can be the same as those of a uniform magnetic shell of a certain strength, the shell having the same boundary as the current circuit: and further, that the e.m. unit of current has been so chosen that the strength ϕ of this equivalent shell is numerically equal to μI, where I is the current in e.m. units, and μ the permeability of the medium. (If the medium be air (or vacuo) ϕ numerically equals I.) This leads to another method of dealing with the preceding facts: brief reference only need be made to them.

8. Work done in Linking a Current

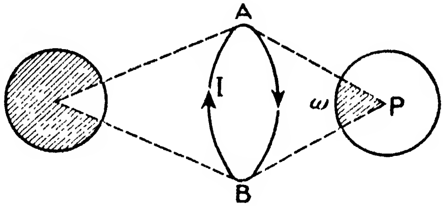


FIG. 306.

Consider the circuit AB (Fig. 306) carrying a current I e.m. units. If it were replaced by its equivalent shell the magnetic potential at P would be $\phi\omega$ where ϕ = strength of the shell and ω = solid angle subtended at P by the shell (page 83): hence the magnetic potential at P due to the current circuit AB = Iω (medium is air).

If P moves up to AB the solid angle ω changes to 2π, and if it moves through the circuit ω changes to 4π (this is perhaps more readily seen if ω is measured by the area intercepted on a sphere of unit radius). If P be brought back again to its original position *without passing through the circuit,* ω will have changed to $(\omega + 4\pi)$: thus the magnetic potential due to the current circuit changes from Iω to I $(\omega + 4\pi)$, *i.e.* by an amount 4πI. But change in potential equals the work done in moving unit pole: hence the work done when unit pole links the circuit once is 4πI ergs, and if it links it n times the work is $4\pi n$I ergs (see Arts. 2, 4).

Note that the magnetic potential at a point *due to a current circuit* may be said to be Iω *or* I $(\omega + 4\pi n)$, where n is any whole number.

9. Magnetic Fields Due to Currents

(1) LINEAR CURRENT (Art. 1).—Let H gauss = field at distance d cm. The work done in moving unit pole once round the wire on the magnetic line of radius $d = \mathrm{H} \times 2\pi d$ ergs. But this work $= 4\pi\mathrm{I}$ ergs, where I = current in the wire in e.m. units. Hence $\mathrm{H} \times 2\pi d = 4\pi\mathrm{I}$;

$$\therefore \mathrm{H} = \frac{2\mathrm{I}}{d} \text{ gauss} \quad \text{or} \quad \mathrm{H} = \frac{2\mathrm{I}}{10d} \text{ gauss.}$$

(2) CIRCULAR CURRENT (Art. 3).—Consider the field at P distance d along the axis of the circular coil AB carrying a current I *e.m.* units (Fig. 289). Replace AB by its equivalent shell of strength ϕ. Now on page 83 it is proved (by change in magnetic potential) that the field at P due to the shell is given by:—

$$\text{Field due to shell} = \frac{2\pi r^2 \phi}{(d^2 + r^2)^{\frac{3}{2}}} \text{ gauss:}$$

hence the field at P due to the current I e.m. units in AB is $2\pi r^2\mathrm{I}/(d^2 + r^2)^{\frac{3}{2}}$ gauss. This must be multiplied by n if there are n turns on the coil and divided by 10 if I is in amperes.

If $d = 0$ the expression becomes $2\pi\mathrm{I}/r$—the field at the centre.

(3) SOLENOID (Art. 4).—The expression for the field of an infinite solenoid may be established by making use of the fact that the work done in linking a current $= 4\pi\mathrm{I}$ ergs: this was referred to in Art. 4 (Fig. 292). By an extension of the method given on page 83, *i.e.* by estimating the rate of variation of the magnetic potential along the axis, the field of a finite solenoid can be determined ($\mathrm{H} = - dv/dx$).

(4) FORCE ON A CURRENT IN A MAGNETIC FIELD (Arts. 5, 6, 7). —Consider again the rectangular current carrying loop of Art. 6 with its *face* at an angle θ with the field. Now replace by its equivalent shell of strength ϕ, the magnetic axis of which will therefore make an angle of $(90° - \theta)$ with the field (the angle α of Art. 6). The couple on it is therefore (see page 63) $\mathrm{MH} \times \sin(90 - \theta)$, where M is the magnetic moment and this is therefore the couple on the coil. Now from the definition of the strength of a shell $\mathrm{M} = \phi\mathrm{A}$, where A is the face area, and since $\phi = \mu\mathrm{I}$ we have $\mathrm{M} = \mu\mathrm{IA}$. Hence:—

$$\text{Couple on the coil} = \mu\mathrm{IAH} \sin(90 - \theta) = \mu\mathrm{HIA} \cos\theta,$$

as obtained in Art. 6. If l be the length of AB (Fig. 299) and F the

force on it, then the couple on the coil (Fig. 299) is $F \times Dk = F \times \overline{AD} \cos \theta$: hence:—

$$F \times \overline{AD} \cos \theta = \mu HIA \cos \theta; \quad \therefore \ F = \mu HIl = BIl \text{ dynes,}$$

for $A = l \times AD$. This becomes HIl dynes if the medium be air.

C—CURRENT MEASUREMENT BY THE MAGNETIC EFFECTS

10. Galvanometers

Galvanometers are instruments used for the detection of currents and for the comparison and measurement of current strengths. Commercial *ammeters* for the measurement of current in practical units, and *voltmeters* for the measurement of P.D. often involve the same principles as the galvanometers (with the exception of the electrostatic voltmeter), and the main point of difference between the two is that an ammeter is arranged to have a low resistance and a voltmeter a high resistance: this is dealt with in Art. 20.

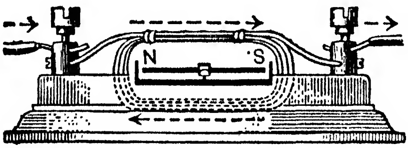


FIG. 307.

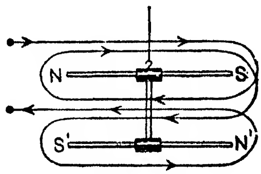


FIG. 308.

One of the simplest galvanometers (Fig. 307) consists of a small magnetic needle pivoted at the centre of a coil of insulated wire. If the coil and needle are in the magnetic meridian and a current be passed, the needle will be deflected from the meridian, and the stronger the current the greater will be the deflection. The "deflecting couple" due to the current which tends to set the magnet at right angles to the meridian is, of course, balanced by the "controlling couple" due to the earth tending to set it in the meridian. The deflection can be read by means of a light pointer attached to the needle and moving over a scale, or by the mirror, lamp, and scale method.

Galvanometers are often wound with two coils, one consisting of few turns the other of many turns of wire. A moderately strong current is passed through the former, but if the current is weak it is passed through the latter, for the greater the number of turns, the greater will be its effect on the needle. The coil of many turns is usually of fine wire to avoid the instrument being very bulky. Sometimes more than two coils are used.

The moving part of a galvanometer tends to oscillate about a mean position when the current is started, stopped, or varied. This is often undesirable, and a **damping** device is frequently introduced to prevent this undue oscillation: the instrument is then said to be **dead-beat.** Damping is secured (1) by having a light vane attached to the moving part, the vane moving in oil and therefore steadying the motion; (2) by having the vane moving in a small air space—often a piston moving in a tube; (3) by induced currents. The last named method is explained later, but briefly the principle is as follows:—When a coil of wire moves in a magnetic field, a *momentary* current is *induced* in it which flows in such a direction that it opposes and "steadies up" the movement of the coil: in the same way if the coil is wound on a metal frame which also moves in the field, currents are induced in the frame which oppose the movement. Similarly, then, if any moving part of an instrument—magnet or coil—has a metal vane (or frame) attached to it, and the vane moves in a magnetic field, induced currents in the vane will steady the movement.

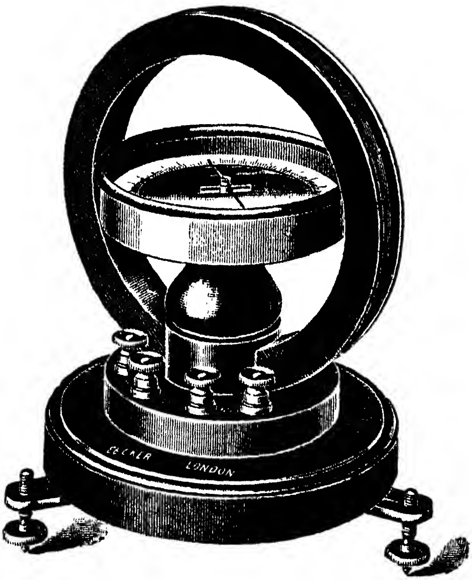


FIG. 309. Three coils, 2, 50, 100 turns.

In the simple galvanometer of Fig. 307 it is clear that the arrangement could be made more sensitive (so that a weak current would give a greater deflection) by lessening the effect of the earth on the moving magnetic needle. This can be done by using an "astatic pair of needles," *i.e.* two magnets of nearly equal moment, fixed parallel with unlike poles adjacent, and with their magnetic axes in the same vertical plane, and in which, therefore, the turning effect of the earth on one is nearly equal but opposite to that on the other. Such is called an *astatic galvanometer.* The earth has little effect on it, so that the "controlling influence" is mainly due to the torsion of the suspension. One method of winding is shown in Fig. 308: the rules of Chapter IX., Art. 3 will show that both needles are urged in the same direction *by the current.*

The **figure of merit** of a galvanometer fitted with a pointer moving over a scale of degrees is usually taken as measured by the current in amperes necessary to produce 1° deflection: in the case of a reflecting galvanometer it is measured by the current (usually in micro-amperes) which will produce 1 mm. deflection on a scale 1 metre from the mirror of the galvanometer (this definition is somewhat modified in modern specifications relating to delicate galvanometers). The **current sensitivity** of a galvanometer is usually measured by the deflection in millimetres on a scale 1 metre away produced by a current of 1 micro-ampere. The **voltage sensitivity** is measured by the deflection in millimetres on a scale 1 metre away when the voltage applied is 1 micro-volt.

11. The Tangent Galvanometer

This is a laboratory and test-room instrument, and a simple form is shown in Fig. 309. It consists of a circular coil of a few turns of insulated wire with a *small* magnetic needle pivoted or suspended at the centre. A light aluminium pointer attached to the needle enables the deflection to be read on a horizontal scale graduated in degrees. The needle is very small, so that even when deflected its ends may be regarded as not moving out of the small uniform field area at the centre of the coil. In working with the instrument, the coil is first set in the magnetic meridian so that the needle and coil are in the same vertical plane; the current to be measured is then passed and the angle of deflection read off from the scale.

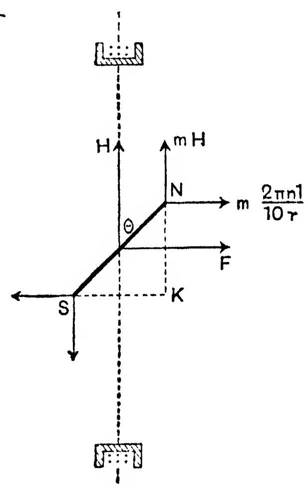


FIG. 310.

Now the needle is under the influence of two magnetic fields which are at right angles, viz. the earth's field H acting in the meridian, and the field F due to the current acting at right angles to the plane of the coil (Fig. 310). If θ be the angle of deflection of the needle from the meridian we have as on page 72, $F = H \tan \theta$. But F, the field due to the current, is $2\pi n I/10r$, where I is the current *in amperes*, *n* the number of turns of wire in the galvanometer coil, and *r* the coil radius in centimetres: hence—

$$\frac{2\pi n I}{10r} = H \tan \theta; \quad \therefore\ I = 10H \frac{r}{2\pi n} \tan \theta \quad \ldots\ldots (16)$$

The factor $2\pi n/r$ which depends on size and turns (it is numerically the field at the centre due to unit e.m. current in the coil) is a constant for the galvanometer: it is the same wherever the instrument is used, and is known as the **coil constant.** Denoting it by G we have

$$I = 10\frac{H}{G}\tan\theta \text{ amperes} \quad \ldots\ldots\ldots\ldots\ldots(17)$$

The factor H/G is called the **reduction factor,** and it depends on the value of H, *i.e.* it varies with the place where the instrument is used; denoting it by K—

$$I = 10K\tan\theta \text{ amperes} = K\tan\theta \text{ e.m. units} \ldots\ldots\ldots(18)$$

(*a*) Note that sometimes $r/2\pi n$ is referred to as the galvanometer constant.

(*b*) As an alternative "proof" of the formula we can write (using m for the pole strength of the needle):—

$$\text{Deflecting couple} = \text{Controlling couple.}$$

$$m.\frac{2\pi nI}{10r} \times NK = mH \times SK;$$

$$\therefore \frac{2\pi nI}{10r} = H \times \frac{SK}{NK} = H\tan\theta,\ i.e.\ I = 10H\frac{r}{2\pi n}\tan\theta,$$

$$i.e.\ I = 10K\tan\theta \text{ amperes.}$$

(*c*) To eliminate the error mentioned on page 98 read *both* ends of the pointer and then *reverse the current* and again read both ends; the mean of the four readings gives the value of θ.

(*d*) When possible it is advisable to obtain deflections in the vicinity of 45°, as in practice the accuracy in reading is then at a maximum, a given variation in the current producing its greatest effect in this region. Thus, if $d\theta$ be a small increase in the deflection produced by a small increase dI in the current, we have, using e.m. units:—

$$I = K\tan\theta; \quad \therefore \frac{dI}{d\theta} = K\sec^2\theta;$$

$$\therefore \frac{dI}{I} = \frac{\sec^2\theta}{\tan\theta}.d\theta = \frac{2}{\sin 2\theta}d\theta.$$

Now dI/I is the relative change in the current, and for this to be *as small as possible for a given value of* $d\theta$, the factor $2/\sin 2\theta$ must be as small as possible, *i.e.* $\sin 2\theta$ must be as large as possible; this is so when $2\theta = 90°$, *i.e.* $\theta = 45°$.

The reduction factor K for a galvanometer is found by passing a suitable *known* current through it, noting the deflection, and working out the value of K from the relation $K = I/10\tan\theta$. A usual laboratory method is to arrange the galvanometer, a copper sulphate voltameter, a battery, and a rheostat in series, pass a steady current for two minutes or so, and get the amount of copper deposited. From this the current I is calculated (page 283) and the value of K determined. We thus know K for all future measurements with the galvanometer used at that place.

Example.—*A tangent galvanometer consists of* 10 *turns of* 22 *cm. radius, and a current of* ·21 *ampere is passed through it at a place where the earth's field is* ·18 *unit. Find* (1) *the magnetic field at the centre of the coil,* (2) *the constant of the coil,* (3) *the reduction factor of the galvanometer,* (4) *the deflection. What current in amperes would give a deflection of* 30°?

$$I = 10H\frac{r}{2\pi n}\tan\theta.$$

(*a*) Field at centre $= \frac{2\pi n I}{10r} = \frac{2 \times 3{\cdot}14 \times 10 \times {\cdot}21}{10 \times 22} = {\cdot}06$ gauss (approx.)

(*b*) Constant $= \frac{2\pi n}{r} = \frac{2 \times 3{\cdot}14 \times 10}{22} = 2{\cdot}85.$

(*c*) Reduction Factor $= K = H\frac{r}{2\pi n} = \frac{{\cdot}18}{2{\cdot}85} = {\cdot}063.$

(*d*) Deflection $I = 10K\tan\theta$; $\therefore \tan\theta = \frac{I}{10K} = \frac{{\cdot}21}{10 \times {\cdot}063} = {\cdot}3333$;

$\therefore$ θ (from tables) $= 18°\ 30'$ approx.

(*e*) Current required $= 10K\tan 30° = 10 \times {\cdot}063 \times \frac{1}{\sqrt{3}} = {\cdot}36$ amp.

12. The Helmholtz Tangent Galvanometer

From (6) of Art. 3 it follows that the field due to unit current (e.m.) in the coil is $2\pi n r^2/(d^2 + r^2)^{\frac{3}{2}}$, and from this the variation of the field along the axis may be graphically represented by working out the value of the field for different values of *d*. The curve in Fig. 311 depicts this variation, the abscissae denoting distances along the axis and the ordinates values of the field. The curve is at first concave towards O, the centre of the coil, but the curvature becomes less and less and soon changes sign, the curve becoming convex towards O.

The point of inflection or change of curvature is at the point where $d = \frac{1}{2}r$, and at this point the curve is, for a short length, practically a straight line. If, therefore, we have two equal coils placed with their axes coincident and *at a distance apart equal to the radius of either*, then, for the same direction along the common axis, the rate of increase of the field due to one coil at a point midway between the two coils (*i.e.* $r/2$ from each) is equal to the rate of decrease of the field due to the other coil at the same point, and the field for a fair

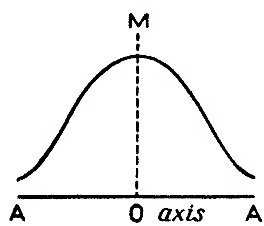


FIG. 311.

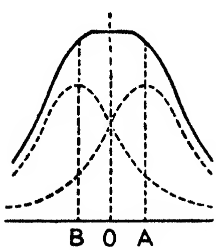


FIG. 312.

distance on each side of this point will be practically uniform. This is shown graphically in Fig. 312. The dotted curves show the fields for the separate coils, and the full line curve, the resultant of the two dotted curves, represents the resultant field due to the two coils; the horizontal part indicates uniformity of field. Fig. 313 shows the field of force between the coils and clearly indicates that, in the middle of the field, there is a region of comparatively fair extent where the field due to the two coils is uniform. For a detailed mathematical proof of this see *Advanced Textbook of Electricity and Magnetism.* The fact is utilised in the Helmholtz galvanometer.

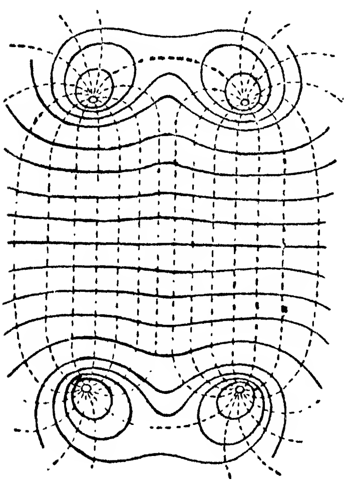


FIG. 313.

The galvanometer consists of two equal coils arranged, as referred to above, at a distance apart equal to the radius r of the coils (Fig. 314). The needle is suspended midway between them (*i.e.* at distance $r/2$ from each) where the field is uniform. The current to be measured passes through both coils and in the same direction, so that their fields at the mid-point are in the same direction. With the galvanometer arranged in the meridian as in Art. 11 we have $F = H \tan \theta$, where F = field due to the coils and θ = the deflection. But from (6), Art. 3, F is clearly given by:—

$$F = 2\left\{\frac{2\pi n r^2 I}{((r/2)^2 + r^2)^{\frac{3}{2}}}\right\} = \frac{32\pi n I}{5\sqrt{5}\cdot r}$$

where I = *current in e.m. units.* Substituting in the above:—

$$\frac{32\pi n I}{5\sqrt{5}\cdot r} = H \tan \theta;$$

$$\therefore\ I = \frac{5\sqrt{5}\cdot r}{32\pi n} H \tan \theta \text{ (e.m. units)} = \frac{50\sqrt{5}r}{32\pi n} H \tan \theta \text{ (amp.)} \quad \text{(19)}$$

13. The Sine Galvanometer

This is an instrument similar in principle and construction to the tangent galvanometer of Art. 11. It differs in the fact that the

coil and needle box can be rotated round a central vertical axis, and a horizontal circular scale is provided on which the amount of this rotation can be accurately read (Fig. 315).

For use the instrument is adjusted in the same way as the tangent galvanometer, but when the needle is deflected the coil is rotated after it until the needle is overtaken by the coil and *in its deflected position* lies in the plane of the coil. The diagram of Fig. 316 shows the conditions of equilibrium of the needle, and as before:—

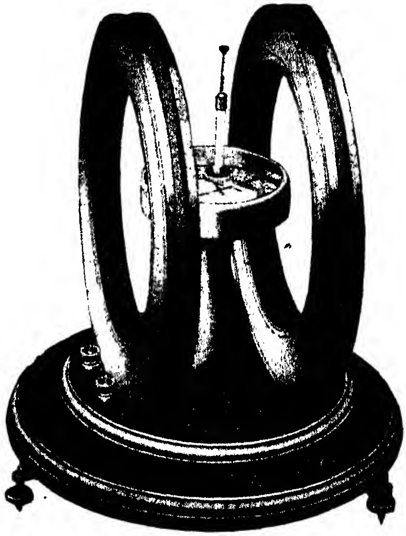


FIG. 314.

Deflecting couple
= Controlling couple;

$\therefore$ $m\text{F} \times \text{NS} = m\text{H} \times \text{NK}$ or $\text{F} = \text{H} \times \text{NK}/\text{NS} = \text{H} \sin \theta$,

where θ = angle the coil is turned through. But $\text{F} = 2\pi n\text{I}/10r$:—

$$\therefore \frac{2\pi n\text{I}}{10r} = \text{H} \sin \theta; \quad \therefore \text{I} = 10\,\text{H}\frac{r}{2\pi n} \sin \theta,$$

i.e. $\text{I} = 10\text{K} \sin \theta$ (amperes)

or $\text{I} = \text{K} \sin \theta$ (e.m. units),

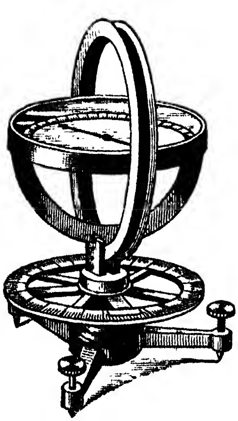


FIG. 315.

where K is, as before, the reduction factor of the galvanometer.

Since $\text{I} = \text{K} \sin \theta$ we have $d\text{I}/d\theta = \text{K} \cos \theta$: hence (cf. Art. 11):—

$$\frac{d\text{I}}{\text{I}} = \frac{\text{K} \cos \theta d\theta}{\text{K} \sin \theta} = \frac{d\theta}{\tan \theta},$$

and it follows that for $d\text{I}/\text{I}$ to be as small as possible, $\tan \theta$ would have to be as great as possible, *i.e.* θ would be 90°. Further, since $\text{I} = \text{K} \sin \theta$ and the maximum value of $\sin \theta = 1$ ($\theta = 90°$) the galvanometer cannot be used to measure currents greater than K e.m. units (10K amperes).

14. Current Method of Finding the Intensity of the Earth's Field

It was stated on page 116 in dealing with the determination of the strength of a magnetic field that such determinations could be done with great accuracy by electrical methods. With the tangent galvanometer, for example, the earth's horizontal field H could be calculated from the relation $I = H(r/2\pi n) \tan \theta$ provided the current I e.m. units could be measured by some other means. There are, however, certain slight defects in this simple method, and improvements have been made by Schuster, Bates, and others.

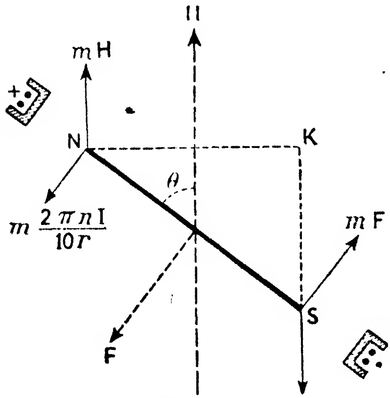


FIG. 316.

The Schuster method employs a galvanometer of the Helmholtz pattern, but in which the coils can be rotated about a vertical axis (after the style of the sine galvanometer). The galvanometer is set up in the usual way, the known current I e.m. units is passed, and the needle is deflected. The coils are then rotated in the same direction as the deflection: as the coils approach the needle it is deflected more, and this is continued until the total deflection of the needle from the magnetic meridian is 90° and the angle $\theta°$ through which the coils have been rotated is noted.

Now (Fig. 317) the couple on the magnet due to the earth's field H is MH, where M is its magnetic moment. The couple due to the field of the coils is $mF \times NK = mF \times NS \sin \theta = MF \sin \theta$, where F is the field due to the coils: hence:—

$$MH = MF \sin \theta; \quad \therefore H = F \sin \theta,$$

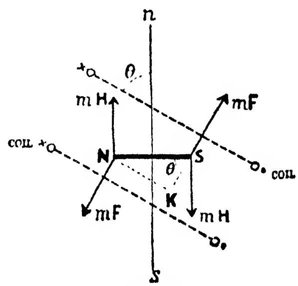


FIG. 317.

$$i.e. \text{ (Art. 12) } H = \frac{32\pi n I}{5\sqrt{5}r} \sin \theta \text{ gauss} = \frac{32\pi n I}{50\sqrt{5}r} \sin \theta \text{ gauss},$$

the latter applying if the current I be in amperes. If the strength of the current be so regulated that θ itself is very nearly 90° any

small error made in reading θ has very little effect on the value of sin θ: thus H is determined with a fairly high degree of accuracy.

Bates also used the Helmholtz principle, but his method is a null method, *i.e.* he found the current necessary to produce a field equal and opposite to the earth's field.

15. Moving Magnet Reflecting Galvanometers

A very simple galvanometer of this type is shown in Fig. 318. It consists of a wooden base and vertical pillar carrying at the top the bobbin B on which the coil is wound. There are two separate coils, one consisting of a few turns of wire, the other of many turns: the ends of one coil are joined to the terminals *a* and *b* and the ends of the other to *b* and *c*, so that either coil (or both) can be used. At the centre of the coil is suspended by a silk fibre a small mirror M carrying at the back three small magnets arranged parallel with *like* poles adjacent. The bobbin is often closed by a glass window in front. The short suspension and confined air space combine in making the instrument to a certain extent "dead beat." In working the galvanometer it is set with the plane of the coil in the magnetic meridian: on passing a current the magnets and mirror are deflected, and the deflection is read by the lamp and scale method. If the actual angle of deflection of the magnets is small we can take the law of the galvanometer to be $I = kD$, where D is the deflection of the image along the scale and k a constant.

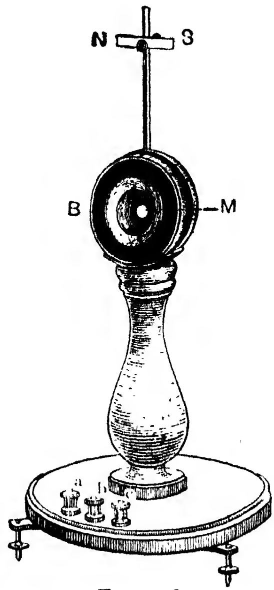


FIG. 318.

On a vertical support above the bobbin is a magnet NS: this is known as the "control magnet," and is for the purpose of altering the sensitivity of the galvanometer. Consider the galvanometer with the coil, needles, and control magnet in the meridian, the control having its north pole northwards. Clearly the earth and control are exerting *opposing* influences on the needles, so that the magnetic field in which the needles hang is weak, and the moving system will be readily deflected by a small current in the coils. This case is shown in Fig. 319. Lowering the control will still further increase the sensitiveness, and finally a position will be found in which the earth's field is

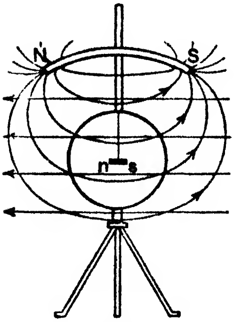


FIG. 319.

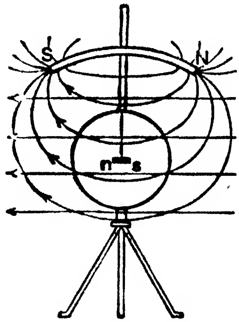


FIG. 320.

cancelled, and the galvanometer extremely sensitive. Raising the control will *decrease the sensitivity*, and this will be more marked if the control be set with its south pole northwards, in which position it will *assist* the earth and create a strong control field in the vicinity of the needles (Fig. 320).

A much more elaborate type of galvanometer than that of Fig. 318 is shown in Fig. 321, in which two sets of needles are used arranged in "astatic order." The figure should explain itself: it is really an outline sketch of *Kelvin's astatic galvanometer.*

In the *Broca galvanometer* an astatic arrangement of magnets is also used, but the magnets are vertical (Fig. 322). These magnets N*ss*N and S*nn*S have poles *ss* and *nn* at their mid-parts, and the coil is arranged with these at its centre.

It will be noted that in these instruments *which are designed for great sensitiveness* the tangent law, *i.e.* that the current is proportional to the tangent of the deflection, does not hold, for the coil is not large compared with the magnetic needle. As already stated when the actual deflection of the magnet is small, the current is proportional to the deflection, and the law of the instrument may be taken as I $= k$D (D $=$ scale deflection, k $=$ constant). If larger deflections are to be used the galvanometers must be calibrated. This is done by passing various *known* currents, noting the deflections on the scale and plotting a "calibration curve," say the current values on the horizontal and deflections vertically. This curve could then be used to find the actual current producing any particular deflection. It

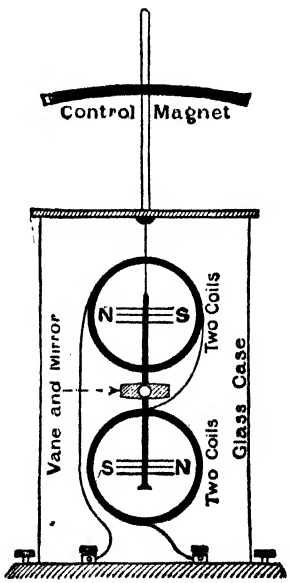


FIG. 321.

is a tedious business if it is to be done *accurately*, and manufacturers usually supply details by which we can "measure" a current when it is necessary. (Much experimental work, however, only involves "comparisons" of currents or uses "null" (no deflection) methods.)

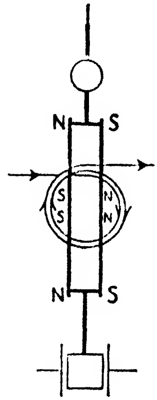


FIG. 322.

Many reflecting moving magnet galvanometers are so sensitive that currents of the small order of one-third *micro-ampere* will deflect the spot of light along 50 cm. of a scale one metre distant, and some are *much more* sensitive than this.

Reference has been made to the fact that galvanometers are often fitted with shields ("screens") of iron to protect the moving parts from external magnetic fields. Fig. 323 will indicate how thoroughly this magnetic shielding is done in some delicate galvanometers. The figure shows the shields used in the *Paschen galvanometer* made by the Cambridge Instrument Co. A is the inner shield of mumetal: this is surrounded by B of mild steel: finally C is a gun-metal outer case.

16. Moving Coil Instruments

Moving coil galvanometers make use of the fact that a current-carrying wire moves (if it is free to do so) when it is suitably placed in a magnetic field (Art. 5). In Fig. 324 N S are the poles of a permanent horse-shoe magnet. A rectangular coil C consisting of a number of turns of fine insulated wire is suspended between these poles by wires or phosphor bronze strips, which also serve to conduct the current to the coil. The lower end is attached to a small spring fixed at the bottom, and the upper suspension to a torsion head at the top of the instrument; in this way the coil is maintained in its

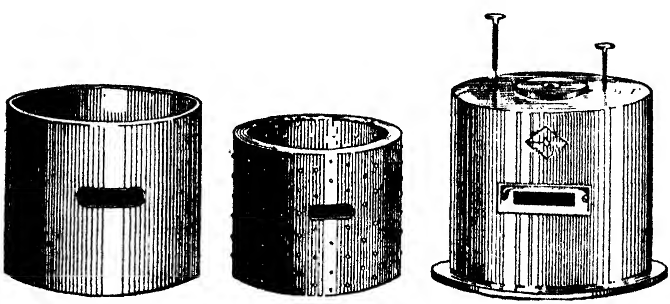


FIG. 323.

normal position, with its plane along the lines of force of the field between the magnet poles (as in the left-hand figure). A cylindrical piece of soft iron I is supported within the coil, and by concentrating the lines results in an intense field in the space at each side in which the vertical wires of the coil move.

Suppose the arrows in Fig. 324 indicate the direction of the (conventional) current: the face of the coil which is towards the reader is a north face. Hence the coil turns, endeavouring to set its plane at right angles to the field with this north face towards the south pole of the magnet so as to embrace as many negative lines as possible (Art. 6): that is the vertical wires on the left move in the direction "out of the paper," and those on the right into it, and this motion is, of course, resisted by the torsion on the suspension and the spring. Further, Fleming's left-hand rule applied to the vertical wires shows that these move as indicated. Deflections are read by the lamp and scale method.

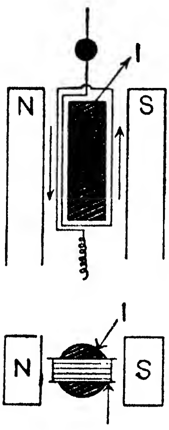


FIG. 324.

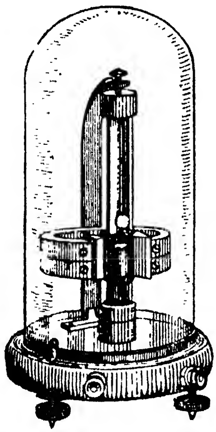


FIG. 325.

Fig. 325 shows a simple type of moving coil galvanometer. The magnet is fitted with soft-iron pole pieces which are "hollowed out." (See below.) The coil is wound on a light rectangular brass frame and suspended by phosphor-bronze: it is the twist on this which provides the *controlling* couple that balances the *deflecting* couple when current passes. Note again the cylindrical iron core. To the frame is suspended a light vane which moves in an oil chamber at the bottom, thus tending to make the instrument dead-beat. Further, when the coil and its brass frame move in the magnetic field, currents are induced in the frame, and these oppose the movement and make the instrument dead-beat. A mirror is attached just above the coil and the deflection is read by the lamp and scale method.

The law of the instrument may be developed as follows:—Let n = number of turns on the coil, A = area of each turn, I = current in coil in e.m. units, H = intensity of the magnetic field, k = torsion

constant for the suspension. Then assuming the field uniform and $\mu = 1$, we have, by Art. 6 (12):—

$$\text{Deflecting couple} = \text{H}n\text{IA} \cos \theta,$$

where θ is the angle of deflection (angle between coil face and field). This is balanced by the controlling couple $k\theta$. Hence:—

$$\text{H}n\text{IA} \cos \theta = k\theta; \quad \therefore \text{I} = \frac{k}{n\text{AH}} \cdot \frac{\theta}{\cos \theta} \quad \ldots\ldots\ldots\ldots (20)$$

This is the simple case of a coil suspended in the field in which case the deflection is not proportional to the current. In practice, however, a **radial field** is used, *i.e.* the pole pieces are *cylindrical* in shape and a *cylindrical* iron core is used, giving a narrow air gap where the magnetic lines are *radial* and the field strong and practically uniform (Fig. 326). The deflecting couple (if deflection does not exceed about 45°) does not therefore depend on the position of the coil, and the equation above becomes simplified to

$$\text{H}n\text{IA} = k\theta;$$

$$\therefore \text{I} = \frac{k}{n\text{AH}}\theta \text{ e.m. units.} \quad (21)$$

and we can therefore write for the law of the instrument $\text{I} = \text{K}\theta$. One result of the construction referred to above is that commercial forms of the instrument have evenly divided scales.

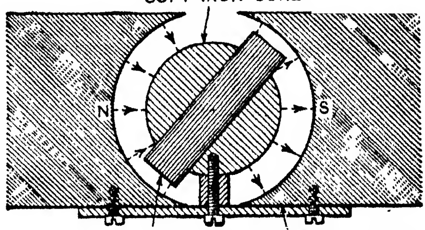


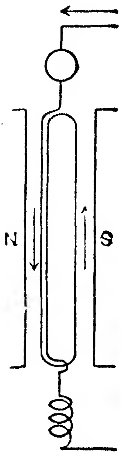
FIG. 326. (See also FIG. 328.)

Note that in the galvanometer of Art. 15, for example, the deflection was greater the *weaker* the earth's field, but of course *that field was the control field.* In the present case the *deflecting* force on a wire is proportional both to the current in it and the field, and thus the stronger the magnet's field the greater the deflection. ($\theta = \text{I}n\text{AH}/k$, *i.e.* H is in the numerator.)

A somewhat different type of moving coil galvanometer is shown in Fig. 327 (*Ayrton and Mather type*). The magnet is nearly a complete cylinder—a very narrow air gap only being between the poles. A narrow rectangular coil is used, the coil being mounted in a thin silver tube which moves with it: thus the damping (induced currents in the silver tube) is very efficient. The very narrow air gap renders a central iron core unnecessary.

Moving coil galvanometers are often fitted with pointers which move over a scale graduated to read the current strength direct,

and of course moving coil ammeters and voltmeters are almost entirely of the pointer type. The construction and action of one of these will be gathered from Fig. 328. The permanent magnet has two soft iron pole-pieces PP screwed to its ends. The inner faces of these pole-pieces are cylindrical in shape, and in the cylindrical space between them is fixed the cylindrical soft iron core I, this core being held rigidly in position by being attached to a plate B of non-magnetic material. The moving coil C embracing the core is wound on a light rectangular frame of aluminium and pivoted in jewelled bearings situated above and below the centres of the ends of the core. Current is led to the coil *via* a flat helical spring *s*, and led from the coil by a similar spring at the other end of the core: these springs are coiled in opposite directions and they furnish the controlling couple which balances the deflecting couple. The coil spindle carries a pointer which moves over the scale.



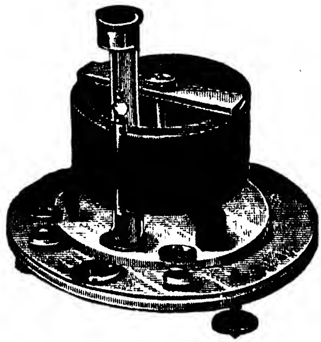


FIG. 327.

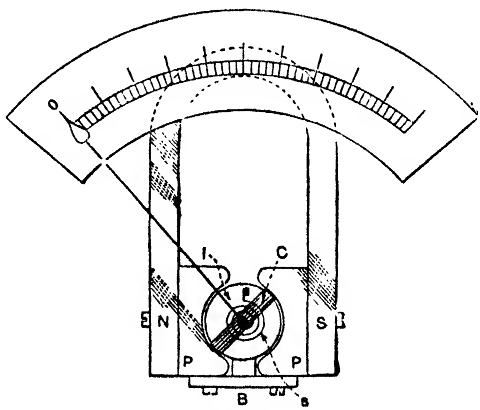
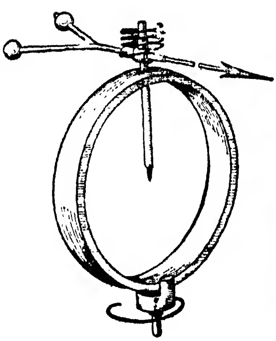


FIG. 328.

Fig. 329 shows another moving coil pointer instrument–the *Cambridge Unipivot Galvanometer*—one of the pole-pieces and the horse-shoe limbs of the magnet being removed to show the working parts. It consists of a circular coil which swings round a spherical core of soft iron in between the pole pieces. The core is

drilled down to the centre, and the coil carries a spindle which goes down into this opening, the bottom of the spindle resting in a jewel cup at the centre of the sphere. A spring at the top of the coil supplies the controlling force. The current is led into the coil by this spring, and out of the coil by a flexible connexion at the bottom.

Since the direction of deflection, as in the case of the moving magnet galvanometer, depends on the direction of the current, these instruments can only be used for direct currents, for an alternating current would tend to move the coil first one way, then the other, and these opposite impulses would occur so rapidly that the coil would not move, or only quiver. It may be noted, however, that owing to the special advantages of moving coil instruments, it is quite common practice now to use them in alternating current work, although the instruments themselves can only be used with direct current. This is done by using rectifiers, as they are called, in conjunction



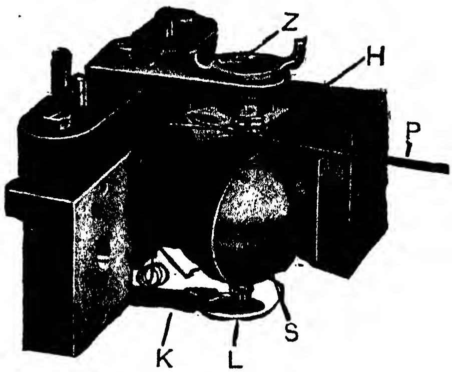


FIG. 329. S = Spherical core; H = Control spring; L = Phosphor bronze ligament to lead current from coil; Z = Zero adjusting device; P = Pointer.

with them: these are so arranged that although the current in the circuit is alternating it always passes through the instrument in one direction.

The advantages of the moving coil galvanometer are (*a*) It is practically independent of the earth's magnetic field since no sensitive magnetic needle is used. (*b*) The field in which the coil moves is so strong that other external magnetic fields do not affect the readings *to any appreciable extent.* (*c*) It is remarkably " dead beat " in its action. (*d*) It may be set up in any convenient position. (*e*) The deflection is directly proportional to the current, and the evenly divided scale is a great advantage in practice.

17. Moving Iron Instruments

These instruments fall into two classes referred to as " attracted iron " and " repulsion iron " instruments respectively. Fig. 330 shows a very simple pattern of the former type. M is a solenoid

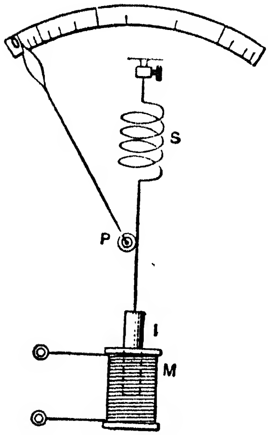


FIG. 330.

through which the current to be measured is passed. I is a soft iron rod or "plunger" which, in its normal position, hangs partly inserted in M. The suspension goes round and thus actuates a pulley P mounted on a horizontal axle, and is attached to a spring S at the top of the instrument. When the current passes I is magnetised by induction and drawn down into M, thus causing P to rotate and the pointer moves over the scale. The deflecting force due to the current is balanced by the controlling force due to the tension of the spring.

Fig. 331 shows a modern and largely used type. A soft iron disc I is pivoted eccentrically just outside a short coil C of oval section. When current passes I rotates into C thereby moving the pointer over the scale. The controlling couple is provided by springs (not shown), and damping is effected by means of a vane V attached to the moving system and moving in the air damping box B.

Fig. 332 will explain the principle of a simple type of repulsion instrument. S is the end view of a solenoid arranged with its

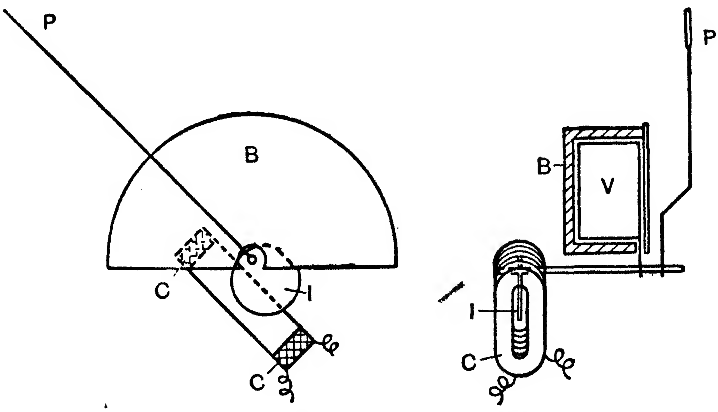


FIG. 331. Attracted Iron Instrument.
I, Moving Iron. C, Coil. V, Damping Vane. B, Damping Box.
P, Pointer. When current passes I rotates into C and deflects P.

axis horizontal. F is a *fixed* soft iron rod lying along the solenoid almost from end to end of it. M is another rod of soft iron lying along the solenoid: it, however, is *movable*, being attached by the curved arm *a* to the pivoted horizontal axis X which carries the pointer. When the current to be measured passes through S, say in the direction indicated by the arrow, the rods F and M are both magnetised, their near ends being both north poles and their far ends both south poles. They therefore repel each other, so that M moves away from F thereby rotating the axle and moving the pointer over the scale. The instrument is "gravity controlled," for when a deflection occurs M is raised against the gravitational pull of the earth.

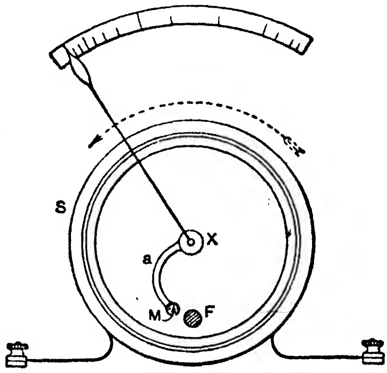


FIG. 332.

It is clear that in these instruments the deflection will be in the same direction whatever the direction of the current in the coil, so that they can be used with alternating currents. Moreover the intensity of the induced magnetisation will be proportional to the field, and the force approximately proportional to the square of the coil current: hence they can be used to *measure* an A.C. as will be understood later (Chapter XVIII.).

Moving iron instruments are usually fitted in an iron case or provided with an iron shield to screen the parts from external magnetic fields. In some types the graduations in the lower parts of the scales are very crowded, but in the best modern makes the use of special nickel-iron alloys combined with special coil design has greatly eliminated this defect.

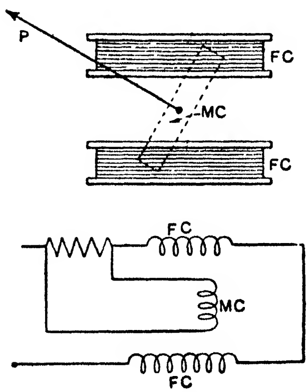


FIG. 333.

18. Dynamometer Instruments

(1) LABORATORY AND COMMERCIAL TYPES.—Dynamometer instruments make use of the facts dealt with in Art. 7, viz. that currents flowing in the same direction attract, but if in opposite directions they repel, and the *general principle* will be gathered from Fig. 333.

Here FC are two fixed coils and MC a movable coil pivoted so that it can rotate inside the fixed coils. MC carries the pointer. The FC coils are in series and the current to be measured passes through them, whilst a *small* fraction of the current passes through MC. The MC coil is so connected that the attractions and repulsions between the currents in MC and FC deflect MC so that the pointer moves clockwise over the scale. The instruments are usually spring controlled. They can be used for alternating current, for when the current reverses it does so in both sets of coils and the deflection is in the same direction.

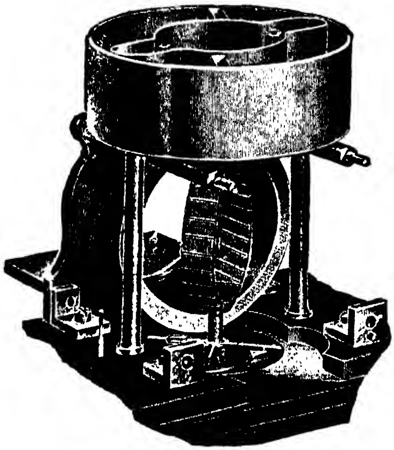


FIG. 334.

Fig. 334 shows an actual instrument pictured standing on its face and with one fixed coil removed to show the working parts. The moving coil carries a light vane fitted with two pistons which move with small clearance in an air-damper box. The latter is shown in the top of the figure: its cover is removed, and the pistons visible.

(2) WEBER'S ELECTRODYNAMOMETER.—This was an early type of dynamometer instrument. It consists of a small coil *in series with* a larger fixed coil and suspended at the centre of the latter by a bifilar suspension. The current is led to and from the small coil by means of the suspension. When no current flows the *axis* of the small coil is in the meridian and at right angles to the axis of the large coil (Fig. 335*a*). When a current flows the small coil tends to set itself coaxially with the large one (page 328), and this tendency is opposed by the action of the earth (if the moving coil has its north face northwards) and the bifilar suspension. The usual method of working will, however, be gathered from the following:—

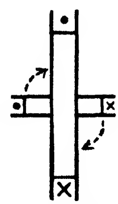


FIG. 335*a*.

Let G be the constant of the large fixed coil, I the current (e.m.), A the face area of the small movable coil (total area of *all* the turns), K the constant

of the bifilar suspension, H the earth's horizontal component, and θ_1 the deflection of the small coil. The deflecting couple on the small coil is given by (15) of Art. 7 which, in this case, is $GI^2A \cos \theta_1$ (for G corresponds to $2\pi I_1/r_1$, whilst $A = \pi r_2^2$ and $I = I_1 = I_2$). The controlling couple due to the earth is $AIH \sin \theta_1$ (for the moment M of the small coil $= AI$) The controlling couple due to the suspension $= K \sin \theta_1$.

Hence:—

$$GI^2A \cos \theta_1 = AIH \sin \theta_1 + K \sin \theta_1;$$

$$\therefore \tan \theta_1 = \frac{GI^2A}{AIH + K}$$

$$\therefore \tan \theta_1 = \frac{GI^2A}{K} - \frac{I^3HGA^2}{K^2} \dots\dots\dots (a)$$

Now reverse the current and let θ_2 be the deflection. The deflecting couple and the controlling couple due to the bifilar will act as before, but the couple due to the earth will be reversed; hence

$$GI^2A \cos \theta_2 = -AIH \sin \theta_2 + K \sin \theta_2;$$

$$\therefore \tan \theta_2 = \frac{GI^2A}{K} + \frac{I^3HGA^2}{K^2} \dots (b)$$

Adding (a) and (b):—

$$I^2 = \frac{K}{2GA} (\tan \theta_1 + \tan \theta_2);$$

$$\therefore I = k\sqrt{\tan \theta_1 + \tan \theta_2}.$$

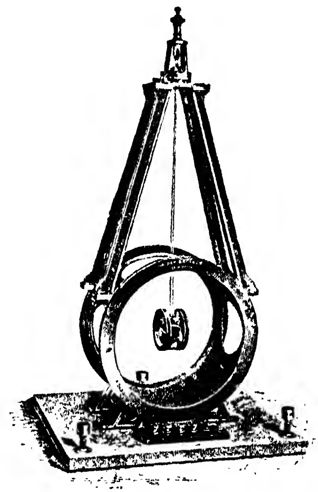


FIG. 335*b*.

By passing and reversing a known current k can be determined. In the actual Weber instrument there are usually two large coils arranged as in the Helmholtz galvanometer (Fig. 335*b*).

(3) THE KELVIN CURRENT BALANCES.—The general principle of these will be gathered from Fig. 336. In them there are four fixed coils, P, Q, R, S, placed horizontally, and between these there are two movable ones, A, B, arranged at opposite ends of a light balance beam, the latter being hung at its centre by a number of very fine copper wires, which also serve to lead the current to and from the movable coils. With current passing as indicated it is clear that the beam will tilt upwards on the right, the left-hand side falling. A horizontal scale is fixed to the beam, and by means of a suitable sliding

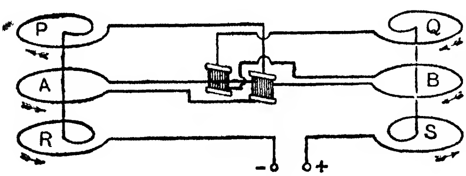


FIG. 336.

weight, which can be moved along the scale by means of cords and a corresponding counterpoise placed in a trough attached to B, the beam can be brought to its initial position and the current calculated from the position of the sliding weight and the known constants of the instrument. Each sliding weight has its own particular counterpoise, the latter being so constructed that it keeps the beam horizontal when no current is passing and the sliding weight is at zero on the scale. Clearly, if the sliding weight be at distance d from zero when the beam is brought into the horizontal, the restoring couple is proportional to d; the couple due to the current is proportional to I^2; hence

$$I^2 \propto d, \quad i.e.\ I \propto \sqrt{d}; \quad \therefore\ I = K\sqrt{d}.$$

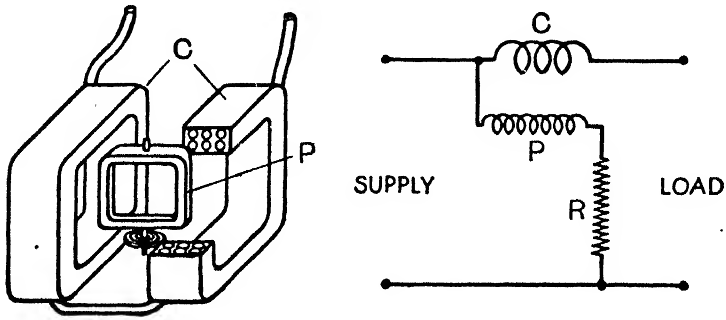


FIG. 337. C, Current coil; P, Pressure coil; R, Resistance (non-inductive).

There are many forms of these balances, all alike in principle but differing slightly in construction according to their purpose. They are not much used in ordinary practice but are valuable as standard instruments: the ampere balance mentioned in the legal definition of the ampere (page 293) is a special type of the Kelvin current balance.

(4) WATTMETERS.—These indicate the rate in watts (watts = volts × amperes) at which energy is being utilised in any part of a circuit, *i.e.* they measure the " power." Wattmeters are not much used on D.C. circuits, for the watts can be obtained by multiplying together the ammeter and voltmeter readings. On A.C. circuits, however, where voltage and current are constantly changing and are *out of phase* (p. 508) they are necessary if *true* power is required.

Wattmeters are usually of the dynamometer type, somewhat similar to the instrument described in (1) above. In the wattmeter (Fig. 337) the two fixed coils C are in series, are put in the main circuit, carry the main current, and are referred to as the " current

coils": the moving coil P, instead of being in series with the C coils as in the ammeter (Fig. 333), is joined *across* the mains (just as a voltmeter is), and it has a *high* (*non-inductive*) resistance R in series with it: it is called the " pressure coil." Since C carries the main current and P a current proportional to the voltage between the mains, and further since the deflection depends on the product of the currents in the two sets of coils, it is clear that the deflection depends on the product of the main current and the P.D. between the mains, *i.e.* it depends on the watts: the scale is graduated to read the power direct in watts or kilowatts. These wattmeters can be used for A.C. as well as D.C.

19. Ballistic Galvanometers

A ballistic galvanometer may be either of the moving magnet or moving coil (usually reflecting) type, but its moving system must have a large moment of inertia. It is used for measuring the *quantity* of electricity passed through it not as a continuous current but as a sudden discharge (*e.g.* such as are met with in condenser discharges and in induced current work—see Chapter XVI.). The moving system has a large moment of inertia, so that it may be slow in beginning to move under the impulse of the sudden discharge and will therefore not have moved appreciably from its position of rest during the time the discharge takes to pass through the galvanometer: in this case the moving coil or magnet receives an impulse due to the whole charge. Further, in these instruments damping must be as small as possible, and for what exists a correction must be made.

For the *moving magnet type* of ballistic galvanometer it is a matter of fairly simple proof that—

$$Q = \frac{Ht}{\pi G} \sin \frac{\alpha}{2} \text{ or } Q = K_1 \sin \frac{\alpha}{2} \quad \ldots\ldots\ldots\ldots (22)$$

where H is the strength of the control field, t the time of swing of the needle, G the constant of the galvanometer coil, α the *first* angular deflection of the needle, and Q the total quantity of electricity discharged through the galvanometer.

For the *moving coil type* of ballistic it can be shown that—

$$Q = \frac{ct}{2\pi HA} \alpha \text{ or } Q = K_2\alpha \quad \ldots\ldots\ldots\ldots (23)$$

where A is effective face area of coil, c a constant for the suspension. In this case the quantity is proportional to the first angular swing

α and not to the sine of half the first angular swing as in the first case. Further, here H is the field to which the deflections are due and it appears in the denominator; in the previous case H is the controlling field and it appears in the numerator.

Note particularly that in working with these galvanometers we do not read a steady permanent deflection, but the *first* angular swing α. Note also that if the galvanometer is a moving coil one, any frame on which the coil is wound must be of *non-conducting* material to avoid damping induced currents in it.

The full proof of these relations would take us beyond the scope of this book (see *Advanced Textbook of Electricity and Magnetism*), but for the benefit of the more mathematically minded student we briefly indicate the proof in the first case above (it may be omitted of course): the second is established similarly.

Let K be the moment of inertia of the magnet and M its magnetic moment. If the current at any instant has the value I, then IG is the strength of the deflecting field, and, if the needle is supposed to be inappreciably deflected from its position of rest during the discharge, IGM is the moment of the couple tending to deflect the needle.

The angular acceleration due to this couple is IGM/K and, therefore, during the very short time, δt, for which the current has the value I, the gain of angular velocity is (IGM/K) δt. During the whole discharge, therefore, the angular velocity imparted to the needle is given by Σ (IGM/K) δt, the summation being for the whole time of the discharge. But:—

$$\Sigma \frac{\mathrm{IGM}}{\mathrm{K}} \delta t = \frac{\mathrm{GM}}{\mathrm{K}} \Sigma \mathrm{I} . \delta t \text{ and } \Sigma \mathrm{I} . \delta t = \mathrm{Q},$$

where Q denotes the total quantity of electricity discharged through the galvanometer. Hence ω, the final angular velocity of the needle, is given by

$$\omega = \frac{\mathrm{MGQ}}{\mathrm{K}} \text{ or } \mathrm{K}\omega = \mathrm{MGQ} \quad \ldots\ldots\ldots\ldots (a)$$

The kinetic energy of the needle is given by $\frac{1}{2}\mathrm{K}\omega^2$, and this must be equal to the work done against the couple due to the control field H during the deflection throw of the needle. Let α be the angular deflection. Then:—

$$\tfrac{1}{2}\mathrm{K}\omega^2 = \mathrm{MH}\,(1 - \cos\alpha);$$

$$\therefore\ \mathrm{K}\omega^2 = 4\mathrm{MH}\sin^2\frac{\alpha}{2} \quad \ldots\ldots\ldots\ldots (b)$$

for MH $(1 - \cos\alpha)$ is the work done in deflecting the small needle through the angle α in the field H (page 66). Now if t = time of swing of needle:—

$$t = 2\pi\sqrt{\frac{\mathrm{K}}{\mathrm{MH}}}; \quad \therefore\ \mathrm{K} = \frac{\mathrm{MH}t^2}{4\pi^2} \quad \ldots\ldots\ldots\ldots (c)$$

Combining the relations (b) and (c) we get:—

$$(\mathrm{K}\omega)^2 = \frac{\mathrm{M}^2\mathrm{H}^2t^2\sin^2\frac{1}{2}\alpha}{\pi^2} \text{ or } \mathrm{K}\omega = \frac{\mathrm{MH}t}{\pi}\sin\frac{\alpha}{2}.$$

Substituting the value of $K\omega$ given by (a) in this, we get

$$MGQ = \frac{MHt}{\pi} \sin \frac{a}{2} \text{ or } Q = \frac{Ht}{\pi G} \sin \frac{a}{2}.$$

20. Shunts and Shunting

To lessen the sensitiveness of a galvanometer or to prevent damage to the instrument, it is often necessary to send only a portion of the current through it: this is done by joining its terminals by a wire termed a "shunt" (Fig. 338a).

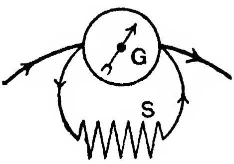


FIG. 338a.

Let I be the total current, I_g the portion in the galvanometer, I_s the portion in the shunt, G the resistance of the galvanometer, and S the resistance of the shunt. Then:—

$$\frac{I_g}{I_s} = \frac{S}{G}; \quad \therefore \frac{I_g}{I_s + I_g} = \frac{S}{G + S}: \quad \text{But } I_g + I_s = I;$$

$$\therefore I_g = \frac{S}{G + S} I \quad \text{and} \quad I = \frac{G + S}{S} I_g,$$

i.e. Current in galvanometer $= \frac{S}{G + S}$ of total current.

Total current $= \frac{G + S}{S}$ times the galvanometer current.

The factor $(G + S)/S$ by which the galvanometer current has to be multiplied to get the total current is called the **multiplying power of the shunt.** Thus to send 1/10 of a current through a galvanometer, the multiplying power of the shunt must be 10: hence $(G + S)/S = 10$; $\therefore$ $G = 9S$ or $S = \frac{1}{9}G$: the shunt resistance would have to be, therefore, $\frac{1}{9}$ of the galvanometer resistance. To send $\frac{1}{100}$ of the current through the galvanometer the required multiplying power must be 100: putting $(G + S)/S = 100$ we get the required shunt resistance as $\frac{1}{99}$ of the galvanometer resistance. Similarly, if $S = \frac{1}{999}G$ the multiplying power is 1000. Some galvanometers are supplied with shunt boxes fitted with three coils which give these three multiplying powers 10, 100, and 1000. Shunt boxes fitted with coils to give as many as 20 different multiplying powers can be supplied in some cases. Manufacturers can frequently supply separate shunts of various values with any particular galvanometer together with the necessary "factors" to apply to the scale reading in order to convert the

deflection to the correct total current value according to the shunt being used. These accessories are referred to as *range multipliers*.

Example.—*A circuit contains a battery* (1 *ohm*), *reflecting galvanometer* (4 *ohms*), *and other resistances* (2 *ohms*), *and the deflection is* 100 *divisions. The galvanometer is then shunted with a* 4 *ohm coil: what will be the deflection assuming it proportional to current? What is the multiplying power?*

Case 1. If I = current in the galvanometer:—

$$I = \frac{E}{1 + 4 + 2} = \frac{E}{7} \text{ amperes.}$$

Case 2. The joint resistance of galvanometer and shunt is $(4 \times 4)/(4 + 4) = 2$ ohms, and the *total* current therefore is $E/(1 + 2 + 2) = E/5$ amperes. If I_2 be the current in the galvanometer now:—

$$I_2 = \frac{S}{G + S} \text{ of total current} = \frac{4}{4 + 4} \times \frac{E}{5} = \frac{E}{10} \text{ amperes,}$$

so that if d be the deflection of the galvanometer:—

$$\frac{d}{100} = \frac{I_2}{I_1} = \frac{E}{10} \div \frac{E}{7} = \frac{7}{10}; \quad \therefore d = 70 \text{ divisions.}$$

$$\text{Multiplying power} = \frac{G + S}{S} = \frac{4 + 4}{4} = 2.$$

Note that the insertion of a shunt really reduces the resistance of the circuit by an amount $G - GS/(G + S)$, *i.e.* by an amount $G - G/m$, where m = multiplying power: this measures the additional resistance which must be put in the main circuit to keep the total current the same as before.

The *principle* of a "universal" shunt, which may be employed with *any* galvanometer, will be understood from Fig. 338 (*b*). With the lever on stud 1 the whole of the shunt coils are employed, and the current in the galvanometer is:—

$$\frac{10000}{G + 10000} \text{ of the total current,}$$

When the lever is moved to stud 10 the 9,000-ohm coil is inserted in the galvanometer branch and the shunt employed is of resistance 1,000 ohms; the galvanometer current is now

$$\frac{1000}{(G + 9000) + 1000}, \; i.e. \; \frac{1000}{G + 10000} \text{ of the total current,}$$

or $\frac{1}{10}$ of the galvanometer current in the first case. Turning the switch to stud 100 will result in a galvanometer current $\frac{1}{100}$ of the first and so on, and clearly this is true whatever may be the value of G.

It has been stated that an ammeter is a *low resistance* instrument. In practice it is put *in* the circuit through which the current to be measured is passing (Fig. 339), and if its resistance were high it would *reduce* the strength of the current to be measured and, even more important, would cause a big "voltage drop" on itself ($V = IR$) and waste much power (watts $= I^2R$). Ammeters are therefore invariably fitted with *shunts*. Such shunts make the

total resistance low, and enable large currents to be measured as only a safe part goes through the instrument.

A voltmeter, on the other hand, is a *high resistance* instrument. In practice it is put *in parallel* with the circuit the P.D. on which is to be measured (Fig. 339), and it is easily seen that its resistance must be high. Consider a coil of 3 ohms resistance in a circuit through which a *constant current* of 10 amperes is maintained; the P.D. at its ends will be 30 volts. Imagine now that a voltmeter of only 1 ohm resistance is joined across the coil; the combined resistance of the two in parallel is $(3 \times 1)/(3 + 1)$, *i.e.* $\frac{3}{4}$ ohm, and the P.D. at the ends of the coil is reduced from 30 volts to $(10 \times \frac{3}{4})$, *i.e.* $7\frac{1}{2}$ volts, whereas, had the resistance of the instrument been high—say 3000 ohms—the P.D., as the reader will readily perceive, would have been practically unaltered by its introduction. Again, consider the above instruments joined across mains kept at a

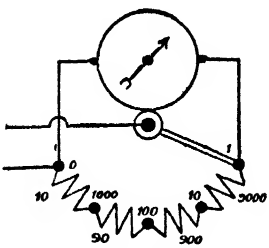


FIG. 338*b*.

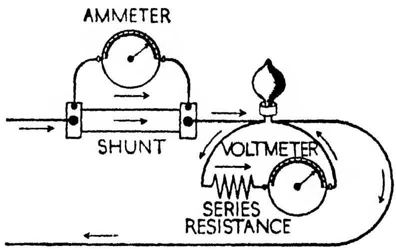


FIG. 339.

constant P.D. of 100 volts. The power wasted in the first instrument is 10,000 watts (V^2/R), and in the second $3\frac{1}{3}$ watts; thus, to render the power consumption as small as possible, the resistance of the voltmeter must clearly have a high value. Voltmeters are therefore often fitted with *series resistances* which not only ensure a high resistance, but enable large P.D.s to be dealt with. Thus if the series resistance was three times the voltmeter resistance, the two could be used for a P.D. four times that for the instrument alone, for three-quarters of the total P.D. would be "used" on the series resistance and only the safe voltage on the instrument. Expressed in general terms if V_1 be the P.D. for the voltmeter, R_1 its resistance, R_2 the value of the series resistance, and V_2 the total P.D. to be measured,

$$\frac{V_2}{V_1} = \frac{R_2 + R_1}{R_1}; \quad \therefore \ V_2 = \frac{R_2 + R_1}{R_1} V_1.$$

Examples.—(1) *An ammeter of resistance 5 ohms gives a full scale deflection when the current is ·03 ampere. What resistance must a shunt have to enable this ammeter to measure current up to 2 amperes?*

First Method. Since ·03 ampere is the greatest current which can be allowed to go through the ammeter and the greatest current in the problem is 2 amperes, we must use a shunt which will take the balance of current, viz. 2 − ·03 = 1·97 ampere. Now:—

$$\frac{\text{Curr. in ammeter}}{\text{Curr. in shunt}} = \frac{\text{Res. of shunt}}{\text{Res. of ammeter}}; \quad \therefore \frac{\cdot 03}{1\cdot 97} = \frac{S}{5}; \quad \therefore S = \cdot 076,$$

so that the shunt must have a resistance of ·076 ohm.

Second Method. When ·03 ampere is going through the ammeter the P.D. at its terminals is given by V = IR, *i.e.* it is ·03 × 5 = ·15 volt, and this is also the P.D. at the ends of the shunt. Hence the shunt resistance (R = V/I) is ·15/1·97 = ·076 ohm.

Third Method. When ·03 ampere goes through the ammeter the total current in the circuit is 2 amperes: hence the *multiplying power* of the shunt must be 2/·03. But the multiplying power is (G + S)/S;

$$\therefore \frac{G + S}{S} = \frac{2}{\cdot 03}, \ i.e.\ 2S = \cdot 15 + \cdot 03S; \quad \therefore S = \cdot 076 \text{ ohm}.$$

(2) *With a P.D. at its terminals of 75 millivolts an instrument gives a full scale deflection and passes a current of 15 milliamperes. What series resistance must be inserted if the instrument is to be used as a voltmeter reading up to 15 volts?*

Resistance of instrument = ·075/·015 = 5 ohms.

The maximum current for the instrument is ·015 ampere, so that for an applied P.D. of 15 volts the *total* resistance must be 15/·015 = 1000 ohms. The instrument resistance is 5 ohms. Hence a series resistance of 1000 − 5 = 995 ohms must be used. Or again:—

$$\frac{V_2}{V_1} = \frac{R_2 + R_1}{R_1}; \quad \therefore \frac{15}{\cdot 075} = \frac{R_2 + 5}{5};$$

$$\therefore \cdot 075\, R_2 + \cdot 375 = 75; \quad \therefore R_2 = 995 \text{ ohms}.$$

CHAPTER XII

ELECTRICITY AND HEAT

THE unit of heat generally employed in scientific work—the calorie—was defined on page 278. Now heat is a form of energy, and can therefore be measured in terms of the usual energy units, ergs, joules, foot-pounds, etc., so that the calorie is equivalent to a definite number of each of these units. Joule of Manchester and others determined the relationship, and their results indicated that 1 calorie $= 4{\cdot}19 \times 10^7$ ergs $= 4{\cdot}19$ joules. Later work by Callendar and Barnes gives $4{\cdot}184 \times 10^7$ ergs (at 15° C.). The number of units of energy which is equivalent to one calorie is referred to as *the mechanical equivalent of heat* (J): we will assume the value $4{\cdot}18 \times 10^7$ ergs, *i.e.* 4·18 joules.

1. The Laws of Heating Effects of a Current

(1) BY EXPERIMENT.—On page 278 it was shown by experiment that if a current I amperes flows through a conducting wire of resistance R ohms for t seconds, the heat produced is proportional to the square of the current, the resistance, and the time, so that if H calories be the heat:—

$$H \text{ is proportional to } I^2Rt; \quad \therefore H \text{ (calories)} = kI^2Rt,$$

where k is a constant still to be determined. Further, since $R = V/I$, where V is the P.D in volts at the ends of the resistance:—

$$H \text{ (calories)} = kI^2Rt$$
$$= kVIt = kV^2t/R.$$

To find k the calorimeter of Fig. 269 may be used.

Join up the apparatus as shown in Fig. 340, where A is an ammeter to measure the current and V a voltmeter joined across the coil to give the P.D. at its ends. The mass of water (say m grammes) in the calorimeter must be known and so must the ***water equivalent*** (say

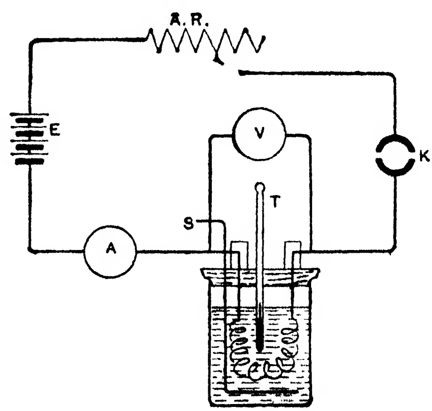


FIG. 340.

w grammes) of the calorimeter apparatus (calorimeter, stirrer, coil, thermometer): the latter is determined by a separate experiment. Pass the current (keeping it constant) for a definite time, t seconds, and note the rise in temperature (θ° C.), the current (I amperes), and the P.D. (V volts). The total heat produced (say H calories) is given by $(m + w)\ \theta^\circ$. But $H = kVIt$; $\therefore\ k = H/VIt$, i.e. $k = (m + w)\ \theta^\circ/VIt$, so that k is determined.

There are several defects in the above simple determination. The coil resistance varies with the temperature, so that it is difficult to keep *both* I and V constant: it is not easy to find the water equivalent of the *whole* apparatus *accurately*: there are heat losses which cannot be accurately estimated. These defects can be largely overcome by using Callendar and Barnes *continuous flow calorimeter* (see *Textbook on Heat*—Archer). Accurate determination gives the value of k as ·24. Thus when a current of I amperes flows for t seconds between two points of a conductor where the P.D. between the points is V volts and the resistance R ohms, the heat produced in calories (H) is given by the expressions:—

$$\text{Heat in calories} = H = \cdot 24 VIt = \cdot 24 I^2Rt = \cdot 24 \frac{V^2}{R} t \quad \ldots .(1)$$

and any of these may be taken as a statement of what is called **Joule's Law** of the heating effects of currents: it is usual to take the second, viz. $H = \cdot 24 I^2Rt$.

Again, since one volt $= 10^8$ e.m. units, we have 1 e.m. unit $= 1/10^8$ volt: hence V e.m. units $= V/10^8$ volts. Similarly I e.m. current units $= 10I$ amperes. *Thus if* V *and* I *be in e.m. units:*—

$$H = \cdot 24 \times (V/10^8) \times 10I \times t = VIt/(4\cdot 18 \times 10^7)$$

since $\cdot 24 = 1/4\cdot 18$. Hence when a current I e.m. units flows for t seconds between two points of a conductor where the P.D. between the points is V e.m. units and the resistance R e.m. units, the heat produced in calories (H) is:—

$$H = \frac{VIt}{4\cdot 18 \times 10^7} = \frac{I^2Rt}{4\cdot 18 \times 10^7} = \frac{V^2t}{(4\cdot 18 \times 10^7)\ R} \quad \ldots . (2)$$

(2) BY CONSIDERATIONS OF ENERGY.—We have seen that whenever a P.D. exists between two points in a circuit an *energy transformation* occurs between them. In the case of a battery the poles of which are joined by an ordinary conductor, the whole of the energy supplied by the battery is transformed into heat partly in the battery and partly in the conductor. Should the external circuit contain a voltameter or motor, a certain amount of the energy is utilised in chemical or mechanical work, the balance again

appearing, however, as heat in the circuit. The transformation of electrical energy into heat always takes place when a current flows through a conductor, and from a consideration of this energy the preceding laws of the heating effect can be deduced. We shall, in this case, deal first with absolute e.m. units.

Consider a current-carrying wire; let V denote the P.D. and I the current, both in e.m. units, and let t denote the time in seconds the current flows. Then (page 285) V = energy (in ergs) appearing as heat when unit e.m. current flows for one second: $\therefore$ VI = energy (in ergs) appearing as heat when I e.m. units flow for one second: and V$\mathrm{I}t$ = energy (in ergs) appearing as heat when I e.m. units flow for t seconds.

Now the mechanical equivalent of heat is $(4{\cdot}18 \times 10^7)$ ergs per calorie; hence if H be the heat in calories in the case above, $(4{\cdot}18 \times 10^7)$ H ergs will represent the total energy appearing as heat in the conductor. Clearly, then,

$$(4{\cdot}18 \times 10^7)\,\mathrm{H} = \mathrm{VI}t; \quad \therefore \; \mathrm{H} = \mathrm{VI}t/(4{\cdot}18 \times 10^7),$$

so that if V, I, and R are in e.m. units and t in seconds the heat (calories) is given by the three expressions in (2) above.

Again, from our definition of the volt we saw (page 285) that if the P.D. in the above case were V volts, the current I amperes, and the time of flow t seconds, the work done or energy transformed would be V$\mathrm{I}t$ joules, and in this case all the energy appears as heat. The mechanical equivalent of heat is 4·18 joules per calorie, so that if H calories be the heat, 4·18 H is the equivalent joules. Clearly:—

$$4{\cdot}18\,\mathrm{H} = \mathrm{VI}t; \quad \therefore \; \mathrm{H} = \mathrm{VI}t/4{\cdot}18, \; i.e. \; \mathrm{H} = {\cdot}24\,\mathrm{VI}t,$$

since $1/4{\cdot}18 = {\cdot}24$. Hence with V, I, and R in volts, amperes, and ohms, and t in seconds the heat (calories) is given by (1) above.

Note that in each case—practical or e.m. units—we can write, taking the second expression in illustration:—

$$\text{Heat in calories} = \mathrm{H} = \frac{\mathrm{I}^2\mathrm{R}t}{\mathrm{J}}$$

where J = mechanical equivalent of heat = $4{\cdot}18 \times 10^7$ (ergs per calorie) if I and R are in e.m. units, and = 4·18 (joules per calorie) if I and R are in practical units: t = seconds.

2. Principle of Least Heat

It should be noted that *when a current divides into branch circuits* (e.g. *wires in parallel*) *it does so in such a way that the heat produced is a minimum.* This may be shown for a simple case as follows.

Consider two wires of resistances r_1 and r_2 ohms in parallel: let the *total* current be I amperes, and suppose I_1 is the portion in r_1 and I_2 that in r_2. Then:—

$$H = \text{Heat produced} = \cdot 24\,(I_1^2 r_1 t + I_2^2 r_2 t),$$

$$i.e.\ H = \cdot 24\,\{I_1^2 r_1 t + (I - I_1)^2 r_2 t\}.$$

$$\text{If this has a minimum value } dH/dI_1 = 0;$$

$$\therefore\ \cdot 24\,\{2I_1 r_1 t + (-1)\,2\,(I - I_1)\,r_2 t\} = 0;$$

$$\therefore\ \cdot 48 I_1 r_1 t = \cdot 48\,(I - I_1)\,r_2 t,\ i.e.\ I_1 r_1 = (I - I_1)\,r_2;$$

$$\therefore\ \frac{I_1}{I - I_1} = \frac{r_2}{r_1},\ i.e.\ \frac{I_1}{I_2} = \frac{r_2}{r_1}.$$

Thus for least rate of heat production the current must divide inversely as the resistances, and this is how it *does* divide.

3. Work Done in the External Circuit other than Heating

First consider a battery with its poles joined by a simple conducting wire. To make our treatment definite we will assume practical units: and to avoid the frequent writing of t we will assume $t = 1$ second. Let the battery E.M.F. be E volts, the *total* resistance (external + internal) R ohms, and the current I amperes. By the action of the battery a quantity of electricity I coulombs is, *in one second*, raised in potential to the extent E volts, and therefore the energy *supplied* by the battery per second is EI joules (*i.e.* the *power* is EI watts). The battery supplies this energy per second at the expense of the chemical energy of the materials consumed in it: as already stated the chemical action in the battery is such that the products of the action possess less chemical energy than the materials from which they are produced, and the difference is the electrical energy supplied.

In the simple case considered this energy is all transformed into heat in the circuit (some outside, some inside), and the energy appearing as heat is I^2R joules per second. Hence $EI = I^2R$; $\therefore\ I = E/R$, and $E = IR$.

Now suppose the external circuit contains a voltameter or motor so that some of the energy is devoted to other work—chemical or mechanical—in this outside appliance. The appliance has a certain resistance, so that we will now call the *total* resistance R_1 and the current I_1. Let w joules be the energy devoted to this *additional* work every second. $I_1^2R_1$ joules per second appear as

heat in the circuit. EI_1 joules per second are produced by the battery. Hence:—

$$EI_1 = I_1^2R_1 + w; \quad \therefore \; E = I_1R_1 + \frac{w}{I_1} \quad \ldots\ldots\ldots (3)$$

But I_1R_1 the product of the *total* current and the *total* resistance in the circuit is, by Ohm's law, the **effective E.M.F.** in the circuit: denoting this by E' we have:—

$$E = E' + w/I_1 = E' + e; \quad \therefore \; E' = E - e,$$

where we are writing e for w/I_1. Thus **in cases where there is work other than the generation of heat,** the effective or resultant E.M.F. (E') is less than the actual E.M.F. (E) of the battery by an amount e, where $e = w/I_1$; in other words, **there is a back E.M.F., e,** and the actual current (I_1) is given by the expression

$$I_1 = \frac{E - e}{R_1} \text{ or } I = \frac{E - e}{R} \quad \ldots\ldots\ldots\ldots\ldots (4)$$

if now we write the usual I and R for total current and resistance. If $w = 0$, then $e = 0$ and $I = E/R$.

Again, since $e = w/I$ we have $w = eI$; that is *the energy in joules expended per second (or the power in watts) in additional work,* i.e. *other than the generation of heat, is given by the product of the back E.M.F. and the current strength.*

Example.—*A copper sulphate vat has a resistance of ·014 ohm and a polarisation or back E.M.F. of ·3 volt. The total current required for the proper working of the vat is* 1000 *amperes. Find the total watts supplied to the vat.*

Watts spent in heat in the vat $= I^2r = (1000^2 \times \cdot 014)$

Watts expended in chemical work $= eI = (\cdot 3 \times 1000)$;

$\therefore$ Total watts supplied to the vat $= (1000^2 \times \cdot 014) + (\cdot 3 \times 1000)$

$= 14300$ watts.

Further details about the back E.M.F.'s which occur in electrolysis are given in Chapter XIII.

4. Rise in Temperature of Conductors

Joule's law refers to the *heat produced* in a conductor, and though the *rise in temperature* depends on the heat, other factors exert an influence. If equal amounts of heat were given to two wires, A and B, of the same material, A being short and thin, B long and thick, the rise in temperature of A would be greater than that of B, since the mass to be raised in temperature is smaller in A than in B. Again, each wire would be losing heat by

radiation, etc., from its surface; experiment shows that, other things being the same, the greater the surface area the greater is this loss, and, further, the rate of loss of heat is affected by the nature of the surrounding gas and its pressure, and by the character of the surface, black bodies, for example, emitting radiation more freely than white or transparent ones.

Yet another factor, the capacity for heat of the material, exerts an influence; thus, if the same current flows through equal pieces of platinum and copper the heat developed in the former is about seven times that in the latter, since the resistivity of platinum is about seven times that of copper (annealed in both cases), but the *initial rate of rise* in temperature as compared with the copper is greater than would be anticipated, for its capacity for heat is only about three-fifths that of copper.

Consider a wire through which a current is flowing. The production of heat is accompanied by a rise in temperature until finally a steady condition is reached when *the heat lost per second (by radiation, conduction, and convection) is exactly equal to the heat gained per second.* Assuming Newton's Law of Cooling, if T° C. be the elevation of temperature of the wire above the enclosure, the heat lost per second is proportional to T°, and it is proportional also to the surface area of the wire. If d cm. be the diameter of the wire its circumference is πd cm., and if l cm. be its length its surface area is πdl sq. cm.; hence—

$$\text{Heat lost per second} \propto \pi dl\mathrm{T} = a\pi dl\mathrm{T} \text{ calories,}$$

where a denotes the "emissivity" of the material, *i.e.* the heat (in calories) lost in one second from unit area when the temperature difference between the body and the enclosure is 1° C.

Again, if I be the current in amperes and R the resistance of the wire in ohms,

$$\text{Heat gained per second} = \cdot 24\mathrm{I}^2\mathrm{R} \text{ calories;}$$

$$\therefore\ a\pi dl\mathrm{T} = \cdot 24\mathrm{I}^2\mathrm{R},\ i.e.\ a\pi dl\mathrm{T} = \cdot 24\mathrm{I}^2\mathrm{S}l/(\cdot 7854d^2),$$

where S is the resistivity of the material; reducing this we get:—

$$\mathrm{T} = \frac{\cdot 097}{a} \times \frac{\mathrm{SI}^2}{d^3} \qquad (5)$$

Strictly the formula applies only to *bare* conductors. From it we see that: (1) With a given current the elevation of temperature above that of the enclosure does not depend on the length of the wire. (2) For the same current the elevation of temperature is directly proportional to the resistivity and inversely proportional

to the *cube* of the diameter (hence the high temperature of very thin wires). (3) The elevation of temperature is proportional to the square of the current.

(*a*) From the above formula for the temperature *elevation* T° C. we get:—

$$I^2 = \frac{Tad^3}{\cdot 097S}; \quad \therefore\ I = d^{\frac{3}{2}}\sqrt{\frac{Ta}{\cdot 097S}},$$

an expression from which, knowing the values of d, a, and S for any conductor and the maximum temperature (and therefore T°) which it can be allowed to attain, the maximum permissible current for that conductor can be calculated. In practice, however, this formula is only very approximate for other factors have to be considered.

(*b*) In practical electrical work, the term carrying capacity of a conductor refers to the greatest current which can be allowed to flow through it without heating it so much that there is risk of the insulation being damaged, or fire risk. It is clear that a thick wire can safely carry a greater current than a thin one, for the thick one has less resistance and a bigger surface area from which it can lose heat. The kind of insulation on the wire—how *it* conducts heat, and what temperature rise *it* can stand—naturally influences the decision as to what is the " safe current " for a wire. Some years ago current at the rate of 1000 *amperes per square inch* cross-section for insulated copper cable was regarded (in practical work) as a general rule, but to-day the carrying capacity is generally taken to be as given in the carefully estimated tables issued by the Institution of Electrical Engineers. These tables are based on an allowable temperature rise of 20° F. (11·1° C.) for vulcanised rubber-insulated cables, 50° F. (27·7° C.) for impregnated paper insulated cables, and 100° F. (55·5° C.) for bare copper.

(*c*) *Fuses* are wires or strips of material the melting points and dimensions of which are such that when placed in a circuit they melt and break the circuit if the current exceeds a certain allowable value. Tin, lead, alloy of tin and lead, strip zinc, copper, and aluminium are some of the materials used as fuses. For currents up to 20 amperes tin or tin-lead alloy (63 per cent. tin, 37 per cent. lead) are largely used: copper is suitable where large currents are employed. Aluminium is not very satisfactory.

5. The Incandescent Electric Lamp

When a body (of sufficiently high melting point—say a platinum wire) is raised to a high temperature, some of the radiation from it falls within the range we term " light ": this range comprises radiation of wave-lengths from ·00075 mm. (red) to ·00039 mm. (violet), *i.e.* frequencies from, say, 4×10^{14} to 8×10^{14} approximately.* When such a body is heated the first colour a dull red appears at about 525° C. This turns to cherry colour at 800° C.,

$$\text{*Frequency} = \frac{\text{Velocity}}{\text{Wave length}} = \frac{3 \times 10^{10}\ \text{(cm. per sec.)}}{\text{Wave length (cm.)}}.$$

and to a bright cherry at 1000° C. Bright orange appears at 1200° C., whitish at 1300° C., and dazzling white at 1500° C. and above.

To use such a device as a fine wire heated by a current for lighting purposes, the first condition then is that the material must have a high melting point to withstand the high temperature necessary for a good light to be produced: one reason that platinum does not make a practical lamp is that the temperature at which it gives out a really good light is too near its melting point. A second condition is that the thin wire must not be in contact with air (or other medium from which it can get oxygen) or it will soon become oxidised and burn out.

The earliest practical incandescent electric lamp consisted of a thin carbon filament in an evacuated glass globe. It was invented practically at the same time by Edison in America and Swan in England (about 1878-1880) but their methods of making the filament differed. The carbon filament lamp only gave a yellowish light, and its power consumption was fairly high ($2\frac{1}{2}$ to 4 watts per candle-power—see below). Another drawback was that the carbon at the high temperature tended, rather readily, to vaporise, which resulted in a weakening of the filament and a black deposit on the glass which cut off light. To-day, the material almost universally used is *tungsten* in the form of a thin filament, and there are two main types of these lamps, viz. the **vacuum lamp** and the **gas-filled lamp**. The melting point of tungsten is about 3600° C.: vacuum lamps are run at about 2100° C., but gas-filled lamps are run at a higher temperature—about 2700° C.

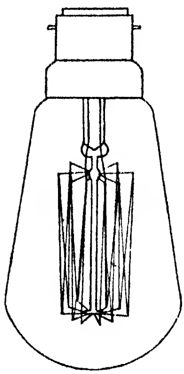


FIG. 341.

A vacuum lamp is shown in Fig. 341. The lead-in wires to the filament consist of an alloy of nickel and iron, thinly copper-plated and with a coating of borate. The expansion of such lead-ins is the same as that of the glass, so that the result is a vacuum-tight joint with no cracking of the glass due to heating and cooling. In operation the tungsten filament disintegrates, thus weakening the filament and producing the " blackening " seen in old lamps. Usually, when a lamp has been in use for about 1000 hours, some 20 per cent. of its light-output will have been cut off. The power-consumption is about 1·1 watts per candle-power. (See below.)

The disintegration difficulty is overcome to a great extent in the gas-filled lamp, in which the bulb is filled with an *inert* gas—

generally argon with traces of nitrogen. The filament does not oxidise for there is no oxygen, whilst the disintegration of the tungsten is much less in the gas than in a vacuum: hence the filament can be run at a higher temperature giving a much better light. Against this, however, there is the fact that in a gas convection currents are set up which carry away heat from the filament thus tending to lower its temperature. This is reduced by having the filament in the form of a **close spiral** suspended from a ring of radial arms in one horizontal plane (Fig. 342): thus only the external surface of the helix is exposed to any extent to the cooling action of the gas. Finally the bulb has a long neck, and the tendency is for any disintegration particles to be carried by the convection currents upwards into the neck, where no useful light is required to pass out. The power consumption of the gas-filled lamp (larger sizes) is about ·5 to ·6 watt per candle-power (see below).

The idea of the coiled filament of the gas-filled lamp is extended in the **coiled-coil filament** where the single spiral is itself coiled into a second spiral (Fig. 343). These lamps, if of the 230 volt type, give 15 to 20 per cent. more light (for the same power consumption): for 115 volt lamps the gain is less.

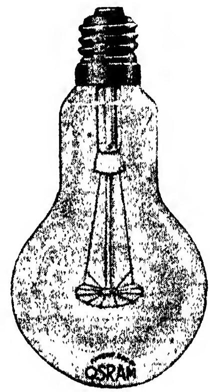


FIG. 342.

The light output of an incandescent lamp varies very much, with small variations of potential difference across the terminals. If a lamp be run at a higher voltage than its normal its light output is certainly much increased, and so is its efficiency (Art. 8), but its "life" is considerably shortened; a 5 per cent. increase in voltage increases the candle-power by about 25 per cent., but it halves the lamp life.

It should be noted that even the best lamps only radiate from 5 to 10 per cent. of their energy in the form of light. Incandescent lamps are generally arranged in parallel on mains kept at a constant P.D.

Use has been made above of the expression "candle-power" in connexion with the emission of light by a lamp. The *intensity* or *luminous intensity* of a lamp in any direction really refers to *the light energy it sends out in one second in that direction* (see Art. 7), and in practice this intensity is estimated by the lamp's power of producing *brightness* in that direction: two lamps, for example, which, when placed in turn at the same distance from a *given screen*,

cause it to appear equally bright, are regarded as having the same luminous intensity in that direction, and are said to be of the same **candle-power** (c.p.). The name *candle-power* originally referred to what was known as the British Standard Candle. This was a sperm candle $\frac{7}{8}$ inch diameter, weighing $\frac{1}{6}$ pound, and burning 120 grains per hour, and it was used as the standard for the comparison and measurement of other light sources, its luminous intensity in any direction being called *one candle-power.*

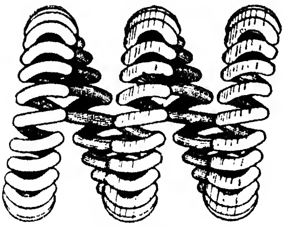


FIG. 343.

A candle of any kind is not an ideal thing to use as a working standard for testing purposes, and substitutes for it were proposed and used, *but these working substitutes were all stated to be, themselves, of so many candle-power.* A standard used in this country is the *Vernon-Harcourt Pentane Lamp*: it burns a mixture of air and pentane vapour and is taken to have an intensity of 10 c.p., so that a lamp is of 1 c.p. if its light intensity is equal to one-tenth of that of the pentane lamp. There were other practical standards, but all these "flame lamps" have now been abandoned as practical *working standards* and an *international candle-power* has been agreed upon between the chief National Standardising Bodies, and the standard lamps now used are certain special tungsten incandescent electric lamps kept at the National Physical Laboratory.

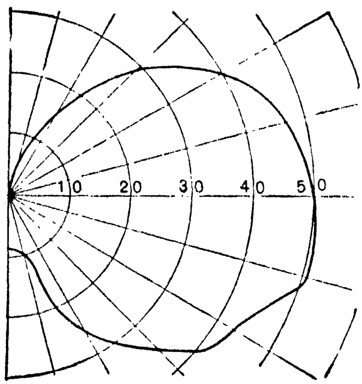


FIG. 344.

A lamp does not give out light equally in all directions: thus a vacuum metal filament lamp sends out light fairly equally in directions at right angles to the axis of the lamp (Fig. 344), but little in the direction of the axis; and it is clear that, owing to the shape and arrangement of the filament of a gas-filled metal lamp, it cannot send out equally in all directions. Strictly, then, in stating in general or comparative terms the candle-powers of lamps, what is called the **mean spherical candle-power** (M.S.C.P.) should be used: this is the mean of the candle-powers in *all* directions from the centre of the light. Curves such as Fig. 344 which give the c.p. in various directions are referred to as *light distribution curves.*

6. Illumination on a Surface

The illumination at a point on a surface is measured by the amount of light (i.e. *luminous energy*) *falling in one second on unit area round about the point.* (Note particularly that it is the amount falling on unit area *per second.*) The laws relating to this are:—(*a*) The illumination on a surface varies directly as the luminous intensity, *i.e.* as the candle-power of the lamp. (*b*) The illumination varies inversely as the square of the distance of the surface from the lamp. (*c*) If the light does not fall normally on the surface the illumination varies directly as the cosine of the angle of incidence (say $\theta°$) of the light. For simplicity in what follows we will assume the light falls normally ($\theta° = 0°$; $\therefore \cos\theta = 1$). It should be noted that law (*b*) is strictly true only for a point source, but it may be assumed true if the source is small compared with its distance: law (*b*) does not apply to a parallel beam of light.

Combining laws (*a*) and (*b*) above, if E = the illumination on a screen due to a source of light of candle-power cp at a distance, say, d ft. (we will use British units here—1 ft. and 1 sq. ft.), the light falling normally, we can write E is proportional to cp/d^2 or $E = k\,(cp/d^2)$ where k is a constant. And if we choose our unit of illumination so that when $cp = 1$ and $d = 1$ ft., the illumination $E = 1$, then k is unity and we have:—

$$E = \text{Illumination on screen} = \frac{cp}{d^2} \text{ units} \quad \dots\dots\dots\dots(6)$$

The British unit of illumination referred to is called the **foot-candle** which is therefore defined as *the illumination produced by a light source of one candle-power at a distance of one foot, the light falling normally.* The international unit (1 metre and 1 sq. metre) is the **lux** which is the illumination produced by a source of one candle-power at a distance of one metre.

Note that the "illumination" on a surface does not depend on the nature of the surface, but the "brightness" does: the latter depends on how much of the incident light the surface reflects, *i.e.* it depends on illumination and coefficient of reflection.

7. Luminous Intensity of a Lamp

The expression "luminous intensity of a source of light in any direction" really refers to *the light energy it emits per second in that direction*: strictly it is the light energy emitted per second along a unit solid angle *in that direction.* The method of measuring solid angles and the meaning of *unit solid angle* were explained on page 81.

In practice, as we have seen, the unit in terms of which we measure the intensities of lights in various directions is an arbitrary one called the **candle-power,** this being the luminous intensity of the British standard candle or $\frac{1}{10}$ of the intensity of the pentane standard lamp. We have also seen (Art. 9) that if E be the illumination on a screen due to a source of light of candle-power (c.p.) at distance d, then $(\text{c.p.})/d^2 = E$, and if $d = 1$ we have $(\text{c.p.}) = E$. Hence we can say that *the luminous intensity of a source of light is represented numerically by the illumination it produces on a screen at unit distance, the light falling normally.*

The International Committee dealing with these matters recommended a few years ago that *the unit for the measurement of the light energy or luminous energy given out by a source of light per second* be taken to be the **lumen,** as it was called. It is usual to call *the total luminous energy* (say Q) emitted by a source of light *in one second* the **flux of light,** and this unit of light flux—the lumen—is defined as the flux of light (light energy *per second*) which falls on unit area of a screen when the illumination is unity. The length unit we are using is the foot (British System) and the illumination unit the foot-candle, so that *a lumen is the light flux which falls on one square foot of a screen when its illumination is one foot-candle.* Put another way, if a standard candle be at the centre of a sphere of 1 ft. radius the flux of light (light energy per second) on each sq. ft. of surface is 1 lumen.

Consider a luminous point P *sending out light equally in all directions,* and imagine a sphere of radius d ft. surrounding P, the latter being at the centre. Let Q be the total light energy given out by P in one second, *i.e.* Q is the *flux of light* in lumens. The uniform illumination (foot-candles) of the sphere is $Q/4\pi d^2$, for this is the luminous energy falling on unit area (1 sq. ft.) of the sphere in one second. But the illumination is c.p./d^2foot-candles:—

$$\therefore \frac{(\text{c.p.})}{d^2} = \frac{Q}{4\pi d^2}, \text{ i.e. c.p.} = \frac{Q}{4\pi}; \quad \therefore Q = 4\pi\,(\text{c.p.}).$$

We thus have that the *total* luminous energy given out per second by a source having an intensity of 1 c.p. in every direction (1 spherical candle) is 4π lumens, and for other light sources

$$\text{Total flux of light} = (4\pi \times \text{M.S.C.P.}) \text{ lumens} \ldots\ldots (7)$$

Methods of determining the luminous intensity (c.p. and lumens) of lamps are dealt with on page 452. Note that the *lumen-hour* is the *total quantity emitted in one hour* when the flux is one lumen.

8. Luminous Efficiency of a Lamp

An expression often met with is the *efficiency* or **luminous efficiency** of a lamp. It is still often defined thus—

$$\text{Luminous efficiency} = \frac{\text{Candle-power of lamp}}{\text{Watts absorbed by lamp}}, \quad \ldots . (8)$$

i.e. as *the candle-power emitted per watt absorbed.* It is not a bad definition although not scientifically perfect, but in modern practice the luminous efficiency is defined thus—

$$\text{Luminous efficiency} = \frac{\text{Lumens output of lamp}}{\text{Watts absorbed by lamp}}, \quad \ldots . (9)$$

i.e. as *the light energy in lumens emitted per watt absorbed.*

Thus taking the power consumption of the gas-filled tungsten lamp to be as given on page 365, viz. say ·6 watt per candle (spherical candle), the luminous efficiency of the lamp would be $1/\cdot 6 = 1\cdot 6$ c.p. per watt $= (4\pi \times 1\cdot 6) = 20\cdot 1$ lumens per watt. General efficiency values may be given as follows:—

Carbon filament: ·22-·36 c.p. per watt = 2·8-4·5 lumens per watt.
Tungsten (Vacuum): ·557-·675 c.p. per watt = 7-8·5 lumens per watt.
Tungsten (Gas): 1·1-2 c.p. per watt = 13·8-25 lumens per watt.

9. Electric Arc Lamps

If two carbon rods in contact, end to end, form part of a *continuous* current circuit at a P.D. of about 50 volts, and if when the current is flowing the carbons be drawn apart to a distance of $\frac{3}{16}$ or $\frac{1}{4}$ inch, a luminous "arc" will be formed between them, the arc forming a conducting path from one carbon to the other. After a time the ends of the carbons become luminous, and more so than the arc; about 85 per cent. of the light is due to the positive carbon, 10 per cent. to the negative, and 5 per cent. to the arc. The positive carbon assumes a crater-like form (Fig. 345), the temperature of which is from 3500° to 4000° C.; the end of the negative becomes gradually pointed, incandescent matter being carried to it from the

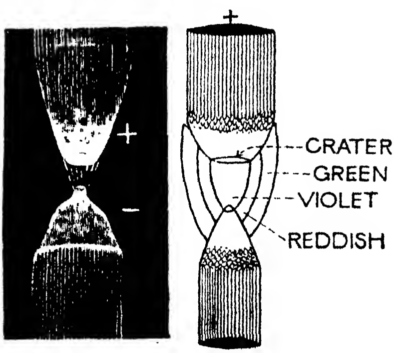


FIG. 345.

positive, and its temperature is about 2500° C. (Arc lamps for *A.C. currents* are also in use: both carbons of these become pointed.)

Modern theory indicates that when the carbons are separated the negative carbon emits a stream of electrons, the space being rendered conducting owing to *ionisation* of the air at the high temperature: conduction is thus effected by electrons moving in the direction negative carbon to positive carbon, and positive ions moving in the opposite direction. The bombarding of the positive carbon by the electrons and the impact of the positive ions at the negative carbon cause considerable heat, but the electrons having a much smaller mass than the ions move with a greater velocity, and produce more heat (and light) at the positive carbon than the ions do at the negative carbon.

The P.D. (V) necessary to maintain an arc was found to be given by a relation of the following type:—

$$V = \left\{ a + bL + \frac{d + eL}{I} \right\} \text{ volts}$$

where a, b, d, e are constants, L the length of the arc in mm., and I the current in amperes. From this it follows that *an increase in the current* I *causes a decrease in the voltage if the length of the arc* L *remains the same.* This curious fact should be noted. It is due to the cross-section of the arc increasing and thus lowering the resistance in a greater ratio than the increase of current.

The c.p. of the arc lamp was high in certain directions (of the order 1000 +) and the power consumption was of the order ·5 to ·8 watt per candle, but complicated devices were necessary to start the arc and keep it at the correct length, and there was the trouble of renewing carbons: thus they are now obsolete as sources of illumination though still used in projection lanterns, searchlights, cinematograph work, arc furnaces, and in arc welding. A modified form—the *flame arc lamp*—has a limited use in lighting.

10. Gas Discharge Lamps

Under ordinary conditions air and other gases are insulators, but when rarefied they become poorer insulators and under certain conditions an electric discharge can be made to pass through them. Moreover, this discharge is accompanied by the emission of light, and thus is utilised in the construction of *electric discharge lamps.*

One of the most familiar is the **neon lamp** or **neon tube** largely used for advertising purposes. They consist of long tubes (sometimes shaped into letters) closed at the ends and with an

iron electrode at each end, and they contain rarefied neon. When joined to an electric supply at high pressure an electric discharge passes through the tube, the whole tube glowing with a rich *red-orange* light. It takes a much greater P.D. to start or " strike " the discharge than it does to keep it going once it is started. Thus a 10 ft. neon tube needs a " striking voltage " of about 3200 volts, but, once started, 1500 volts will keep it going: it is usual, therefore, to run them on A.C. circuits. In addition, the larger the discharge current the lower the voltage across the tube. This behaviour is, of course, similar to that of the ordinary arc, and, as already mentioned, is due to the fact that the discharge is its own conductor: the tube thus has a " negative resistance characteristic," and in order to prevent an excessive flow of current destroying the tube an " inductance " as it is called (termed a choke) is always connected in series with them: this will be explained later (Chapter XVI.).

The light of the neon lamp penetrates fog very well, and they are much used in aerodrome beacons for the guidance of aircraft. They are not suitable for general illumination, partly on account of the colour, and partly on account of the low candle-power per foot run which can be obtained from them. Different shades of colour can be obtained by using different coloured tubes, and such lamps are being used for illumination. Different coloured lights are also obtained by using different gases or a mixture of gases and vapours. Thus mercury vapour gives a distinct blue (it also sends out ultra-violet rays). A usual type employs mercury vapour and neon giving a bluish-white light; if these be used in a yellow uranium glass tube the light is green.

Why the discharge in a rarefied gas is *luminous* has already been briefly referred to, and further details are given in Chapter XIX. The rarefied gas in the tube under the big P.D. is *ionised* (page 130). In the subsequent movement of the positive ions and electrons in the tube numerous collisions will take place with gas particles, in some cases resulting in further electrons being ejected from atoms and in other cases positive ions joining up with electrons and becoming temporarily neutral again. These two processes ionisation and recombination are constantly going on. Now in ionisation work is done, *i.e.* energy is absorbed, and when recombination takes place this energy is given out, *i.e.* there is a *radiation given off*, and some of this radiation comes within the radiation we call light. The " kind " of light (*i.e.* the colour) depends on the " kind " of radiation, and this depends on the " kind " of ions recombining, *i.e.* depends on the nature of the gas: thus recombining mercury vapour ions result in one kind of radiation and therefore one colour, recombining neon ions in another colour, and so on. Thus it is that whilst the radiation from a hot body gives a continuous spectrum, that of a luminous and ionised gas gives a line (or a band) spectrum.

The latest type of discharge lamp is known as the **hot electrode discharge lamp,** which has been developed for lighting purposes. The electrodes of these lamps are rods or filaments which are themselves incandescent. Now a hot filament in a vacuum or rarefied gas ejects electrons itself and these, by collisions, assist in the ionisation of the rarefied gas. In these lamps, then, much larger tube currents are used which results in higher candle-powers and lower striking and working voltages: they are largely used for street lighting, factory lighting, etc. Fig. 346 shows one form—the *Osira lamp* (G.E.C.). There are two cylindrical bulbs, one inside the other, the space between them being a vacuum. The inner bulb contains rarefied gases and a trace of mercury, and the two electrodes are rods of alkaline earth oxides. When the voltage is applied a discharge takes place, and as the bulb warms up the mercury is volatilised. The discharge then contracts into a narrow column stretching from one electrode to the other. The radiation is deficient in red, but the efficiency is from 2 to 3 times that of the gas-filled incandescent lamp. Their voltage (A.C.) is 200-260. Other types are on the market.

FIG. 346.

11. Electric Heating

Electric heaters usually consist of high resistance wire wound in a spiral and mounted on an insulating fire-clay base. It is essential that the wire should not oxidise very much when hot, for if the oxide penetrates deeply the wire burns out. To ensure this wires of special alloys are employed. A favourite one is "Dullray" (its trade name) which consists of about 63 per cent. iron, 34 per cent. nickel, and 3 per cent. chromium: its temperature range is up to 200° C. "Glowray" (20 per cent. iron, 65 per cent. nickel, 15 per cent. chromium) has a range up to 850° C. Other alloys are also used. Some heaters employ rods made of non-metallic compounds with carbon.

Steel hardening, annealing, and enamelling are often done by electrical methods, in "resistance furnaces."

In *resistance welding* the two metals to be welded are clamped together and a very strong current is passed across the contact surfaces. The heat produced raises the junction to a white heat, so that on further increasing

the clamping pressure the two metals are joined together. In *spot welding* (which is supplanting rivetting), the overlapping metals are clamped into close contact, a metal rod is pressed against each side, and a heavy current is passed from one rod to the other. The heat produced causes the plates to join at the contact area in line with the tips of the rods.

In one method of *arc welding* an arc is struck between the material to be welded and an electrode in the form of a rod of the same material, this rod being the anode. The heat of the arc raises the temperature of the part up to a welding heat thus resulting in a " join," and at the same time the end of the rod electrode melts and is deposited in the join. In *arc furnaces* the heat is produced by an electric arc in the furnace: they are used for the extraction of metals from their ores, production of alloys, the fixation of atmospheric nitrogen in the production of soil fertilizers, etc.

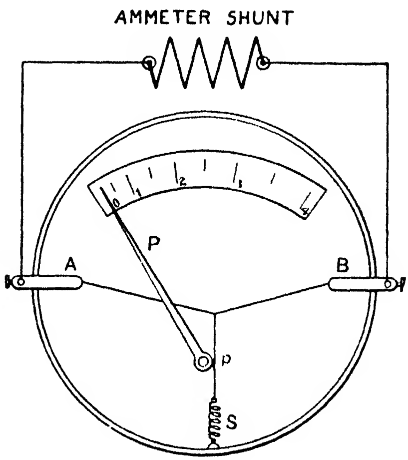


FIG. 347.

Hot-Wire Ammeters and Voltmeters

The essential *principle only* of a well-known type will be gathered from Fig. 347. The current passes through a stretched platinum-silver wire AB. To the middle of this is attached a filament which passes over a pulley *p* and is then fastened to a spring S. The axle of the pulley also carries the pointer P which moves over the scale. When current passes the wire is heated and expands, and the " slack " is immediately taken up by S which pulls the filament, rotates the pulley, and moves the pointer over the scale, which is graduated to give the current in amperes, etc. (or the pressure in volts, etc., as the case may be). Mounted on the axle there is also an aluminium disc (not shown) which, when the pointer is deflected, moves between the poles of a small magnet, thus making the instrument dead beat (induced currents in the disc). The voltmeters are similar to the ammeters but are fitted with series resistances in place of shunts. They can be used for A.C. as well as D.C., for the heat is proportional to the square of the current and does not depend on current direction.

CHAPTER XIII

ELECTRICITY AND CHEMISTRY

SOLUTIONS and substances in the liquid state, except metals, are in general either non-conductors, *e.g.* certain oils, a sugar solution, or they conduct *but at the same time are decomposed* by the passage of the current through them: the latter type are known as *electrolytes* (pages 21, 130, 274), and it is with them we are mainly concerned in this chapter.

1 Faraday's Laws of Electrolysis

Based on the results of many experiments, Faraday in 1833 stated the following laws relating to electrolysis:—

(1) For any given substance, the mass liberated, removed, or deposited at an electrode is proportional to the quantity of electricity which passes.

(2) The mass of any substance liberated, removed, or deposited by a given quantity of electricity is proportional to the chemical equivalent of the substance.

The first law was given on pages 282-4: thus with the usual notation, if w grm. be the mass liberated:—

$$w \propto Q \text{ or } w \propto It \text{ or } w = zIt,$$

where z = a constant = the electro-chemical equivalent.

Coming to the second law, suppose a current be passed through four voltameters in series containing respectively acidulated water and solutions of copper sulphate, silver nitrate, and hydrochloric acid. The atomic weights of oxygen, hydrogen, copper, silver, and chlorine are 16, 1·008, 63·57, 107·88, and 35·457 respectively, and the valencies are 2, 1, 2 (cupric), 1, 1, so that the chemical equivalents are 8, 1·008, 31·78, 107·88, and 35·457. Thus if the current were passed until 1·008 milligrammes, say, of hydrogen were liberated, in the first and fourth voltameters, about 8 mgrm. of oxygen, 31·78 mgrm. of copper, 107·88 mgrm. of silver, and 35·457 mgrm. of chlorine would be liberated in the other voltameters.

Again, the electro-chemical equivalent of hydrogen, *i.e.* the mass of it liberated when 1 coulomb passes, is ·000010446 grm. *One gram-equivalent* of hydrogen, *i.e.* the number of grammes

numerically equal to the chemical equivalent, is 1·008 grm. Hence the number of coulombs which will pass to liberate 1·008 grm. of hydrogen or one gram-equivalent is evidently 1·008/·000010446 = 96496 coulombs. (The figure 96500 is accepted as sufficiently accurate.) By the second law this same quantity will pass to liberate 8 grm. of oxygen, 107·88 grm. of silver, and so on, and these masses in grammes are the respective gram-equivalents. Hence **96500 coulombs will have passed if one gram-equivalent of any substance has been liberated.** This quantity of electricity, 96,500 coulombs (9650 e.m. units), is called a **faraday**.

It was early surmised from the preceding laws that *the electricity was actually "carried" through the electrolyte by the materials themselves which appeared at the electrodes*—that the current was carried by the atoms or groups of atoms derived from the electrolysis, such atoms or groups being charged, some positively, some negatively. Faraday called these charged atoms or groups **ions** (wanderers).

Now the number of atoms in one gram-equivalent of hydrogen may be taken to be about $6{\cdot}06 \times 10^{23}$, and a liberation of this amount of hydrogen in electrolysis means a passage of 96,500 coulombs. If, then, the charge is carried by the hydrogen atoms (strictly, *ions* as we are now supposing them charged), it follows that each hydrogen ion carries a charge $96500/(6{\cdot}06 \times 10^{23})$ $= 1{\cdot}59 \times 10^{-19}$ coulomb $= 1{\cdot}59 \times 10^{-20}$ e.m. unit $= 4{\cdot}77 \times 10^{-10}$ e.s. unit (see again page 160). *It is important to note that this is numerically the same as the charge of an electron*—the *unit* charge referrred to on pages 6, 266.

Further, in the case of any univalent substance the number giving the chemical equivalent is the same as the number giving the atomic weight, so that the gram-equivalents of all *univalent* substances contain the same number of atoms. But the gram-equivalent carries 96,500 coulombs in each case: hence **the charge carried by all univalent ions is numerically equal to the unit electron-charge** $1{\cdot}59 \times 10^{-19}$ coulomb.

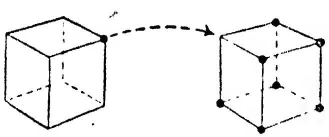
Take now the case of a bivalent substance, say copper. Its atomic weight is 63·57, so that 63·57 grm. of it will contain the same number of atoms as 1·008 grm. of hydrogen. Its chemical equivalent is, however, *half* its atomic weight, *i.e.* the gram-equivalent is 31·78 grm. The gram-equivalent of copper carries the same charge as the gram-equivalent of hydrogen (96,500 coulombs), but it only contains half the number of atoms, so that each (charged) copper atom (ion) must carry double the charge that the (charged) hydrogen atom (ion) carries. Thus **the charge carried by a bivalent ion is**

numerically equal to two unit electron-charges, *i.e.* 2 ($1{\cdot}59 \times 10^{-19}$) coulomb. Similarly *a tervalent ion carries three unit charges*, and so on. Clearly, the number of unit charges carried by an ion of a substance is the same as its valency number.

Remember that H^+, for example, denotes a hydrogen *ion* with a unit positive charge (*i.e.* the H *atom* has lost *one* electron): SO_4^{--} denotes a sulphate ion with two unit negative charges (*i.e.* the SO_4 has gained *two* electrons), and so on.

2. The Modern (Dissociation) Theory of Electrolysis

It has been noted that the ordinary types of chemical change consist in the transfer or sharing of valency electrons between two or more atoms, and the student should again read page 12, which deals with this. Thus sodium combines readily with chlorine to form the compound sodium chloride or common salt (a " molecule " of which has the symbol NaCl), and in the combination the sodium atom's single outer electron passes over to the chlorine atom *so that both become ions*, the Na positive and the Cl negative, and the " combination " is maintained by the electrostatic attraction between the oppositely charged ions. (This was diagrammatically indicated in Fig. 6: see also Fig. 348.) Compounds formed in this way are spoken of as **hetero-polar compounds.**



OUTER SHELL OF SODIUM ATOM OUTER SHELL OF FLUORINE ATOM

FIG. 348. Sodium atom (electrons 2, 8, 1) combining with fluorine atom (2, 7) to form sodium fluoride ((2, 8) (2, 8)).

But a point may be noted about this combination to form sodium chloride. Many atoms when they combine to form compounds do certainly form single " molecules " which exist separately. In the case, however, of sodium and chlorine combined in the solid state as sodium chloride, a *crystal structure* results in which all the ions of sodium (+) are bound to all the ions of chlorine (−) to form a symmetrical three dimension " space-lattice ": this orderly arrangement of the sodium and chlorine ions is indicated in Fig. 349 (black dots = chlorine: white dots = sodium). *Thus there are no " molecules " of sodium chloride as such* in the solid, but ions of sodium and chlorine built up into a cubical lattice. Similar remarks apply to other compounds of the hetero-polar type *when in the solid state.* All this is a detail at present but it is worth noting. The positive ions in a metal also form a lattice structure (page 129), the " free " electrons drifting through the avenues between the positive ions when current flows.

Many compounds, especially organic, are not formed by the *transfer* of electrons as indicated above, but by the *sharing* of electrons between the combining atoms. The case of a chlorine molecule (Cl_2) also formed in this way was pictured in Fig. 7.

Now returning to our sodium chloride, *when this is dissolved in water* some of it is immediately "dissociated" or "ionised," or split up into sodium ions which have a unit positive charge and chlorine ions which have a unit negative charge, these two sets of ions being now able to move about independently in the liquid. This dissociation is caused by the electrical forces due to the water's charged groups neutralising the forces by which the Na and Cl ions are held together in the solid substance. It is reasonable to surmise that this may be possible since the force of attraction between two charges is *reduced* to $1/K$ of its value in air if they are put into a medium of dielectric constant K (page 160), and the dielectric constant of water is *very* high, viz. 80—hence the lessening of the forces binding the Na and Cl ions together. Such dissociation renders the solutions conducting and they are *electrolytes.*

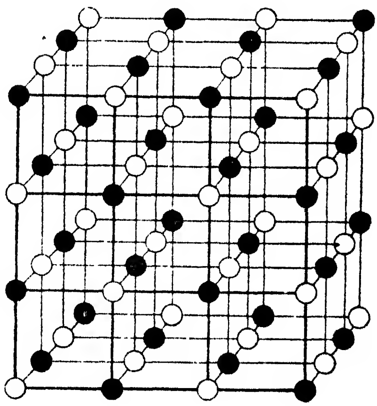


FIG. 349.

Electrolytes are classified into **acid, alkali,** and salt electrolytes. Substances, which when dissolved in water dissociate and produce hydrogen ions are known as *acids*: thus sulphuric acid (H_2SO_4) dissociates into two hydrogen ions each with a unit positive charge and one sulphate ion or sulphion with two unit negative charges, viz. $H_2SO_4 \rightleftharpoons 2\ H^+ + SO_4^{--}$. Substances which dissociate and produce hydroxyl or OH ions are *alkalis*: thus caustic soda (NaOH) dissociates into a sodium ion with a unit positive charge and a hydroxyl ion with a unit negative charge, viz. $NaOH \rightleftharpoons Na^+ + OH^-$. Substances which produce neither hydrogen nor hydroxyl ions are *salt* electrolytes: thus copper sulphate dissociates thus: $CuSO_4 = Cu^{++} + SO_4^{--}$.

Remember that such an expression as "a strong acid" has no connexion with the *concentration.* The strength of an acid is measured by its capacity to produce hydrogen ions (H^+) in solution. Similarly the strength of an alkali is measured by its capacity to produce hydroxyl ions $(OH)^-$.

The explanation of electrolysis is now clear. The positive and negative ions are moving about in a haphazard fashion. Now

suppose the electrodes are inserted and a battery joined up to them. The anode is at a high potential (say positively charged), and the cathode at a lower potential (say negatively charged), and the haphazard movement becomes a *directed* movement. The positive ions move through the liquid towards the (negative) cathode, *take electrons from the cathode* to neutralise their positive charges, and the ions thus become ordinary atoms with their usual chemical properties. (The battery, of course, maintains a "flow" of electrons —the current—in the connecting wire from its negative pole to the cathode.) The negative ions move through the liquid towards the (positive) anode, *give up their negative charges* (*electrons*) *to the anode*, and thus become ordinary atoms with their usual chemical properties. (The battery maintains a "flow" of electrons in the connecting wire in the direction anode to positive pole.) How this "discharging" of the ions at the electrodes occurs, and what the final result will be, depend on (*a*) the nature of the ions, (*b*) the material of the electrodes, and (*c*) the solution there. Two particular cases were noted on page 275, and further cases are given in Art. 3.

It was urged against the dissociation theory that the *mere fact of dissolving in water* would be insufficient to break down the firm bond between, say, the sodium and chlorine atoms in sodium chloride: it must be remembered, however, that it is the affinity between oppositely charged *ions* that is involved (for sodium chloride is already in *ion* form in the solid state), and the possibility of "dissociation" on dissolving in water is indicated by the high dielectric constant (80) of water.

Again, it is known that Ohm's law applies to electrolytes, and that some electrolysis occurs however small the applied P.D. provided there is no back or polarisation E.M.F.: hence it is clear that none of the electrical energy supplied by the battery to the voltameter can possibly be used in bringing about the dissociation into ions—the ions must already exist in the dissociated condition.

It was also argued that it would be difficult for a sodium ion to exist in an aqueous solution since a sodium atom reacted so vigorously with water, but this again was assuming ions and atoms to have the same chemical properties, which, of course, is not the case, as the electronic theory very clearly indicates: in fact we now know that this very activity of the sodium *atom* in water is mainly due to its tendency to lose an electron and become an *ion*.

The colour of electrolytic solutions is always an additive property, a fact which seems to support the theory. Thus, the colour of copper chloride "molecules" is yellow, a concentrated

solution of the salt is green, and a dilute solution blue. This is explained by assuming that in the concentrated solution dissociation is incomplete, and the green colour results from the combination of yellow due to undissociated copper chloride molecules and blue due to the copper ions, while in the dilute solution where the dissociation is complete the colour is the blue of the copper ions. Certain other properties of electrolytes are also additive.

There is additional and ample evidence outside the domain of "Electricity" to support the dissociation theory, but for details some work on *Physical Chemistry* must be consulted. We have, for example, mentioned the fact (page 264) that substances in solution exert a pressure (*osmotic pressure*—see also Art. 15), and work by Pfeffer and Van't Hoff showed that the pressure was in many cases proportional to the concentration of the solute (dissolved substance), *i.e.* proportional say to the number of molecules of the dissolved substance present in 1 c.c. Now in the case of electrolytes, abnormally high results were obtained in osmotic pressure determinations, but these are at once explained if it is assumed that the number of active "particles" per c.c. of the solution is given, not by the number of molecules of the dissolved substance per c.c., *but by a greater number, viz. by the number of free ions* (*and undissociated molecules*) *per c.c.*

Again, allied to the above, is the lowering of the freezing point, the raising of the boiling point, and the change in vapour pressure brought about by dissolving substances in water—and all these factors can be shown to depend on "concentration." By measuring the lowering of the freezing point or the raising of the boiling point brought about by dissolving a known mass of substance, the molecular weight can be found. In the case of electrolytes the results seemed to indicate that too many molecules were present, which, of course, would be explained if some or all the molecules were dissociated into ions. At great dilution the molecular weight found was, in some tests, only a half of the true molecular weight, thus indicating that every molecule had then dissociated into two ions (in some cases the value was about a third, indicating three ions).

3. Some Cases of Electrolysis

(1) DILUTE SULPHURIC ACID WITH PLATINUM ELECTRODES.—A simple type of voltameter (or coulometer) for this experiment is shown in Fig. 350: the vertical tubes placed over the electrodes enable the gases given off to be collected and examined.

The solution contains positive hydrogen ions and negative sulphate ions or sulphions (there may also be some OH^- and HSO_4^- ions—see below), the total "charges" on the opposite ions being equal as the solution is not charged. When the battery is joined up the positive hydrogen ions (H^+) travel to the negative cathode: here they take electrons from the cathode, become neutralised or

ordinary hydrogen atoms (H) which join up to form molecules (H_2), and hydrogen is given off. The negative sulphate ions (SO_4^{--}) travel to the positive anode where they give up their electrons and become neutralised. But an *uncharged* group SO_4 is incapable of a separate existence: it reacts with water, forming sulphuric acid and liberating oxygen which is given off. (In recent years another way of regarding these *reactions at the electrodes* has been found which leads to the same final conclusions: this will be briefly referred to later.)

Thus the net result is hydrogen given off at the cathode and oxygen at the anode, and it will be found that the volume of hydrogen is about twice the volume of oxygen (actually it is just as if water (H_2O) itself had been electrolysed). Of course, for every 1 grm., say, of hydrogen liberated about 8 grm. of oxygen are liberated, but the density of oxygen is about 16 times that of hydrogen under like conditions, so that the *volume* of oxygen is about half that of hydrogen as stated. The case, then, of dilute sulphuric acid may be represented by the following scheme in which *e* denotes the unit (electron) negative charge:—

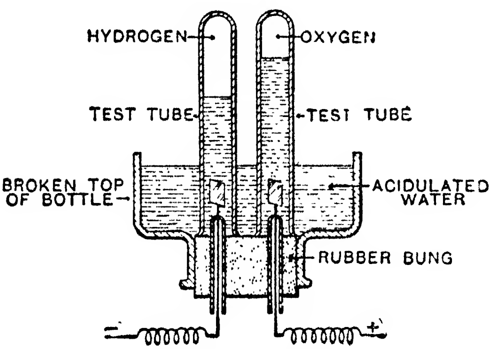


FIG. 350.

$$H_2SO_4 \rightleftharpoons 2\,H^+ + SO_4^{--}$$

Cathode ← ——— ——— → *Anode*

$$2\,H^+ + 2e\,(-\text{ve}) = 2\,H \qquad\qquad SO_4^{--} - 2e\,(-\text{ve}) = SO_4$$

$$2\,H = H_2 \qquad\qquad 2\,SO_4 + 2\,H_2O = 2\,H_2SO_4 + O_2$$

Note that electrons are *taken from* the cathode and electrons are *given to* the anode, and the battery keeps up its circulation of electrons which constitutes the current in the connecting wires, viz. electrons in the direction negative pole to cathode, anode to positive pole.

If the acid is about 50 per cent., there is a large proportion of HSO_4^- ions, and if a high current density is used, say by employing

a point anode, the conditions are favourable for the production of *persulphuric acid* ($H_2S_2O_8$), thus:—

$$H_2SO_4 \rightleftharpoons H^+ + HSO_4^-$$

Cathode ← *Anode* →

$$H^+ + e\ (-ve) = H \qquad HSO_4^- - e\ (-ve) = HSO_4$$

$$2\,H = H_2 \qquad 2\,HSO_4 = H_2S_2O_8$$

Even a dilute solution of the acid may dissociate or ionise in two stages thus:—

$$H_2SO_4 \rightleftharpoons H^+ + HSO_4^- \rightleftharpoons 2\,H^+ + SO_4^{--}$$

and many substances dissociate in two or more stages in this way.

For most practical purposes *pure water* is regarded as undissociated and non-conducting, but as a matter of fact it is slightly ionised into positive hydrogen ions and negative hydroxyl ions thus: $H_2O = H^+ + (OH)^-$ (more strictly $2\,H_2O = H_3O^+ + OH^-$) and is therefore *slightly* conducting. (H_3O^+ = *hydroxonium ion.*)

(2) COPPER SULPHATE SOLUTION WITH PLATINUM ELECTRODES. —Read again pages 275-6. The case may be represented by the following scheme:—

$$CuSO_4 \rightleftharpoons Cu^{++} + SO_4^{--}$$

Cathode ← *Anode* →

$$Cu^{++} + 2e\ (-ve) = Cu \qquad SO_4^{--} - 2e\ (-ve) = SO_4$$

$$2\,SO_4 + 2\,H_2O = 2\,H_2SO_4 + O_2$$

(3) COPPER SULPHATE SOLUTION WITH COPPER ELECTRODES.— The scheme is as follows:—

$$CuSO_4 \rightleftharpoons Cu^{++} + SO_4^{--}$$

Cathode ← *Anode* →

$$Cu^{++} + 2e\ (-ve) = Cu \qquad SO_4^{--} - 2e\ (-ve) = SO_4$$

$$SO_4 + Cu = CuSO_4$$

The anode action is perhaps better put thus: The anode dissolves, *i.e.* copper (as positive copper ions Cu^{++}) passes from it into solution forming, with the SO_4 (*i.e.* SO_4^{--}), copper sulphate. The change might, in fact, be depicted thus: Cu (of anode) = Cu^{++} (into solution) + $2e$ (—ve) (to battery): $Cu^{++} + SO_4^{--} = CuSO_4$. For further details see pages 275-6.

(4) CAUSTIC SODA (SODIUM HYDROXIDE) SOLUTION WITH PLATINUM ELECTRODES.—In (2) and (3) *secondary reactions* as they

are called have occurred only at the anode. In this case they take place at both electrodes: hydrogen is given off at the cathode and caustic soda formed again, whilst oxygen is given off at the anode.

$$NaOH \rightleftharpoons Na^+ + OH^-$$

Cathode ←   → *Anode*

Cathode	Anode
$Na^+ + e\ (-ve) = Na$	$OH^- - e\ (-ve) = OH$
$2\ Na + 2\ H_2O = 2\ NaOH + H_2$	$4\ OH = 2\ H_2O + O_2$

4. Back E.M.F. in Electrolysis

In many cases the ions appearing at the electrodes alter the character of the latter. Thus when current passes through dilute sulphuric acid between platinum electrodes, some hydrogen forms a layer on the cathode and oxygen appears at the anode. The observed effect is an apparent increase in resistance, but the real effect is to produce an E.M.F. which is opposite in direction to that maintaining the current. The value of this **back** or **polarisation E.M.F.** in the case considered is about 1·49 volts: that it does exist can be shown by the arrangement of Fig. 351 (see also page 361).

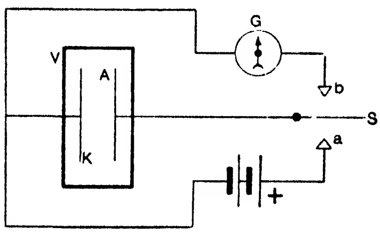


FIG. 351.

V is the voltameter with platinum electrodes, and G is a galvanometer. With the key S on stud *a* current passes through V in the direction A (anode) to K (cathode), and electrolysis ensues. After a time S is moved to *b*, thus cutting out the battery and joining V to G. A current flows through G in the direction ASGKA, *i.e.* through V *in the opposite direction* to the previous current: this continues, but decreases, until the gases at the electrodes are used up. A modification of this simple experiment is necessary if it is desired to measure the back E.M.F.

In a copper sulphate voltameter *fitted with platinum electrodes* copper is deposited on the platinum cathode and the back E.M.F. is about 1·17 volts. Consider, however, this voltameter *fitted with copper electrodes, the latter being in such a condition that they are readily acted on by the sulphion* SO_4; the "equations" for the chemical actions at the electrodes may be written:—

(Cathode) $CuSO_4 = Cu + SO_4$ .... (*a*)

(Anode) $Cu + SO_4 = CuSO_4$ .... (*b*)

Equation (*b*) is of a similar type to that of Art. 1, page 261. As in that case *energy is liberated*, and further, as in the simple cell, *a forward E.M.F. is the result.* Equation (*a*) is the converse of (*b*); it represents a condition in which *energy is absorbed*, and, like the polarisation of the high-potential plate in the simple cell, *an opposite E.M.F. is the result.* These effects are equal, and cancel each other; hence *when copper sulphate is subjected to electrolysis, the electrodes being copper plates which can readily be acted on by the* SO_4, *there is on the whole no back E.M.F.* (There may be a slight E.M.F. due to changes in concentration, the latter slightly increasing near the anode and decreasing near the cathode as electrolysis proceeds.)

It is found that in the case, say, of dilute sulphuric acid, whilst a *small* current will flow and a *little* decomposition will occur when the applied P.D. on the voltameter is less than 1·49 volts, very little gas will be given off and the current will soon cease. This is, of course, owing to the existence of the back E.M.F. for the applied P.D. must evidently be greater than the maximum value of this back E.M.F. if a continued current is to flow.

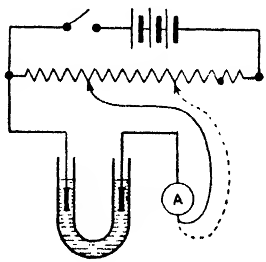


FIG. 352.

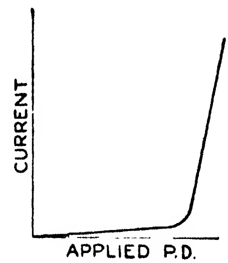


FIG. 353.

It is instructive to apply various P.D.'s to the voltameter, note the steady current at each step, and plot a P.D.-Current graph. Fig. 352 shows the method, and Fig. 353 the result. From the latter figure it looks as if Ohm's law does not apply, but the explanation is that the back E.M.F. gradually increases and at any stage is nearly equal (and, of course, opposite) to any applied P.D. less than about 1·5 volts.

5. Calculation of Back E.M.F. from Thermo-Chemical Data

As an illustration we will take the electrolysis of dilute sulphuric acid. When 1 grm. of water is formed by the combustion of hydrogen in oxygen there is a liberation of about 3800 calories of heat. We may therefore take this as the amount of energy required to bring about the complete "electrolysis" of one gramme of water. If 1 ampere flows through the electrolytic cell for 1 second 0·000094014 grm. of water is decomposed (for the electro-chemical

equivalent of hydrogen is ·000010446 and of oxygen it is ·000083568). Hence the energy required for this decomposition is:—

$$3800 \times 0{\cdot}000094014 \times 4{\cdot}18 \times 10^7 \text{ ergs} = 1{\cdot}49 \times 10^7 \text{ ergs}.$$

Again, the work done in opposition to the back E.M.F. (e volts) is $e \times I \times 10^7$ ergs per second for I amperes, or $e \times 10^7$ ergs per second for 1 ampere; hence:—

$$e \times 10^7 = 1{\cdot}49 \times 10^7; \quad \therefore e = 1{\cdot}49 \text{ volts}.$$

This is the minimum E.M.F. necessary to balance the maximum back E.M.F., and it is thus seen why a cell of less voltage than this is not sufficient for the electrolysis of dilute sulphuric acid (or water). It would, of course, start a current, but when polarisation begins, the back E.M.F. due to it rises until it equals the E.M.F. of the cell, when the current will cease.

6. The Conductivity of Electrolytes

The *specific resistance* or *resistivity* (S) of an electrolyte is defined in the same way as for a metallic conductor (page 301), viz. as the resistance of a "cm. cube" of it.

The **specific conductivity** or **conductance** (κ) is the reciprocal of the resistivity, viz. 1/S (the unit of conductivity is the *mho*—page 292). Since the ions of the dissolved substance carry the current, it is to be expected that the specific conductivity will depend on the concentration, for if we consider a column of the electrolyte from anode to cathode, it is clear that if the number of carriers in the column be increased, more will be driven upon the electrodes in a given time, and the conductivity will be greater. Put another way, the specific conductivity (κ) will decrease with increasing dilution.

Since, then, the conductivity depends on concentration, in order to properly compare different electrolytes their conductivities must be dealt with at corresponding, *i.e.* equivalent, concentrations. It is usual therefore to deal with what is termed the **equivalent conductivity** (γ) which is defined as measured by *the specific conductivity* (κ) *multiplied by the volume* V *in c.c. containing one gram-equivalent of the electrolyte* (the *c.c.* is chosen as unit of volume because κ is defined in terms of the *cm.*-cube); *i.e.*—

$$\text{Equivalent conductivity} = \gamma = \kappa V.$$

Again, as the *concentration* (c) is the number of gram-equivalents per *c.c.*, V is clearly $1/c$ and $\gamma = \kappa/c$: hence:—

$$\text{Equivalent conductivity } (\gamma) = \kappa V = \frac{\kappa}{c} = \frac{\text{Specific conductivity}}{\text{Concentration}}$$

Frequently the **molecular conductivity** (μ) is used: it is defined as measured by the specific conductivity (κ) multiplied by the volume (V^1) in *c.c.* containing one *gram-molecule* of the electrolyte: thus $\mu = \kappa V^1$ (μ and γ are, of course, the same for an electrolyte in which the ions are univalent).

Although, as already stated, the specific conductivity (κ) gets less as the solution is progressively diluted, it is found that the equivalent conductivity (γ)—and the molecular conductivity (μ)—*increase* with dilution until a maximum limiting value is reached. The limiting value approached by γ as the solution is diluted is called the *equivalent conductivity at infinite dilution* (γ_∞) To calculate γ_∞ (it cannot be determined *experimentally* at *infinite* dilution) the value of γ is determined over a range of dilutions, the values plotted on a graph, and γ_∞ read off by extrapolation.

Now the current passing through an electrolyte clearly depends on (*a*) the number of ions present, (*b*) the charge they carry, and (*c*) their mobility or the ease with which they travel through the liquid. Arrhenius assumed that (*c*) did not vary with dilution (this is nearly true for weak electrolytes but it does not hold for strong electrolytes—see below), and of course (*b*) is constant for any particular electrolyte. Hence the only reason for the increased equivalent conductivity on dilution seems to be that *more ions are produced on increasing the dilution.* At infinite dilution Arrhenius supposed the electrolyte completely dissociated.

The conductivity *increases* with rise in *temperature* (page 304). The effect is probably due to a change in mobility of ions, not to a change in their number. Also the change in mobility may be affected by a change in viscosity of the solution.

As an illustration the following table gives the conductivities of potassium chloride solution at 18° C. and at various dilutions. Note that κ gets less, but γ increases with increasing dilution.

CONDUCTIVITIES OF KCl SOLUTION AT 18° C.

CONCENTRATION, GRAM.-EQUIV. PER C.C. c	VOLUME CONTAINING 1 GRAM.-EQUIV. V.	SPECIFIC CONDUCTIVITY κ	EQUIVALENT CONDUCTIVITY γ
0·001	1,000	0·0983	98·3
0·0001	10,000	0·0112	112·0
0·00001	100,000	0·001224	122·4
0·000001	1,000,000	0·0001273	127·3
0·0000001	10,000,000	0·00001291	129·1
0	∞	—	(131)

It has been stated that the increase in equivalent conductivity with increasing dilution is due, in the case of weak electrolytes, mainly to the production of more ions. Strong electrolytes are, however, completely dissociated at all dilutions, and changes in conductivity on dilution are not therefore due to changes in the number of ions but to changes in their speed caused by changes in the electrical forces (attraction) between ions of opposite sign: on dilution the ions will be further apart and these interionic forces less, so that the ions will be less retarded in their movement by the " pull back " of opposite ions, their speed will be greater, and the conductivity will increase. At infinite dilution the interionic forces will be negligible and the conductivity a maximum.

Strong electrolytes include all salts and many acids and bases: weak electrolytes include most organic acids and bases and a few inorganic, *e.g.* hydrogen sulphide, carbonic acid, ammonium hydroxide. (Refer to some book on Chemistry.)

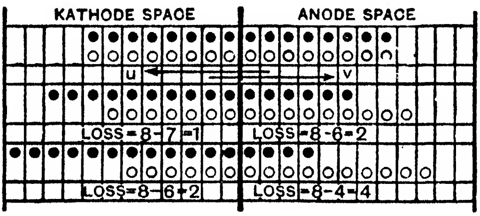


FIG. 354.

The experimental methods of finding the conductivities of electrolytes are dealt with in the chapter on Electrical Measurements.

7. Ratio of Ionic Velocities. Transport Numbers

It is known that changes in concentration occur in an electrolyte during the process of electrolysis; Hittorf has explained this in terms of the difference between the velocities of the positive and negative ions, and has shown how the *ratio* of the velocities may be determined from the changes in concentration.

Consider Fig. 354, in which the dots represent positive ions (cations) and the circles negative ions (anions), and imagine the velocity of the cation to be double that of the anion. The first two rows indicate the condition before the current passes; there are eight molecules in each compartment. The second two rows indicate the condition an instant later, when the current is passing. At the cathode 3 ions (cations) have been deposited, the cathode space contains 7 molecules and *has therefore lost* 1 *molecule*; at the anode 3 ions (anions) have also been deposited, but the anode space contains only 6 molecules and *has therefore lost* 2 *molecules*. Still

later, the third two rows indicate the condition: 6 ions have been deposited at each electrode; the cathode space has 6 molecules and *its loss is therefore 2 molecules*; the anode space has 4 molecules and *its loss is therefore 4 molecules.* Clearly—

$$\frac{\text{Loss in concentration at cathode}}{\text{Loss in concentration at anode}} = \frac{1}{2} = \frac{\text{Velocity of anion}}{\text{Velocity of cation}};$$

$$\therefore \frac{\text{Cathode space loss}}{\text{Anode space loss}} = \frac{v}{u} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where v is the velocity of the negative ion (anion) and u the velocity of the positive ion (cation). Further:—

$$\frac{\text{Cathode space loss}}{\text{Total loss}} = \frac{v}{u+v}; \quad \frac{\text{Anode space loss}}{\text{Total loss}} = \frac{u}{u+v}.$$

Since u and v are the speeds of migration of the cation and anion, the total quantity of electricity carried is *proportional to* $(u + v)$. Of this total, *if n is the fraction carried by the anions,* $(1 - n)$ will be the fraction carried by the cations: then

$$n = \frac{v}{u+v} \quad \text{and} \quad 1 - n = \frac{u}{u+v} \quad \ldots\ldots\ldots\ldots (2)$$

and the values obtained for n and $(1 - n)$, *i.e.* for $v/(u + v)$ and $u/(u + v)$ are called the **transport** or **migration numbers** of the anions and cations respectively. Further, $n/(1 - n) = v/u$.

From (1) above it is clear that a chemical analysis of the concentrations of the solutions in the neighbourhood of the cathode and anode after the passage of a current will give the transport numbers and the *ratio* of the ionic velocities $v : u$.

The method is slightly modified when one of the ions dissolves in solution. Thus take the case of a solution of copper sulphate with copper electrodes: here copper is deposited on the cathode, and an equal amount passes into solution at the anode, so that there is increase in concentration at the anode and decrease at the cathode (see Art. 8).

8. Experimental Determination of Transport Numbers

The common form of transport number tube used in elementary work is shown in Fig. 355. The longer limb, fitted with a tap, contains the anode, and the shorter limb the cathode. The two limbs are connected by a short wide tube. The form of the apparatus is sufficient to prevent mixing to any marked extent during the experiment. The anode and cathode are in the form of

flat metal spirals. The leads to the electrodes are insulated within the vessel by glass tubes.

As an illustration we will take the case of the determination of the transport numbers of the silver and nitrate ions in a silver nitrate solution: we will assume both electrodes are silver so that silver from the anode will pass into solution in this case. The cell is filled with a solution of silver nitrate. The electrodes A and C are connected with the positive and negative terminals of a source of direct current at about 110 volts. In series with the cell is an ammeter B, a silver voltameter D, and an adjustable resistance R. A small current of about 0·01 ampere is passed. After, say, three hours about half the solution from the *anode* compartment is withdrawn and analysed. The total amount of silver deposited in the silver voltameter D is also noted.

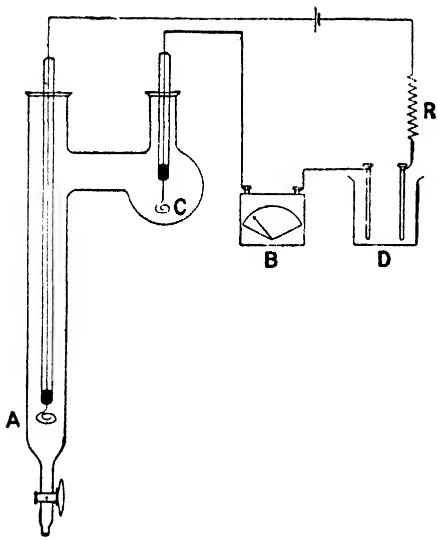


FIG. 355.

In an actual experiment the silver nitrate solution in the transport tube originally contained 25·382 mgrm. of silver in 25 c.c. of solution. After electrolysis 25 c.c. of the anode solution contained 39·041 mgrm. of silver. In the silver voltameter 25·92 mgrm. of silver were deposited on the cathode.

Now the increase in the concentration of the silver in the anode limb should have been 25·92 mgrm., due to the discharge of the NO_3 ions at the anode and silver going into solution (page 276). But the increase was only $39{\cdot}041 - 25{\cdot}382 = 13{\cdot}659$ mgrm. Therefore $25{\cdot}92 - 13{\cdot}659 = 12{\cdot}261$ mgrm. of silver must have migrated out of the compartment. Hence:—

$$\text{Transport number for silver ion} = 1 - n = \frac{u}{u+v} = \frac{12{\cdot}261}{25{\cdot}92} = 0{\cdot}473,$$

$$\text{Transport number for nitrate ion} = n = \frac{v}{u+v} = 1 - {\cdot}473 = 0{\cdot}527,$$

$$v/u = {\cdot}527/{\cdot}473 = 1{\cdot}1.$$

TRANSPORT NUMBERS OF ANIONS (n)

SALT	TRANSPORT NUMBER OF ANION (n)	SALT	TRANSPORT NUMBER OF ANION (n)
KCl	0·508	NaCl	0·611
$AgNO_3$	0·528	$\frac{1}{2}CuSO_4$	0·632
KI	0·508	HCl	0·172

The transport numbers of the cations are, of course, $(1 - n)$, *e.g.* for sodium (Na) $1 - \cdot611 = \cdot389$. The figures are for M/10 solutions at 18° C.

9. Actual Velocities of Ions

As already noted, the facts of Art. 8 merely enable the *ratio* of v to u to be determined, but a further step by Kohlrausch provides a means of finding $(u + v)$, and from the two sets of results u and v can be separately determined. It is clear that the *actual* velocity of an ion in any experiment will depend on the P.D. applied, *i.e.* it will depend on the *potential gradient* in the solution. This must therefore be specified, and it has been agreed to state *actual velocities in cm. per sec. for a potential gradient of* 1 *volt per cm.*

We have seen that by the ionic theory the current through an electrolyte depends on (1) the number of ions involved, (2) the charge carried by each ion, (3) the velocity of the ions. Thus, in a dilute solution of potassium chloride containing c gram-equivalents per c.c., let the charges carried by a gram-equivalent of the K and Cl ions be q and $-q$ units, and let the speed of these ions be u and v cm. per sec. respectively. The current across one square centimetre in the electrolyte will then consist of the transfer of cqu units of positive in one direction, and cqv units of negative in the opposite direction, that is, the current is measured by $cq(u + v)$ or $96500c(u + v)$, since q, the charge carried by a gram-equivalent $=96500$ coulombs. The current is also given by $\kappa\frac{dV}{dx}$, where κ is the specific conductivity of the liquid and dV/dx the gradient of potential in the direction of the current. Hence:—

$$96500c\,(u+v) = \kappa\,\frac{dV}{dx}; \quad \therefore\ (u+v) = \frac{\kappa}{c}\cdot\frac{1}{96500}\cdot\frac{dV}{dx},$$

and for the required potential gradient of 1 volt per cm., we have

$$(u+v) = \frac{\kappa}{c} \times \frac{1}{96500} = \frac{\kappa}{c} \times 0{\cdot}000010362\ldots\ldots\ldots \quad (3)$$

Hence $(u + v)$, the sum of the actual velocities of the ions, can be calculated from κ/c, the equivalent conductivity (γ) of the

electrolyte: and this latter is known since c is the known concentration and $\kappa = 1/S$, where S is the resistivity found by the methods of page 441. Knowing $(u + v)$ and v/u, the separate values v and u are determined.

VELOCITIES OF IONS

(In cm. per sec. under a potential gradient of 1 volt per cm.)

K = 0·00067	Cu = 0·00046	SO_4 = 0·00071
H = 0·00326	Cl = 0·00068	NO_3 = 0·00064
Ag = 0·00057	OH = 0·00181	I = 0·00068

To be exact, the preceding theory only applies to cases where *all* the molecules are dissociated (infinite dilution). Further, these " drift " velocities are small: they are superimposed on the movements due to thermal agitation and must not be confused with the speeds of ions indiscriminately in all directions with which " osmotic pressure " is associated. The velocity of an ion in cm. per sec. under a potential gradient of one volt per cm. is referred to as its *actual mobility*.

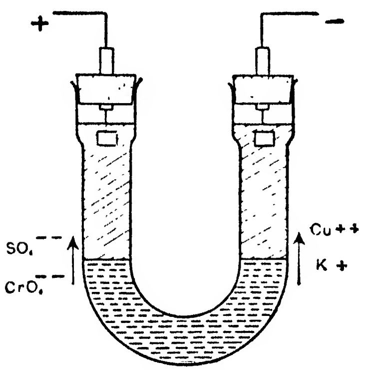


FIG. 356. Migration of ions.

10. Observing the Migration of Ions. Direct Determination of Velocities

The actual migration of ions under the influence of an applied P.D. can be observed by the apparatus of Fig. 356. A moderately dilute solution of a coloured electrolyte such as copper sulphate or potassium chromate containing 3-5 per cent. agar agar is introduced into a U-tube and allowed to set to a jelly. The position of the coloured boundary in each limb is marked and a solution of a colourless salt such as potassium sulphate introduced above the jelly. Ions can travel through the jelly at almost the same speed as through water; the jelly is used to prevent diffusion of one solution into the other. A current of 0·3-0·4 amps. (200 volts) is passed through the U-tubes, platinum electrodes being used.

In the case of copper sulphate solution the copper ion is blue in solution, and a blue boundary can be seen moving towards the cathode. In the case of potassium chromate the chromate ion is yellow, and a yellow boundary is seen to move towards the anode.

Using a modified apparatus (Fig. 357) it is possible to *measure* the velocities with some accuracy. The first determination was made by Lodge in 1885. The two vessels containing dilute sulphuric acid were connected by a tube containing a slightly alkaline agar agar jelly solution of sodium chloride and a trace of phenol phthalein. A current was passed, and as electrolysis went on the velocity of transfer of the H ion was indicated and measured by the rate at which the phenol phthalein indication of the formation of HCl travelled along the tube. Whetham and Orme Masson used similar methods with various electrolytes and indicators.

11. Ionic or Partial Conductivities

Returning to the question of the conductivity of electrolytes (Art. 6) Kohlrausch, from an exhaustive study of conductivities, came to the conclusion that the equivalent conductivity at infinite dilution (γ_∞) was made up of two parts, one (say x) due to the cation, and one (say y) due to the anion (x and y are, of course, in the same units as the conductivity). He found also that for any one salt the ratio of the terms was also the ratio of the velocities of the ions. We can therefore write–

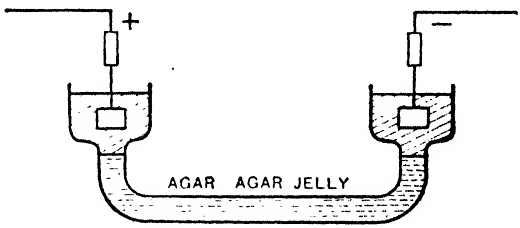


FIG. 357. Measurement of ionic mobility.

$$\gamma_\infty = x + y,$$

where $x : y = u : v$. The terms x and y are called the **ionic** or **partial conductivities** of the cation and anion respectively.

We have seen that the transport number of an anion (n) is equal to $v/(u + v)$, and as $v/u = y/x$ we can write $n = y/(x + y)$: hence $y = n\,(x + y)$; $\therefore y = n\gamma_\infty$. Now γ_∞ can be obtained by extrapolating, while n can be obtained from a determination of transport numbers. From these data the ionic or partial conductivities can be calculated. For example in the case of potassium chloride the equivalent conductivity at 18° = 131, and the transport number (n) for the anion chlorine (Cl^-) is ·508.

$$\therefore \text{ Ionic conductivity of Cl} = n\gamma_\infty = \cdot 508 \times 131 = 66{\cdot}55;$$

$$\text{Ionic conductivity of K} = (1 - n)\,\gamma_\infty = \cdot 492 \times 131 = 64{\cdot}45.$$

IONIC OR PARTIAL CONDUCTIVITIES (18° C.-INFINITE DILUTION)

CATIONS		ANIONS	
H	318	Cl	66·5
Na	44·5	I	67
K	64·4	NO_3	61
Li	35·5	OH	174
Ag	56	$\frac{1}{2}SO_4$	68·5

By means of such values as these we can calculate, of course the equivalent conductivity of an electrolyte at infinite dilution Thus the equivalent conductivity of potassium sulphate ($\frac{1}{2}K_2SO_4$) at infinite dilution $= 64{\cdot}4 + 68{\cdot}5 = 132{\cdot}9$.

Again, from (3), Art. 9, we have κ/c $(= \gamma_\infty) = 96500\ (u + v)$, where u and v are the *actual* velocities of the cations and anions in cm. per sec. under a P.D. of 1 volt per cm. Comparing with the above, viz. $\gamma_\infty = x + y$ (and $x/y = u/v$), we have $96500u = x$, or $u = x/96500$, *i.e.*

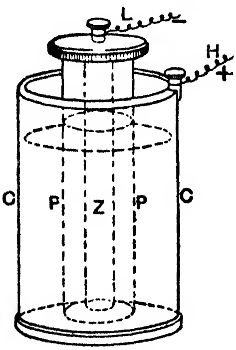


FIG. 358.

Absolute velocity of an ion

$$= \frac{\text{Ionic conductivity}}{96500}.$$

Thus the ionic conductivity of the hydrogen ion is 318, so that its actual velocity $=318/96500 = {\cdot}0033$ cm. per second (approx.).

12. Various Forms of Primary Cells

In a "primary cell" the cell materials are used up by the chemical action, so that the active materials have to be renewed. The various primary cells were mainly devices to eliminate that form of "polarisation" caused in the simple cell by the hydrogen appearing at the high potential plate (see again pages 261-8).

(1) THE DANIELL'S CELL.—This cell (Fig. 358) consists of an outer copper pot C forming the high-potential plate. The pot contains a strong solution of copper sulphate used as the **depolariser,** *i.e.* the substance which prevents polarisation. In this stands an unglazed earthenware pot (a "porous" pot) P containing dilute sulphuric acid (or zinc sulphate) and a rod of amalgamated zinc Z, which is the low-potential plate. When H and L are joined by a wire current flows, the *conventional* direction being from H to L outside and Z to C inside.

Considering first the *equations* for the chemical action we have:—

$$Zn + H_2SO_4 = ZnSO_4 + H_2$$

so that the zinc is used up, zinc sulphate formed, and hydrogen liberated. This hydrogen encounters the copper sulphate with the result:—

$$H_2 + CuSO_4 = H_2SO_4 + Cu:$$

copper instead of hydrogen is deposited on the pot and polarisation prevented.

In the outside wire the current is really a " flow " of electrons from L to H. Inside the cell the solutions are dissociated, so that in the porous pot there are positive H ions and negative sulphions, SO_4, whilst in the outer compartment there are positive Cu ions and negative SO_4 ions. H^+ ions in the inner compartment move through P towards the copper plate (page 266), but they combine with SO_4^{--} ions from the copper sulphate forming sulphuric acid. The Cu^{++} ions also move towards the copper, " give up their charge " (*i.e.* are neutralised by electrons drawn from the copper plate, such electrons being the current in the wire " flowing " towards H), and are deposited there. SO_4^{--} ions in both compartments move towards the zinc, and those reaching it join with positive zinc ions from the plate forming zinc sulphate: the electrons of the zinc plate corresponding to the positive zinc ions which have gone into solution " flow " in the wire in the direction L to H constituting the current outside. The " carriers " in the cell are thus positive hydrogen ions and positive copper ions in one direction (towards the copper) and negative sulphions, SO_4, in the other direction (towards the zinc). The E.M.F. of the cell is about 1·1 volts, and is fairly constant.

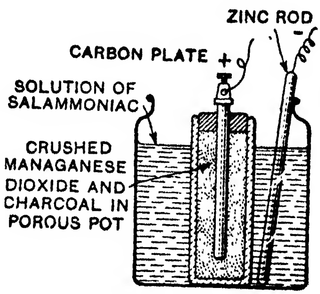


FIG. 359.

Note that when the cell is giving current Zn passes into solution from the zinc plate, and Cu is deposited from the solution on the copper. If an equal current be passed for the same time through the cell in the opposite direction an equal amount of Cu will pass from the copper plate into solution, and an equal amount of Zn will be deposited from the solution on the zinc. The Daniell is therefore spoken of as a **reversible** cell (the Joule heating effect in it is, however, irreversible—see pages 289, 290).

(2) THE LECLANCHÉ CELL.—The low-potential of this cell (Fig. 359) is a rod of zinc in a solution of ammonium chloride (sal-ammoniac) contained in a glass vessel. A rod of carbon forms the high-potential: this is in a porous pot, and is surrounded by

broken carbon and black oxide (dioxide) of manganese which is the depolariser. The chemical reactions are:—

$$\underset{\text{Ammonium Chloride}}{Zn + 2\,NH_4Cl} = \underset{\text{Zinc Chloride}}{ZnCl_2} + \underset{\text{Ammonia}}{2\,NH_3} + H_2 \quad \ldots\ldots\ldots(1)$$

$$H_2 + \underset{\text{Black Oxide of Manganese}}{2\,MnO_2} = \underset{\text{Brown Oxide of Manganese}}{Mn_2O_3} + H_2O \quad \ldots\ldots\ldots\ldots(2)$$

The hydrogen, however, is liberated quicker than the MnO_2 can use it up, so that after a time polarisation sets in and the current falls off. If allowed to rest for a few minutes the MnO_2 performs its work as per equation (2), and the cell regains its strength: thus they are suitable for intermittent work (bell-ringing and telephone calls). The E.M.F. is about 1·5 volts. There are various modified forms of the cell on the market, *e.g.* the *agglomerate block* and the *sack* types, but we need not describe them here.

The student should now think out the action from the ionic point of view as in (1) above.

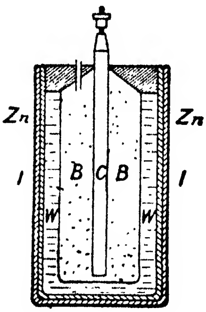


FIG. 360.

(3) THE DRY CELL.—Nearly all the various forms of so-called dry cells are modifications of the Leclanché. They are really not "dry": if they were they would not work. A well-known type (Fig. 360) consists of a zinc cylinder, next to which is a paste, W, composed of plaster of Paris, flour, zinc chloride, sal-ammoniac, and water. Adjoining this is a second paste, B, of carbon, manganese dioxide, zinc chloride, sal-ammoniac, and water. C is a rod of carbon, forming the high-potential element of the cell. The whole is covered with a case of millboard, is sealed with pitch, and is provided with a vent for the escape of gas. The E.M.F. is about the same as that of an ordinary Leclanché.

(4) THE WESTON STANDARD CELL.—On page 293 reference was made to the Weston cell being used as an E.M.F. standard at the National Physical Laboratory. One form of this Weston Cadmium Standard Cell, as it is usually referred to, is shown in Fig. 361. Here mercury M is the positive pole, an amalgam of mercury and cadmium (A) the negative pole, cadmium sulphate (C and S) the electrolyte, and mercurous sulphate paste (P) the depolariser. (C = crystals of, and S = a saturated solution of, cadmium sulphate.) Expressed in international units the E.M.F. of the

Weston is 1·0183 volts at 20° C., and its E.M.F. is slightly less if its temperature is above 20° C. At temperature t° C. the E.M.F. is:—

$$E_t = E_{20} - \cdot 0000406\ (t - 20) - \cdot 00000095\ (t - 20)^2 + \cdot 0000000\ (t - 20)^3.$$

The *Clark standard cell* uses mercury as the positive electrode, and a zinc rod as the negative electrode. Upon the mercury (which is at the bottom of the glass container) is a paste made of mercurous sulphate and a saturated solution of zinc sulphate. Upon this rests a saturated solution of zinc sulphate into which dips the zinc rod. Some crystals of zinc sulphate rest on the paste to ensure that the solution is saturated. Its E.M.F. at 15° C. is usually taken as 1·434 volts, and falls with rise in temperature: at t° C. it is:—

$$E_t = 1\cdot 434\ \{1 - \cdot 00077\ (t - 15)\}\ \text{volts}.$$

13. Calculation of E.M.F. of Cell from Thermo-Chemical Data

It has been indicated that the source of energy of the current from a cell is the chemical action which takes place in it. Consider, for example, the Daniell's cell in which zinc is dissolved at one electrode and copper liberated at the other electrode.

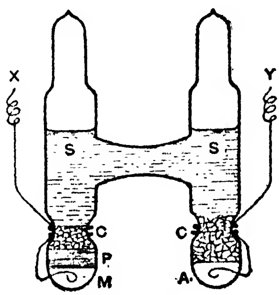


FIG. 361.

When 1 coulomb (say 1 ampere for 1 sec.) passes through the cell ·0003388 grm. of zinc (electro-chemical equivalent) is dissolved and ·0003293 grm. of copper is liberated.

Now when 1 grm. of zinc is dissolved in sulphuric acid in a calorimeter 1630 calories of heat, *i.e.* $1630 \times 4\cdot 18 \times 10^7$ ergs of energy, are liberated: hence if ·0003388 grm. of zinc be dissolved $(1630 \times 4\cdot 18 \times 10^7) \times \cdot 0003388 = 2\cdot 31 \times 10^7$ ergs will be liberated. Thus from the point of view of the zinc action in the cell, when 1 coulomb passes the energy *liberated* is $2\cdot 31 \times 10^7$ ergs.

Again, 1 grm. of copper *dissolved* liberates 881 calories of heat, *i.e.* $881 \times 4\cdot 18 \times 10^7$ ergs of energy, so that if ·0003293 grm. be dissolved the energy liberated would be $(881 \times 4\cdot 18 \times 10^7) \times \cdot 0003293 = 1\cdot 21 \times 10^7$ ergs. But in the cell this amount of copper ·0003293 grm. is *separated out and deposited on the plate* when 1 coulomb passes: hence the coulomb *will require* this amount of energy, $1\cdot 21 \times 10^7$ ergs, to deposit the copper from solution. Clearly then, the balance of the energy, viz.—

$$(2\cdot 31 \times 10^7) - (1\cdot 21 \times 10^7) = 1\cdot 1 \times 10^7\ \text{ergs},$$

represents the output energy when 1 coulomb passes.

Now if E volts be the E.M.F. of the cell, E joules, *i.e.* $E \times 10^7$ ergs, is the energy output when 1 coulomb passes;

$$\therefore\ E \times 10^7 = 1 \cdot 1 \times 10^7,\ i.e.\ E = 1 \cdot 1 \text{ volts.}$$

The above assumes that the net energy liberated by the chemical changes is entirely changed into energy of current. In the case of the Daniell's cell this assumption is very nearly correct. In some cells, however, some of the energy is liberated directly in the form of heat, and only the remainder is available for current: such cells become hotter when running, and decrease in E.M.F. as the temperature rises, as in the case of the Clark and Weston cells. In some cells, on the other hand, heat energy is drawn directly from the cell and goes to increase the energy available for current: the cell then becomes colder when running, and its E.M.F. increases with rise in temperature. For a fuller mathematical treatment of this see *Advanced Textbook of Electricity and Magnetism.*

In fact it can be proved that the E.M.F. is given by the expression $E = h + T dE/dT$, where h = heat *in energy units*, due to the chemical changes when unit quantity passes, dE/dT = rate of change of E.M.F. with temperature, and T = absolute temperature. If dE/dT be positive $E > h$; thus the heat of the cell itself is drawn upon to maintain the current, and the cell cools when running. If dE/dT be negative the opposite will be the case. If $dE/dT = 0$ we have $E = h$: this was assumed in the calculation above. (The equation is known as the equation of Helmholtz or of Willard Gibbs—or as the *Gibbs-Helmholtz* equation.)

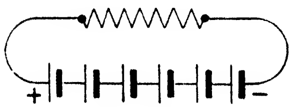


FIG. 362.

14. Grouping of Cells

In connexion with this section, note that *the E.M.F. of a cell does not depend on size of plates nor on their distance apart*: it depends only on the materials employed in its construction (and on its temperature—and concentration of solutions). *The internal resistance does depend on size*: the further the plates are apart and the smaller the plates the greater the resistance.

(1) SERIES GROUPING (Fig. 362).—Here the negative pole of one cell is joined to the positive pole of the next. If there are n cells each of E.M.F. E and resistance r, the combined E.M.F. is nE and the total internal resistance nr; hence, if I be the current and R the external resistance,

$$I = \frac{nE}{nr + R}.$$

Extreme Cases.—(*a*) If the external resistance be very large compared with the internal, the latter may be neglected and $I = nE/R$; this is n times the current that one cell would give. (*b*) If the external resistance be very small compared with the internal, the former may be neglected and $I = E/r$; this is the current which one cell alone would give. *Thus a series grouping lends itself to a large external resistance.*

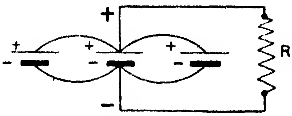


FIG. 363.

(2) PARALLEL GROUPING (Fig. 363).—Here all the high potential plates are connected, forming, as it were, one large plate, and similarly all the low potential plates are connected. Since the E.M.F. does not depend on size of plates the combined E.M.F. is simply that of one cell, viz. E. The total internal resistance is, however, $1/n$ that of one cell, viz. r/n, and the *external* (i.e. *total*) current I is given by

$$I = \frac{E}{(r/n) + R}.$$

Extreme Cases.—(*a*) If the external resistance be very large this becomes E/R, the current which one cell alone would give. (*b*) If the external resistance be very small I becomes nE/r; this is n times the current that one cell would give. *Thus a parallel grouping lends itself to a low external resistance.*

(3) MIXED GROUPING (Fig. 364).—Using the above facts,

(*a*) E.M.F. of each row $= 4E$. Resistance of each row $= 4r$.

(*b*) E.M.F. of 3 rows in parallel $= 4E$. Resistance $= 4r/3$;

$$\therefore I = \frac{4E}{(4r/3) + R}, \text{ i.e. } I = \frac{sE}{(sr/p) + R},$$

if there are s cells in series per row and p rows in parallel.

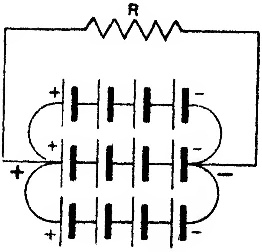


FIG. 364.

(4) GROUPING FOR MAXIMUM CURRENT.—Dividing numerator and denominator of (3) by s, we have

$$I = \frac{E}{(r/p) + (R/s)}.$$

The numerator is constant, hence I will be a maximum when the denominator

is least. But the latter is the sum of two terms whose product is constant, and therefore will be least when the terms are equal; hence I will be a maximum if

$$\frac{R}{s} = \frac{r}{p}, \text{ i.e. if } R = \frac{sr}{p}.$$

Thus to secure the greatest current the resistance of the battery (sr/p) *must be made as near as possible equal to the external resistance* (R).

As an illustration let us find the grouping for maximum current from 24 cells each of resistance 4 ohms, the external resistance being 6 ohms.

$$sp = 24; \quad \therefore p = 24/s.$$

$$\text{Now } \frac{sr}{p} = R \text{ or } sr = pR; \quad \therefore 4s = 6p;$$

$$\therefore s = \frac{6p}{4} = \frac{6}{4} \times \frac{24}{s} = \frac{36}{s}; \quad \therefore s^2 = 36, \text{ i.e. } s = 6.$$

The grouping is 6 cells in series per row, 4 rows in parallel.

(5) CELLS OF DIFFERENT E.M.F.'S IN PARALLEL.—In the above cases the cells have been of equal E.M.F.'s. If cells and batteries of unequal E.M.F.'s are in *parallel*, application of Kirchhoff's Laws will enable the solutions to be obtained. The method will be best understood from an example.

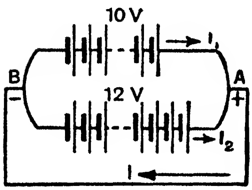


FIG. 365.

Example.—*A battery of 10 volts and internal resistance ·5 ohm is connected in parallel with one of 12 volts and internal resistance ·8 ohm. The poles are connected by an external resistance of 20 ohms. Find the current in each branch.*

Let Fig. 365 represent the details. Taking the top battery and the outside circuit, and applying Kirchhoff's law (page 297) we get:—

$$20I + \cdot 5I_1 = 10 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Taking now the bottom battery and the outside circuit we get:—

$$20I + \cdot 8I_2 = 12 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Dividing (1) by ·5 and (2) by ·8 gives:—

$$40I + I_1 = 20 \text{ and } 25I + I_2 = 15$$

Adding these and remembering that $I_1 + I_2 = I$, we get:—

$$66I = 35; \quad \therefore I = \tfrac{35}{66} \text{ ampere.}$$

Substituting this in (1)—

$$\cdot 5I_1 = 10 - \tfrac{700}{66} = -\tfrac{40}{66}; \quad \therefore I_1 = -\tfrac{80}{66} \text{ ampere,}$$

and substituting in (2)—

$$\cdot 8I_2 = 12 - \tfrac{700}{66} = \tfrac{92}{66}; \quad \therefore I_2 = \tfrac{115}{66} \text{ amperes.}$$

Thus the current through the stronger battery is 115/66 amperes, of which 35/66 ampere flows through the external resistance and 80/66 amperes flow *back through the weaker battery.*

(6) BATTERY EFFICIENCY.—The efficiency of a battery is measured by *the ratio of the power in the external circuit to the total power developed.* If the external circuit does not contain any arrangement involving additional work (*e.g.* a voltameter) the whole of the power outside appears as heat; hence if r be the internal and R the external resistance,

$$\text{Efficiency} = \frac{I^2R}{I^2(R+r)} = \frac{R}{R+r} = \frac{\text{Terminal P.D.}}{\text{E.M.F.}},$$

and this is nearer unity (100 per cent.) the smaller the value of r.

If the battery has to do work apart from heating, the efficiency *of the system* refers to *the ratio of the power spent in this useful work to the total power developed*: hence if e be the back E.M.F.,

$$\text{Efficiency} = \frac{eI}{EI} = \frac{e}{E} = \frac{\text{Back E.M.F.}}{\text{Battery E.M.F.}}$$

Since $EI = I^2R + eI = I^2R + w$, where $w = eI$ = power spent in useful work (page 361) we get on solving the equation:—

$$I = \frac{1}{2}\frac{E}{R} \pm \sqrt{\frac{(E^2/4R) - w}{R}}.$$

Now if w exceeds $E^2/4R$ the expression under the root sign becomes negative; hence the greatest value which w can have is $E^2/4R$, in which case $I = \frac{1}{2}E/R$. Thus *when there is maximum useful power* (w) *in the external circuit the efficiency* (w/EI) *is* $\frac{1}{2}$ *or* 50 *per cent. and the back E.M.F.* (e) $= \frac{1}{2}E$.

15. Electrode Potentials. Measuring Electrode Potentials

We saw in Chapter IX. that when a metal plate is placed in a solution containing ions of the same metal, the **osmotic pressure** of the ions in solution tends to drive positive ions to the plate, whilst the **solution pressure** of the ions of the plate tends to drive positive ions to the solution, and that when equilibrium is established there is a P.D. at the boundary (page 264). As explained, the P.D. between the metal and the solution is called an **electrode potential,** and the E.M.F. of a cell, as usually measured, is numerically equal to the algebraic sum of all the single electrode potentials (and other P.D.'s due to contacts of dissimilar substances) occurring in it.

To measure a single electrode potential, say that between zinc and sulphuric acid (H^+ is the +ve ion in the solution here), another metallic junction must be put in the liquid to connect it to the measuring instrument, and this introduces another contact P.D.—a difficulty and a source of error. One method was to take, say,

Zinc | Sulphuric acid | Mercury,

and as the **capillary electrometer** was considered to give the potential difference $H_2SO_4 \mid Hg$ and the total P.D. between the zinc and mercury can be determined, the remaining single potential difference $Zn \mid H_2SO_4$ was found.

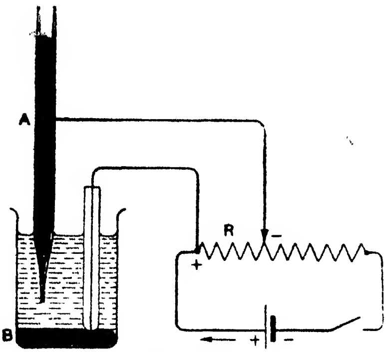


FIG. 366.

The *capillary electrometer* is an application of the effect of electric currents on the surface tension of the boundary between mercury and an acid. At the contact a P.D. is set up such that the mercury is positive to the acid, and the surface tension depends upon this P.D., becoming a maximum when this P.D. is reduced to zero. In Fig. 366 the vessel B contains mercury and dilute sulphuric acid; A is a tube drawn out to a fine point and containing mercury. B is joined to the high potential end and A to the low potential end of an adjustable resistance through which a current is passing, so that this *applied* potential difference, viz. current × resistance, may be varied. Now if the surface tension increases the meniscus will rise in A, while if the surface tension decreases it will fall. By adjusting the resistance the applied P.D. is altered until the mercury in A reaches its highest point, in which case the mercury *in A* and the acid are at the same potential; the P.D. given by the product of the resistance and the current is then equal to the P.D. between mercury and acid since the two are exactly balancing. The electrode potential $H_2SO_4|Hg$ is thus known.

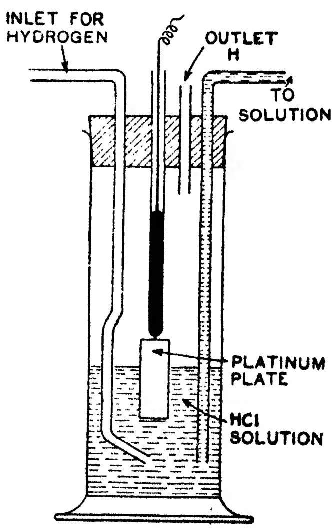


FIG. 367.

Earlier determinations of single electrode potentials showed, however, wide discrepancies, and it was realised that we must base our values of electrode potentials upon a *standard electrode potential which we would agree arbitrarily to regard as zero*, and Nernst suggested the **hydrogen electrode** for this purpose. Hydrogen gas in contact with a solution containing hydrogen ions acts like a metal in a solution containing ions of

the same metal, *i.e.* there is an electrode potential. If the gas is at a pressure of one atmosphere and the solution has a concentration of hydrogen ions of one gram-ion per litre the electrode potential is taken as being zero, and all other electrode potentials are measured from this. It is clear that by suitably combining this electrode with the $Zn|H_2SO_4$ above, the electrode potential between Zn and H_2SO_4 relative to the hydrogen standard as zero will be determined. The normal hydrogen electrode standard itself (Fig. 367) consists of a strip of platinum coated with finely divided platinum and immersed in a solution of hydrochloric acid, and a stream of hydrogen gas is bubbled through the solution.

The hydrogen standard electrode is not an ideally convenient one to use in *practical* testing, and usually what is called a standard **calomel electrode** (Fig. 368) is employed in experiment, this having been standardised in terms of the hydrogen standard. The bottom of the tube in Fig. 368 is covered with a layer of dry mercury, above this a layer of a paste made by working together in a mortar mercury, calomel (Hg_2Cl_2), and a little potassium chloride (KCl) solution, and then a normal solution of potassium chloride which has previously been saturated with calomel. Metallic connexion with the mercury is made by a platinum wire sealed through a glass tube. The side tube is a syphon tube, which is filled with normal potassium chloride solution, by means of which the electrode may be combined with other electrodes in solutions (*e.g.* in the case of our $Zn \mid H_2SO_4$ test the end of this tube is put in the H_2SO_4). The P.D. of the calomel electrode referred to the hydrogen standard is 0·2816 volt at 25° C., the mercury being positive (the P.D. is essentially at the calomel-potassium chloride junction). Thus to find the P.D. (E_1) for a metal M and its solution we combine with the calomel electrode:—

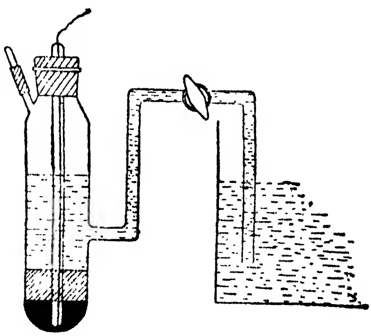


FIG. 368.

$$\begin{array}{ll} M \mid M \text{ solution} \,\|\, KCl \mid Hg_2Cl_2 \mid Hg & \\ E_1 & \longrightarrow E_2 = \cdot 2816, \end{array}$$

and as the total P.D. between M and Hg is readily determined, the magnitude (and direction) of E_1 is found.

The electrode potentials of a few elements in equilibrium with their ions in a normal solution are given below. These values are based on the *hydrogen standard* as explained above.

ELECTRODE POTENTIALS—REFERRED TO HYDROGEN STANDARD
(Metal in contact with a normal solution of its ions)

METAL	ION	POTENTIAL VOLTS	METAL	ION	POTENTIAL VOLTS
H	H^{+}	0·00	Fe	Fe^{++}	− 0·44
K	K^{+}	− 2·92	Sn	Sn^{++}	− 0·14
Na	Na^{+}	− 2·71	Cu	Cu^{++}	+ 0·34
Mg	Mg^{++}	− 1·55	Hg	Hg_2^{++}	+ 0·80
Zn	Zn^{++}	− 0·76	Ag	Ag^{+}	+ 0·80

The above table enables us to calculate the E.M.F. of a cell constructed of two metals in contact with solutions of their salts, the concentration being 1 gram-ion per litre. Take a Daniell's cell of this type

$$Cu \mid CuSO_4 \mid ZnSO_4 \mid Zn \mid Cu \text{ wire.}$$

The contacts $CuSO_4|ZnSO_4$ and Zn|Cu have small P.D.'s and may be neglected. The electrode potential $Cu|CuSO_4$ is + ·34, *i.e.* the copper is above the solution: the value for $Zn|ZnSO_4$ is − ·76, *i.e.* the zinc is *below* the solution. The E.M.F. is therefore

$$·34 + ·76 = 1·1 \text{ volt.}$$

(1) Mathematically it can be shown that the P.D. at the boundary of a metal and a *normal* solution of its salt (at ordinary room temperature 17° C.) is given by the formula:—

$$E = \frac{0·058}{n} \log_{10} \frac{P}{p} \text{ volts}$$

where n = valency of metal, P = solution pressure of metal, and p = osmotic pressure of ions in solution. It follows that for every tenfold increase or decrease in the osmotic pressure (which is proportional to the concentration) there is a *change* in the P.D. of ·058|n volt. To obtain the E.M.F. of a Daniell (or other reversible cell) we would find the *algebraic sum* of two expressions of this type, one for the $Zn|ZnSO_4$, the other for the $CuSO_4|Cu$. For the concentration cell of Fig. 263 we would have:—

$$\text{E.M.F.} = \frac{·058}{n}\left(\log_{10}\frac{P}{p_1} - \log_{10}\frac{P}{p_2}\right) = \frac{·058}{n}\log_{10}\frac{p_2}{p_1} = \frac{·058}{n}\log_{10}\frac{c_2}{c_1}$$

where c_2 and c_1 = concentrations of the stronger and weaker solutions respectively (strictly an expression for the P.D. at the contact of the solutions should also be included). For a full treatment and proofs see *Advanced Textbook of Electricity and Magnetism.*

(2) Now consider again the *electrolysis* of copper sulphate with copper electrodes. Before joining up the battery there is an electrode potential at the surface of each plate, the copper being $+ \cdot 34$ volt relative to the solution. If one plate be made more negative, *copper ions will be deposited on it* from the solution to keep up the equilibrium P.D. If the other plate be made more positive *copper ions will pass from it into solution* for the same reason. Thus we can electrolyse copper sulphate in this voltameter merely by keeping one electrode (anode) at a little higher potential and the other (cathode) at a little lower potential than the equilibrium potential, *i.e.* only a small P.D. need be applied.

There are H^+ ions in the solution as well as Cu^{++} ions, but copper will be deposited at the cathode rather than hydrogen unless the cathode be made at least $\cdot 34$ volt more negative. Further, there are OH^- as well as SO_4^{--} ions, but copper will be dissolved at the anode rather than the liberation of these, for both of these would necessitate the anode being much more positive (about $\cdot 9$ and nearly 2 respectively). With a platinum anode no copper (or platinum) can be dissolved at the anode, and OH^- ions liberated join up to form water and oxygen, viz.

$$4\,OH = 2\,H_2O + O_2.$$

16. Accumulators or Secondary Cells

In the usual accumulator lead plates are used: the following experiment will show the principle (compare with Art. 4).

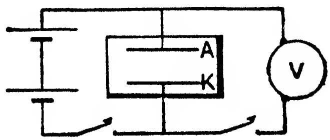


FIG. 369.

Consider two lead plates, A and K (Fig. 369), in dilute sulphuric acid, and let a current be passed in at A. The solution is decomposed, oxygen appearing at the anode A, hydrogen at the cathode K. Some oxygen at A combines with the surface lead (Pb) forming a dark brown peroxide of lead (PbO_2); the hydrogen at K mostly rises to the surface.

When the "charging" process, as it is termed, has continued for some time, disconnect the battery and join A and K through a voltmeter V; this will read about 2 volts at first, a (conventional) current will flow through the outside circuit in the direction A to K, and, decreasing gradually, will after a time cease. On examining the plates it will be found that the peroxide has disappeared, and both plates have lead sulphate ($PbSO_4$) on them.

The charging process may now be repeated by passing a (conventional) current in at A. The liberated oxygen at A will convert the lead sulphate there into lead peroxide, while the hydrogen at K will reduce the lead sulphate there to the metallic state: thus the plates are again in their "formed" condition, viz. PbO_2 at A and Pb at K, and the cell is again ready to supply a current (Fig. 370).

Such an arrangement is called an *accumulator* or *secondary cell* or storage cell: the anode A is the *positive plate* and the cathode K the *negative plate* of the accumulator.

It was noted that on the first " charge " the negative K remained more or less in the initial condition and as such it does not very readily combine to form lead sulphate when the cell is discharging. This defect is overcome (and also more PbO_2 produced at the positive) by using the " alternate " charge process introduced by *Planté*. The current is first passed through in the direction A to K, and A is peroxidised; it is then reversed, in which step the oxygen at K forms PbO_2 there, while the hydrogen at A reduces the existing PbO_2 to *porous spongy lead*. This is several times

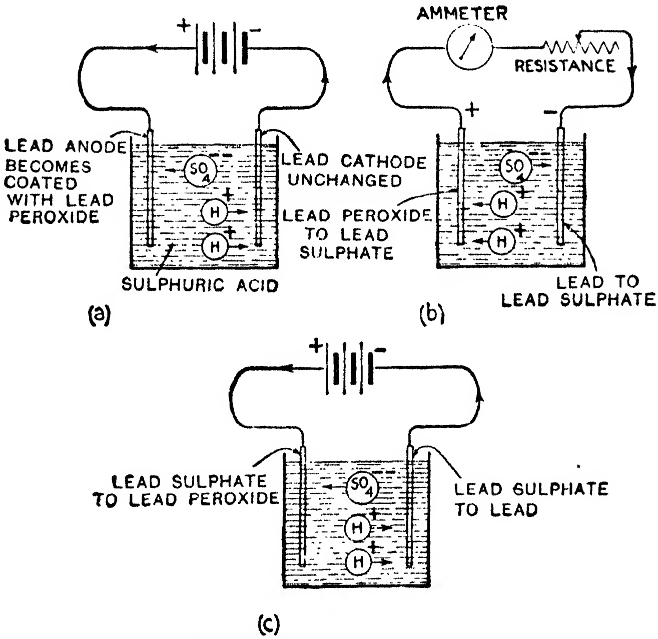


FIG. 370. (*a*) Charging; (*b*) Discharging; (*c*) Recharging.

repeated, with the final result that the last anode or positive has a *thick* coating of dark brown lead peroxide, while the last cathode or negative is *thickly* coated mainly with metallic lead of a greyish colour in a *porous spongy condition*, readily acted on during discharge.

To obviate the tedious formation of the Planté plates *Faure* coated the plates, prior to charging, with a paste of red lead (Pb_3O_4) and sulphuric acid; on charging, the red lead on the cathode becomes quickly reduced to spongy metal, that on the anode becomes peroxidised. [Later, litharge (PbO) was used, and

it is always used for negative plates nowadays.] Then the *Sellon-Volckmar* plates were introduced, which, being constructed in the form of grids, more effectively secured the paste. Thus accumulators follow in general two main types—(1) the Planté "formed" cell, and (2) the Faure or pasted grid cell; frequently they are of a composite character, having Planté positives and pasted negatives, since pasted positives, particularly in power station batteries, are liable to disintegration (falling of paste).

17. Chemical Changes During Discharge and Charge

We need only take one type of cell, say one of the Planté type. Consider a charged cell: the positive is coated with the peroxide PbO_2, while the negative is coated with spongy lead. (The + plate is chocolate-brown in colour, and the − plate greyish.)

(1) *Chemical Changes during "Discharge."* In discharging the current is taken from the cell in the direction positive to negative plate in the *outside circuit*; hence, *in the cell*, hydrogen appears at the positive and oxygen at the negative. The hydrogen at the positive combines with the peroxide:—

$$PbO_2 + H_2 = PbO + H_2O \quad \ldots\ldots (1)$$

$$PbO + H_2SO_4 = PbSO_4 + H_2O \quad \ldots\ldots (2)$$

The oxygen at the negative acts on the lead thus:—

$$2\,Pb + O_2 = 2\,PbO \quad \ldots\ldots (3)$$

$$2\,PbO + 2\,H_2SO_4 = 2\,PbSO_4 + 2\,H_2O \quad \ldots\ldots (4)$$

At the end of "discharge" therefore, we have $PbSO_4$ at both plates (and traces of that PbO, but we need not consider this here).

(2) *Chemical Changes during "Charge."* In charging, *i.e.* passing a current from the positive to the negative plate *in the liquid*, oxygen is liberated at the former and hydrogen at the latter plate. The oxygen at the positive combines with the $PbSO_4$ thus:—

$$2\,PbSO_4 + O_2 + 2\,H_2O = 2\,PbO_2 + 2\,H_2SO_4 \quad \ldots\ldots (5)$$

The hydrogen at the negative combines with the $PbSO_4$ thus:—

$$PbSO_4 + H_2 = Pb + H_2SO_4 \quad \ldots\ldots (6)$$

At the end of "charge" therefore, we have PbO_2 at the positive plate and Pb at the negative as stated above.

It will be observed that the actions represented by the charging equations (5) and (6) increase the density of the electrolyte, while the discharging actions represented by (2) and (4) weaken it. The specific gravity of the acid is a good test of the condition of the cells; when fully charged the specific gravity ranges from 1·205 to 1·215, and when discharged from 1·17 to 1·19. The actual figures vary with the type of cell and are given by the makers.

18. A Few Details about Accumulators

A few plate details will be gathered from Figs. 371 and 372. One type of accumulator made by the Chloride Co. employs a paste negative, but the positive is rather unusual. For this a plate with circular holes is cast, the material being an alloy of lead with antimony. Pure lead in the form of a tape is passed through gimpers, which rib and cut it into definite lengths. These are rolled into coils and pressed into the circular holes in the plate, the whole being next subjected to a pressure of over 70 tons in a hydraulic press (Fig. 372).

To keep the internal resistance of *any* cell low the plates should be large and near together. Fig. 373 shows how the plates are arranged to secure

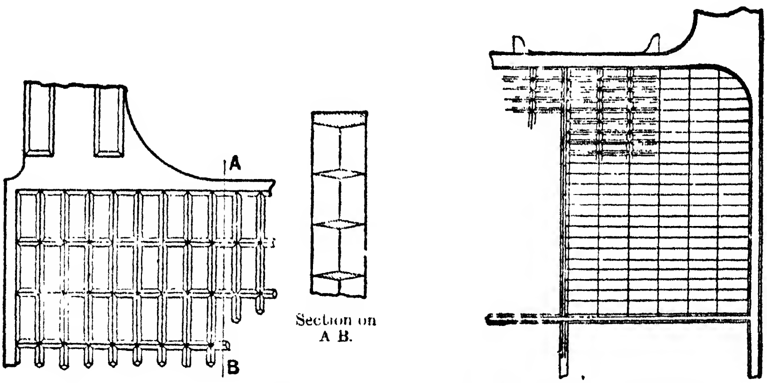


FIG. 371. *Left:* "Tudor" negative. *Right:* "E.P.S." negative.

both these conditions: glass tube separators (or ebonite forks, or celluloid, or specially prepared wood) are put between the plates to prevent internal contact and short circuit.

The E.M.F. is about 2 volts: when freshly charged it exceeds this, but it soon drops to about 2 volts, keeps fairly steady at this for some time, and then falls. When the E.M.F. has dropped to about 1·85 volts the cell should be regarded as discharged; with an E.M.F. below this it should not be worked, for the formation of a large amount of a *hard* lead sulphate (not to be confused with the *ordinary* sulphate formed during normal discharge) will make the recharge difficult, will be conducive to buckling or bending of the plates with the consequent risk of short circuit, and will shorten the life of the cell.

The capacity of an accumulator is measured in *ampere-hours*: thus if a cell has a capacity of 704 ampere-hours and the greatest discharge current as stated by the makers is 64 amperes, it will be able to give this current for 11 hours. Such a cell would, however, give a smaller current of, say, 32 amperes for *more than* 22 hours, for the capacity is greater if the discharge current is less.

The **quantity efficiency** is the ratio of the ampere-hours given out at discharge to the ampere-hours put in at charge: it is of the order 90 per cent. The **energy efficiency** is the ratio of the watt-hours discharged to the watt-hours of charge: it is of the order 75 per cent.

There must be a good space between the plates and the bottom of the containing vessel to hold any active material which may fall from the plates.

Quite a different type of cell is the *Edison accumulator* which does not employ lead. The positives are perforated nickel plated steel tubes filled with alternate layers of nickel flakes and nickel hydroxide. The negatives are nickel plated steel frames provided with rectangular holes which contain the active material, viz. powdered iron oxide and graphite. The electrolyte is a solution of caustic potash with a little lithium hydrate. E.M.F. is 1·33 volts.

19. Some Practical Applications of Electrolysis

Amongst the practical applications of electrolysis are such

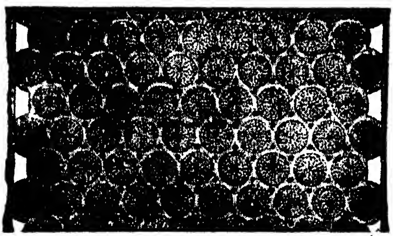


FIG. 372. Portion of "Chloride" Positive.

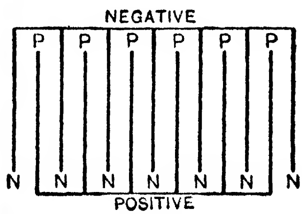


FIG. 373.

processes as electro-plating, electrotyping, the production of certain chemicals, the extraction and refining of certain metals, etc. For full details, however, some book on *Applied Electricity* or *Chemistry* must be consulted.

(*a*) *Electro-plating.* Silver plating for example is arranged thus:—Articles to be silver plated are first cleaned and then immersed in a weak solution of mercuric chloride: this produces on them a slight coating of mercury which is found to take a better deposit of silver. They are then arranged as cathodes in a plating vat (voltameter) the anode being a silver plate and the electrolyte a silver solution (see below). Small currents are used—about ·012 ampere per square inch of cathode surface. Good quality silver plate is made by first depositing a layer of copper and then the silver: the best quality uses first a coating of nickel, for, being

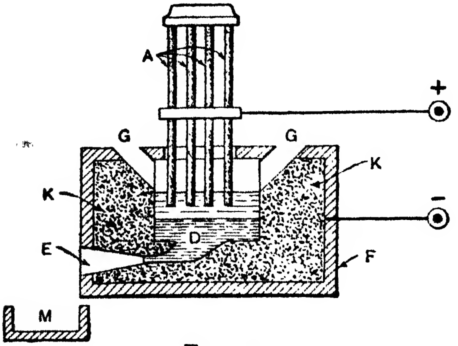


FIG. 374.

white, it is not so noticeable as copper when the silver begins to wear off. The following points may be noted:—

Silver plating. Anode = Silver; Electrolyte = Solution of double cyanide of silver and potassium.

Copper plating. Anode = Copper; Electrolyte = Solution of copper sulphate.

Nickel plating. Anode = Nickel; Electrolyte = Solution of nickel-ammonium sulphate and ammonium sulphate.

Chromium plating. Anode = Chromium; Electrolyte = Solution of chromic acid and chromic sulphate with small quantities of chromium carbonate.

Electro-gilding. Anode = Gold; Electrolyte = Solution of double cyanide of gold and potassium.

(*b*) *Extraction of Metals.* The ores of many metals can be subjected to electrolysis *when they are in a fused condition* and the *pure* metal extracted. In the important case of aluminium, for example, the electrolyte is a fused mixture of alumina (aluminium oxide), cryolite, and fluorspar, only the alumina, however, being decomposed. The electrolysis is carried out in an iron box furnace F (Fig. 374), the bottom and sides of which are thickly lined inside with blocks of carbon K. The anode consists of a number of carbon blocks A dipping into the open top of F, and the carbon lining K serves as the cathode. A very heavy current is passed, and the heat produced fuses the mixture. The alumina is electrolysed and aluminium in a molten condition collects at the bottom D of the furnace, from whence it is drawn at suitable intervals through E into the casting trough M.

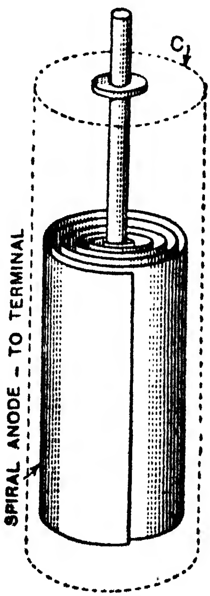


FIG. 375. Principle of Wet Electrolytic Condensers.

(*c*) *Metal Refining.* To obtain metals in a high state of purity, *e.g.* copper for electrical work, the process of electrolysis is often used.

(*d*) *The Electrolytic Condenser.* Mention was made of this on page 217. If two platinum plates be put a short distance apart in certain alkaline solutions and a current be passed, it is found that the current gets less and finally drops almost to zero. This is due to the fact that the electrolysis causes a deposit *on the anode* of a thin film *which is an insulator*, and in a short time the deposition is sufficiently complete to stop the current. The electrolyte is still, of course, a conductor, so that we now have a condenser, the anode being one plate, the electrolyte (and case) the other " plate," and the film the dielectric. The film is very thin so that the capacitance can be great. Plates are often formed into rolls as shown in Fig. 375. In wireless receivers a type is frequently used in which the electrolyte is ammonium borate contained in a paste of a fatty organic compound.

CHAPTER XIV

THERMO-ELECTRIC EFFECTS

BEFORE proceeding with this chapter the student should again read page 270 (2), where it is explained that when two different metals are in contact, electrons diffuse in both directions across the junction, and finally, when equilibrium is established, a P.D. exists at the contact, the magnitude of which depends on the metals and the junction temperature. If two metals make a circuit as in Fig. 204 (*b*) the two junction P.D.'s will cancel each other, but if one junction be heated the contact P.D. there alters, the two no longer balance, and current flows: this current is known as a *thermo-electric current*.

1. The Seebeck Effect

Fig. 376 represents pieces of copper and iron joined at A and B, and G is a mirror galvanometer. If both junctions be initially at 0° C., and B be then gradually heated, a current will flow in the direction indicated, viz. copper to iron through the hot junction, and iron to copper through the cold junction (thermo-electric current).

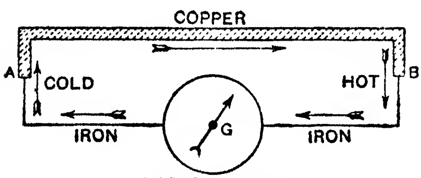


FIG. 376.

This current will increase until the hot junction is at a temperature of about 270° C. (different for different specimens), at which stage the maximum current will be flowing. If heating be continued the current will decrease, and will be zero when the hot junction is at 540° C. On heating B still further, the current will increase again, but *it will be reversed in direction, i.e.* it will flow from iron to copper through the junction B. The production of currents in this way was discovered by *Seebeck* in 1821 (the phenomenon of ***inversion*** of E.M.F. was discovered in 1823 by Cumming), and the effect is referred to as the **Seebeck effect.**

If both junctions of the copper-iron couple be initially at 10° C. (say) instead of at 0° C., there will again be maximum current when the hot junction is at 270° C., but there will be zero current and

reversal in this case when the hot junction is at 530° C. *The temperature of the hot junction at which maximum current flows is a constant for a given couple, and is known as the* **neutral temperature** *for that couple.* The temperature of the hot junction at which there is zero current and reversal is a variable one, being always as much above the neutral temperature as the cold junction is below it.

Other pairs of metals will indicate similar results, the numerics being, of course, different: best results are with bismuth and antimony (current from bismuth to antimony through the hot junction). These E.M.F.'s are small; thus with a copper-iron couple with junctions at 0° C. and 100° C. the E.M.F. is only about ·0013 volt.

Suppose the two junctions of a couple are initially at a temperature T° C., that one of the junctions is then raised in temperature by a very small amount dT, and that dE is the corresponding small E.M.F. developed: the ratio dE/dT measures what is termed the **thermo-electric power** *of the two metals*, i.e. *of the couple, at temperature* T. We might also define it as the rate of change of E.M.F. with the temperature of one junction (dE/dT); or a simple definition might be given as follows: Consider a couple with its "cold" junction at temperature $(T - \frac{1}{2})$° C. and its "hot" junction at temperature $(T + \frac{1}{2})$° C., that is, the mean temperature is T° C. and the temperature difference between the junctions is 1° C.: then the E.M.F. developed may be taken to measure the *thermo-electric power of the couple at* T° C.

It will be seen presently that if one junction of a couple be at T_1° C. and the other at T_2° C., then, *provided that one of these is not above and the other below the neutral temperature* θ, the total E.M.F. is given by the product of the thermo-electric power at the mean temperature $\frac{1}{2}(T_1 + T_2)$ and the difference in temperature $(T_2 - T_1)$. If T_1 be *below* and T_2 *above* the neutral temperature θ, the actual E.M.F. in the circuit is found thus: Total E.M.F. = (E.M.F. with junctions at T_1 and θ) − (E.M.F. with junctions at θ and T_2).

Two simple laws which have been established by experiment may be briefly noted here. The *law of intermediate temperatures* states that the E.M.F. for a couple with junctions at temperatures t_1 and t_n is equal to the sum of the E.M.F.'s for any number of successive steps into which the given temperature range may be divided: that is:—

$$E_{t_1}^{t_n} = E_{t_1}^{t_2} + E_{t_2}^{t_3} + E_{t_3}^{t_4} + \ldots + E_{t_{n-1}}^{t_n}$$

where t_2, t_3, t_4, etc., are successive temperatures between t_1 and t_n. The *law of intermediate metals* practically states that the insertion

of another metal into a couple circuit does not change the total E.M.F. *provided that the added metal is entirely at the temperature of the part of the circuit where it is inserted:* thus in Fig. 376 the insertion of the copper coil of the galvanometer into the iron wire may be taken as causing no change in the E.M.F. in the couple circuit: this is evident from the facts given on page 270.

2. Graphical Representation of E.M.F. in a Couple Circuit

(1) Consider a copper-iron couple with one junction kept at 0° C. and the other gradually heated, and suppose the total E.M.F. is measured at various stages during the experiment. If the results be plotted, temperatures along the horizontal and **total E.M.F.'s** vertically, a curve such as ONB (Fig. 377) is obtained. The point N indicates maximum E.M.F. (= NK) with the hot junction at the neutral temperature 270 °C., and B indicates zero E.M.F. and reversal with the hot junction at 540° C. In normal cases the E M.F. goes on increasing in the reversed direction, the curve beyond B being a geometrical continuation of the parabolic segment ONB. The experimental method of measuring the total E.M.F. in a couple circuit is described in Chapter XV.

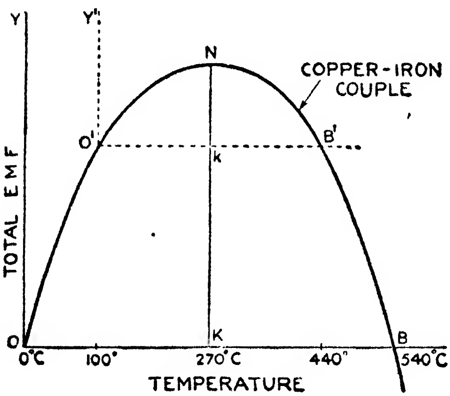


FIG. 377.

If with this couple the cold junction be at 100° C. instead of 0° C. then the E.M.F. curve can be obtained from ONB simply by changing the origin O to O′ and taking O′B′ as axis of temperature and O′Y′ as axis of E.M.F. N again indicates maximum E.M.F. (= N*k*) at 270° C., but B′ now indicates zero E.M.F. and reversal at 440° C.

(2) To include the total E.M.F. parabolic curves such as Fig. 377 for *all possible couples* on one diagram would result in considerable complication, and another method suggested by Prof. Tait is usually employed. Instead of mapping total E.M.F. against temperature as above, the **thermo-electric powers** (dE/dT), *i.e.* say the E.M.F.'s with unit difference of temperature between the

junctions are marked off on the vertical. Thus in Fig. 378 the ordinate TR represents the thermo-electric power of a couple at T° C., *i.e.* TR represents the E.M.F. when the cold junction is at $(T - \frac{1}{2})°$ and the hot junction at $(T + \frac{1}{2})°$, the mean temperature being T° C. Similarly T_1P is the thermo-electric power for the couple at temperature $T_1°$ and T_2Q that at temperature $T_2°$, and so on. For nearly all couples the curve obtained by joining the tops of the ordinates is a straight line such as LN. The point N evidently corresponds to the neutral temperature for the couple, for the thermo-electric power *at this temperature* is nil (remember the E.M.F. is always nil when the neutral temperature for the couple is the *mean of the hot and cold junction* temperatures: when the *hot junction itself* is at the neutral temperature the E.M.F. is a maximum).

The *total E.M.F.* in the couple circuit when the cold junction is, say, at $T_1°$ and the hot junction at $T_2°$, can be found from the diagram. Considering first the narrow dotted strip, the vertical TR is the E.M.F. when the junctions are $(T - \frac{1}{2})°$ and $(T + \frac{1}{2})°$. The area of the strip is TR × width of strip, and as the width is unity the area is numerically the same as TR, *i.e.* the area of the strip represents the E.M.F. For junctions at T_1 and T_2 the total E.M.F. will be obtained by adding up a number of such narrow strips from T_1P to T_2Q, and the sum of these will be the whole area T_1PQT_2: thus the area T_1PQT_2 represents the total E.M.F. with junctions at temperatures $T_1°$ and $T_2°$ respectively.

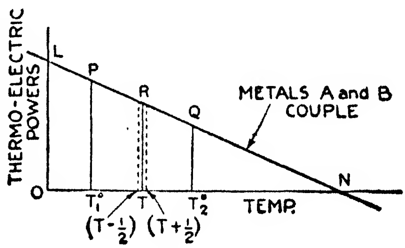


FIG. 378.

In fact at any temperature T the value of the ordinate is dE/dT (Art. 1). For a small increment of temperature dT at this temperature the increment (dE) in the E.M.F. is given by $(dE/dT)\,dT$, the product of the rate of change of E.M.F. with temperature and the change in temperature. In the figure the width of the dotted strip corresponds to dT and the area to the E.M.F., viz. $(dE/dT)\,dT$. For the total E.M.F. with junctions $T_1°$ and $T_2°$ we add up a number of these very narrow areas giving the total area T_1PQT_2.

For reasons which will be explained later, curves such as LN are drawn for all the metals *using lead as the other metal of the couple,* and the diagram obtained is called a **thermo-electric diagram**: from

such a diagram the total E.M.F. in a couple consisting of *any two metals* with junctions at *any two temperatures* can be found. The complete thermo-electric diagram is explained in Art. 7, but one or two points may be briefly noted at this stage in an elementary way.

Suppose A_1A_2 (Fig. 379) is the thermo-electric power line for a couple, say, of a metal A and lead, and B_1B_2 the line for a couple of metal B and lead. T_1D_1 is therefore the thermo-electric power at temperature $T_1°$ for the metal A and lead: T_1D_2 is the thermo-electric power at temperature $T_1°$ for the metal B and lead: hence D_1D_2 is the thermo-electric power at $T_1°$ for a couple composed of metals A and B, and similar remarks apply to other temperatures.

Again from the preceding the area $D_2T_1T_3C_2$ represents the total E.M.F. for a couple consisting of metal B *and lead* when the cold junction is at $T_1°$ and the hot at $T_3°$. Similarly the area $D_1T_1T_3C_1$ represents the total E.M.F. for a metal A and lead couple with the same junction temperatures $T_1°$ and $T_3°$. Hence the difference of these two areas, viz. the area $D_1D_2C_2C_1$ represents the total E.M.F. for a couple consisting of metals A and B with a cold junction at $T_1°$ and a hot junction at $T_3°$. Further, $D_1D_2C_2C_1$ is a trapezium, the area of which is given by $\overline{aa} \times \overline{bb}$: but $\overline{aa}$ represents the thermo-electric power for the A and B couple at the mean temperature of the junctions ($T_2°$), and $\overline{bb}$ is the temperature difference between the junctions ($T_3 - T_1$), so that as stated in Art. 1:—

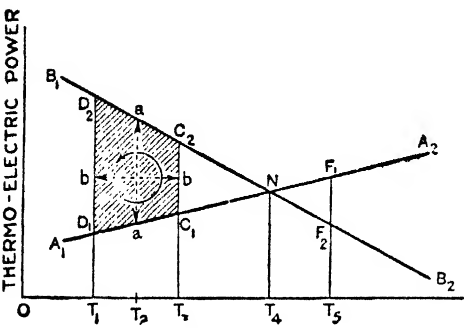


FIG. 379.

$$\text{Total E.M.F.} = \frac{\text{Thermo-electric power}}{\text{at mean temperature}} \times \frac{\text{Difference in temperature}}{\text{of junctions.}}$$

The lines are so drawn on a thermo-electric diagram that the E.M.F. acts *counterclockwise* round the circuit. Thus in Fig. 379 with a couple consisting of the metals A and B with junctions at $T_1°$ and $T_3°$ the E.M.F. acts as indicated by the arrow, *i.e.* it acts from A to B (lower line to upper line) across the hot junction and B to A across the cold.

The crossing point N of the A and B lines indicates the neutral temperature ($T_4°$) for the A.B couple (the thermo-electric power at this temperature for this couple is zero). If the cold junction for the couple be kept at $T_1°$ and the other junction be gradually heated, the area representing the total E.M.F. increases: when the hot junction is at $T_4°$ the E.M.F. area is the triangle D_1ND_2 and the E.M.F. is a maximum. If the hot junction be raised above $T_4°$ the E.M.F. decreases for the B line is now below the A line instead of above it as before and E.M.F.'s added now are opposite: thus if the hot junction is at $T_5°$ the triangle area on the right of N (viz. triangle NF_1F_2) must be subtracted from the triangle area (D_1ND_2) on the left to obtain the resultant E.M.F. area. If the hot junction be raised as much above $T_4°$ as the cold junction is below $T_4°$ the triangle on the right becomes equal to that on the left and the resultant E.M.F. is zero. Raising the hot junction beyond this brings on an E.M.F. again, but it will be in the opposite direction.

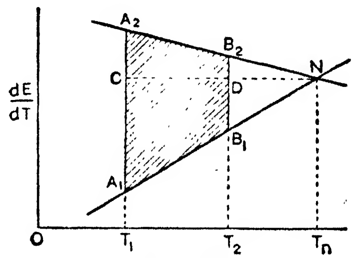


FIG. 380.

From the diagram we can obtain an expression for the total E.M.F. in terms of the various temperatures. From Fig. 380 we have (denoting the neutral temperature by T_n and the two junction temperatures by T_1 and T_2):—

$$\text{E.M.F.} = \text{Area } A_1A_2B_2B_1 = \text{Triangle } A_1A_2N - \text{Triangle } B_1B_2N;$$

$$\therefore \text{E.M.F.} = \tfrac{1}{2}(A_1A_2 \times CN) - \tfrac{1}{2}(B_1B_2 \times DN).$$

$$\text{But (similar triangles) } A_1A_2 \propto CN \text{ and } B_1B_2 \propto DN;$$

$$\therefore \text{E.M.F.} \propto CN^2 - DN^2.$$

$$\text{Now } CN = (T_n - T_1) \text{ and } DN = (T_n - T_2);$$

$$\therefore \text{E.M.F.} \propto (T_n - T_1)^2 - (T_n - T_2)^2,$$

$$\textit{i.e.} \text{ E.M.F.} \propto (T_1^2 - 2T_1T_n - T_2^2 + 2T_2T_n) \propto (T_1 - T_2)(T_1 + T_2 - 2T_n);$$

$$\therefore \text{E.M.F.} = k(T_1 - T_2)\left(\frac{T_1 + T_2}{2} - T_n\right) \ldots\ldots\ldots \quad (1)$$

where k is a constant depending on the metals (see Art. 6). Clearly, the E.M.F. is zero: (1) if $T_1 = T_2$, *i.e.* junctions at the same temperature, and (2) if $(T_1 + T_2)/2 = T_n$, *i.e.* if the *mean* of the hot and cold junction temperatures is equal to the neutral temperature: both these facts have been previously mentioned.

3. The Peltier Effect

Fig. 381 again depicts a copper-iron couple, but with a battery in the circuit. Experiment shows that with current passing as indicated heat is *absorbed* at the junction B and *generated* at junction A, and by comparing Figs. 376 and 381 it will be seen that that particular junction is cooled which must be heated in order to give a thermo-current in the same direction as the battery current; thus in Fig. 381 the junction B would have to be *heated* in order to give a thermo-current in the direction of the arrows, and when a battery is joined up so as to give a current in this direction the junction B is *cooled*. This discovery was made by *Peltier* in 1834, and it is known as the **Peltier effect.** In practice the whole circuit of Fig. 381 would be heated by the usual Joule (or I^2Rt) heating effect for these Peltier effects are usually very small in comparison and special arrangements have to be made to detect and measure them (see below). It follows directly from the above that when a thermo-electric current flows in a circuit such as Fig. 376 heat will be absorbed at the hot junction and liberated at the cold junction owing to these Peltier junction effects.

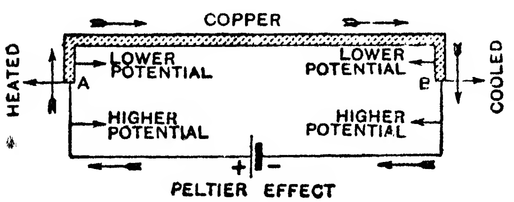


FIG. 381.

As in Art. 1 the explanation of the Peltier effect also lies fundamentally in the fact that contact of dissimilar substances gives rise to potential differences at the contact. When iron touches copper there is, as we have seen, a contact PD, the iron being *above* the copper; thus at B (Fig. 381) the battery current traverses a junction where there is a slight up-gradient of potential due to the contact, so that it gains energy which is absorbed as heat and a cooling effect results; at A the current traverses a junction in the direction of the contact down-gradient of potential, so that it gives out energy and a heating effect results.

The energy (ergs) absorbed or evolved at a junction when unit e.m. current flows for one second, *i.e.* when unit e.m. quantity passes measures what is termed the **Peltier coefficient** (P) of the junction. If a current I e.m. flows for t seconds the energy absorbed or evolved at the junction will be PIt ergs. But if V be the contact P.D. at the junction in e.m. units, this energy is equal to VIt ergs

(page 306). Hence P = V, and thus the Peltier coefficient of a junction is numerically equal to the contact P.D. at the junction in e.m. units. We have expressed the coefficient in *ergs per e.m. quantity unit:* if the coefficient be expressed in *joules per coulomb* it will be numerically equal to the contact P.D. in volts.

Note that the Joule heating effect is proportional to the square of the current and is *not reversible* (page 290). The Peltier effect is proportional to the current and is *reversible*: thus with current going as in Fig. 381 energy is absorbed at B and there is a cooling effect, but if the current be reversed in direction energy will be liberated at B and there will be a heating effect.

The existence of the Peltier effect may be shown by the apparatus of Fig. 382(*a*), in which fairly thick rods of bismuth and antimony are used. When current passes the Joule heating is the same at both junctions, but the Peltier effect causes a generation of heat at one junction (on the left) and an absorption of heat at the other junction (on the right), and the index moves as indicated, *i.e.* away from the junction where more heat is developed. If the current is reversed the index will move in the opposite direction.

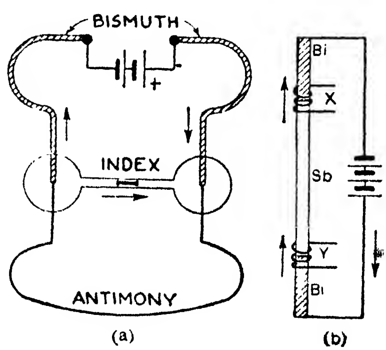


FIG. 382.

Fig. 382(*b*) depicts another method which will be understood on reading the next chapter. Two equal coils X and Y are wrapped round the junctions, joined to a Wheatstone bridge and a balance obtained. Current is then passed through the rod, the junction at X is heated more than that at Y, the resistance of coil X therefore increases more than that of Y, and the bridge balance is upset.

Similarly the value of the Peltier coefficient (P) may be determined by an experiment of the following type: Let (say) a copper-iron junction be immersed in water, and let a current I pass for t seconds in the direction iron to copper; if H_1 be the heat produced and J the mechanical equivalent:

$$JH_1 = I^2Rt + PIt.$$

If H_2 be the heat produced when the same current passes for the same time in the opposite direction,

$$JH_2 = I^2Rt - PIt.$$

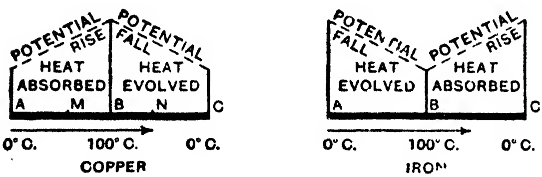
The Peltier effect is a heating effect in the first case and a cooling effect in the second case. By subtraction

$$P = \frac{J(H_1 - H_2)}{2It}.$$

4. The Thomson (or Kelvin) Effect

From theoretical considerations which are referred to later and by experiment, Lord Kelvin (then *Sir William Thomson*) showed (in 1856) that there is an absorption or evolution of energy (heat) if a current flows along a conductor when different parts of the conductor are at different temperatures, the absorption or evolution depending on the direction of the current. Thus if one end of a copper wire be at 0° C. and the other end at 100° C., and a current be passed from the cold to the hot end, heat will be absorbed, but if the current be passed in the opposite direction heat will be evolved (it is therefore a *reversible* effect). This—the absorption or evolution of energy (heat) when a current flows in an unequally heated conductor—is known as the **Thomson effect** (or Kelvin effect), and the fundamental explanation is somewhat similar to that of the Peltier.

In an unequally heated conductor different parts are at different potentials In the case of copper the hotter parts are at a higher potential than the colder; hence if a current passes as indicated along the copper wire of Fig. 383 (*a*) heat will be absorbed in the part AB and evolved in the part BC; on the whole the Joule heating effect will predominate, but if two points M and N equidistant from B be selected, N will be at a higher temperature than M owing to the effect we are considering. In the case of iron the colder parts are at the higher potential; hence if a current passes as indicated along the iron wire of Fig. 383 (*b*) heat will be evolved in the part AB and absorbed in the part BC.



(*a*) FIG. 383. (*b*)

To summarise: heat is *absorbed* when a current flows from "cold" to "hot" in copper, and heat is *evolved* when the current flows from hot to cold; heat is *evolved* when a current flows from cold to hot in iron, and heat is *absorbed* when the current flows from hot to cold. The Thomson effect in silver, zinc, antimony, and cadmium resembles that in copper, and is said to be positive; bismuth, cobalt, platinum, and nickel resemble iron, and the Thomson effect in them is said to be negative. In lead the Thomson

effect is *nil* (for this reason lead is used as the base line in thermo-electric diagrams and thermo-electric powers plotted with respect to lead—pages 413, 421-6).

The heat energy in ergs absorbed or evolved (by Thomson effect) when unit e.m. current flows for one second (*i.e.* when unit e.m. quantity passes) between two points of the conductor which differ in temperature by 1° C. measures what is called the **Thomson coefficient** (σ) or Kelvin coefficient. From the definition of the P.D. between two points it is clear that the Thomson coefficient may also be defined as numerically equal to the difference of potential in e.m. units per degree Centigrade. If the coefficient be expressed in joules per coulomb per degree Centigrade it will be numerically equal to the P.D. in volts per degree Centigrade.

To determine the energy, say, absorbed in this way *by unit quantity* of electricity in passing along a conductor from a point where the temperature is T_1 to one where the temperature is T_2 it is evident that, since σ is not a constant but varies with the temperature, the difference of temperature $T_2 - T_1$ must be divided into infinitely small steps each denoted by dT; then the energy gained at each step is $\sigma . dT$, where σ has its proper value for each step and $\int_{T_1}^{T_2} \sigma dT$ is the energy gained by unit quantity of electricity in passing from a point at temperature T_1 to one at temperature T_2.

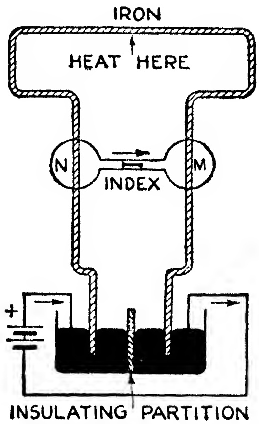


FIG. 384.

The existence of the Thomson effect may be shown by the apparatus of Fig. 384. The Joule heating is the same at M as at N. On the left, however, the current is flowing " cold " to " hot " in iron and heat is evolved, whilst on the right heat is absorbed (Thomson effect): the index moves away from the hotter part, *i.e.* away from N. If the current be reversed the index moves in the opposite direction. An exact determination of the Thomson coefficient is difficult.

5. Origin of the E.M.F. in a Thermo-Electric Circuit

Consider again a couple, one junction " hot " the other " cold," and therefore a thermo-electric current flowing in the circuit. It has been indicated that when this current is flowing heat (energy) is absorbed at the hot junction and evolved at the cold junction

(Peltier). Moreover, the current is flowing along one metal from its high temperature end to its low temperature end, and heat will again be either absorbed or evolved (Thomson) according to the kind of metal. Finally, the current flows along the other metal from its low temperature end to its high temperature end, and heat is again either evolved or absorbed (Thomson) according to the kind of metal. *On the whole, more energy is absorbed than is evolved, and the difference represents the source of the E.M.F.*

It was reasoning of this sort which led Thomson to predict the existence of the Thomson effect. If the hot junction of a couple is at the neutral temperature the thermo-electric power there is nil, there is no contact P.D., no Peltier effect, and no energy *absorbed*. But at the cold junction there is a heating effect, *i.e.* energy is *evolved* there. Hence he showed that energy must be absorbed *in one or both of the wires* in virtue of the difference of temperature between the ends, or that *energy may be absorbed in one wire and generated in the other*, but that the quantity of energy absorbed in the couple circuit must on the whole be greater than that given out.

By considering the amounts of energy absorbed and evolved in a thermo-electric circuit in the manner indicated above, we can find an expression for the total E.M.F. in the circuit in terms of the Peltier and Thomson coefficients and the temperatures. Take the case of a couple circuit of two metals A and B with junctions at temperature T_1 and T_2, the latter being the higher. Let P_1 and P_2 denote the Peltier coefficients at T_1 and T_2, and σ_A and σ_B the Thomson coefficients for the metals A and B. Then, assuming the current to pass from A to B at the hot junction, the energy gained by unit quantity of electricity in passing round the circuit (we assume *unit quantity* to pass as E.M.F. is numerically equal to "energy per unit quantity"—page 285) is:—

P_2 for the Peltier effect at junction T_2,

$-P_1$ for the Peltier effect at junction T_1 (energy *evolved*);

$\int_{T_1}^{T_2} \sigma_A dT$ for the Thomson effect in metal A,

$-\int_{T_1}^{T_2} \sigma_B dT$ for the Thomson effect in metal B (energy *evolved*).

Hence total gain by unit quantity for the complete circuit is:—

$$P_2 - P_1 + \int_{T_1}^{T_2} (\sigma_A - \sigma_B)\, dT,$$

and since this total is numerically equal to the E.M.F. we have:—

$$\text{E.M.F.} = \text{E} = \text{P}_2 - \text{P}_1 + \int_{T_1}^{T_2} (\sigma_{\text{A}} - \sigma_{\text{B}})\, d\text{T} \,\ldots\ldots\ldots (2)$$

If we consider a circuit with junctions at T and T + dT, where dT is infinitely small, the result is more simply expressed, for the Peltier coefficients may be written as P and P + dP, where dP is the difference in P corresponding to the difference in temperature dT, and σ_{A} and σ_{B} are the values of the Thomson coefficients for the temperature T. Hence the four gains of energy given above are respectively P + dP, − P, $\sigma_{\text{A}}d$T, and − $\sigma_{\text{B}}d$T, and we therefore have for dE the infinitely small electromotive force in the circuit

$$d\text{E} = d\text{P} + (\sigma_{\text{A}} - \sigma_{\text{B}})\, d\text{T} \,\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Note that in the above the Thomson coefficients for A and B *have the same sign* (both positive); thus energy is *absorbed* in A (current going from cold to hot) and *evolved* in B (current going from hot to cold). In the copper iron couple energy is absorbed in the copper (cold to hot) and *also absorbed* in the iron (hot to cold); their Thomson coefficients *have opposite signs.*

6. Obtaining further Relations by Application of Thermodynamics

In a thermo-electric circuit the Peltier and Thomson effects are, as we have seen, reversible, so that if we can neglect the Joule effect in the conductors (which is irreversible) we can regard such a circuit as being one in which all operations are completely reversible, and can apply to it the laws of the reversible heat engine which the student encounters in his study of Heat and Thermodynamics. And as the Joule heating is proportional to the *square* of the current it will be small and negligible if the E.M.F. in the circuit (and therefore the current) is small.

Now it is shown in Thermodynamics that round a reversible cycle the algebraic sum of the expressions:—

$$\frac{\text{Heat energy taken in (or given out)}}{\text{Absolute temperature at which it is taken in (or given out)}}$$

for all parts of the cycle is zero, *i.e.* $\int \frac{d\text{H}}{\text{T}} = 0$. (The student should note particularly that **absolute temperatures are now being used**: T° C. = 273 + T Absolute.) Hence in a thermo-electric circuit, with junctions at **absolute** temperatures T and T + dT, since

quantities of energy $P + dP$, $-P$, $\sigma_A dT$, and $-\sigma_B dT$ are absorbed at temperatures $T + dT$, T, T, and T, it follows that:—

$$\frac{P + dP}{T + dT} - \frac{P}{T} + \frac{(\sigma_A - \sigma_B)}{T} dT = 0,$$

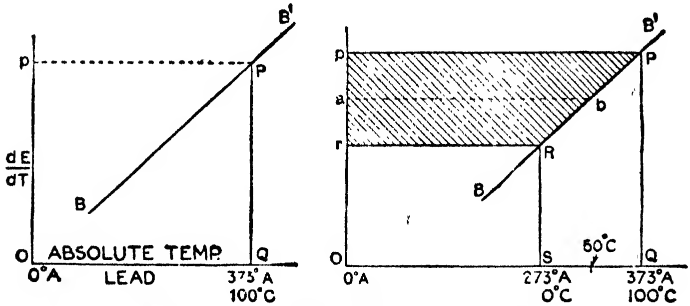
$$\text{i.e. } dP - \frac{P}{T} dT + (\sigma_A - \sigma_B)\, dT = 0;$$

$$\therefore\ (\sigma_A - \sigma_B)\, dT = \frac{P}{T} dT - dP \quad \ldots\ldots\ldots\ldots\ldots (4)$$

and substituting this value in the expression given in Art. 5, for dE, viz. $dE = dP + (\sigma_A - \sigma_B)\, dT$, we get

$$dE = \frac{P}{T} dT \text{ and } P = T\frac{dE}{dT} \quad \ldots\ldots\ldots\ldots\ldots (5)$$

and the last relation means that for any two substances forming a



(a) FIG. 385. (b)

thermo-electric couple circuit *the Peltier coefficient at any temperature is given by the product of the thermo-electric power at that temperature and the absolute temperature.*

Now consider a couple consisting of a metal B and lead with, say, the hot junction at 100° C. or 373° Absolute. In the diagram (Fig. 385(*a*)) BB′ is the thermo-electric power line of B with respect to lead which is taken as the base or zero line, so that PQ measures the thermo-electric power (dE/dT) for the couple at 100° C. (373° A.), the temperature of the hot junction. *Absolute* temperatures are marked on the horizontal so that the origin O is absolute zero (− 273° C.) and not 0° C. OQ measures, then, the *absolute* temperature (T) of the hot junction, and by (5) above the Peltier

coefficient at the hot junction (TdE/dT) is measured by OQ × PQ, *i.e.* by the area of the rectangle OQPp.

Again from (4) we have: $\sigma_A - \sigma_B = \dfrac{P}{T} - \dfrac{dP}{dT}$.

Now $P = T\dfrac{dE}{dT}$; $\therefore$ Differentiating: $\dfrac{dP}{dT} = T\dfrac{d^2E}{dT^2} + \dfrac{dE}{dT}$

$$\text{Hence } \sigma_A - \sigma_B = \frac{dE}{dT} - \left(T\frac{d^2E}{dT^2} + \frac{dE}{dT}\right) = -T\frac{d^2E}{dT^2}.$$

and if the metal A is lead for which σ_A is zero, we have:—

$$\sigma_B = T\frac{d^2E}{dT^2}; \text{ or denoting } \frac{dE}{dT} \text{ by } y \text{ we have:—}$$

$$\sigma_B = T\frac{dy}{dT}; \quad \therefore \sigma_B dT = T.dy \quad \ldots\ldots\ldots\ldots (6)$$

The last expression indicates that the Thomson effect in the metal B with its ends at temperatures T° and T° + dT (Absolute) is given by the product of the absolute temperature T and the difference in the thermo-electric powers at the two ends. Further, a simple application of the Calculus will show that *the Thomson effect in the metal with ends at* T_1 *and* T_2 $\left(viz.\ \int_{T_1}^{T_2} \sigma dT\right)$ *is given by the product of the difference of the thermo-electric powers (with respect to lead) at the ends and the mean absolute temperature.*

Thus in Fig. 385(b) the Thomson effect in B with its ends at 0° C. and 100° C. is measured by the area pPRr for this area = $pr \times ab$ = $(Op - Or) \times ab$ = difference in thermo-electric powers (with respect to lead) at 0° C. and at 100° C. × mean *absolute* temperature.

TREATING THE THERMO-ELECTRIC COUPLE ANALYTICALLY. The mathematically minded student should note the following *brief indication* of an analytical treatment of the thermo-electric couple:—

(1) In the total E.M.F. curve of Fig. 377 the origin was at 0°C. Now assume the origin at absolute zero (− 273°C.). The curve, being practically a *parabola*, may be expressed by the equation:—

$$E = aT + bT^2 \quad \ldots\ldots\ldots\ldots (m)$$

where E is the E.M.F. in a couple with one junction at absolute zero and the other at absolute temperature T, and a and b are constants depending upon the metals. Hence the E.M.F. with junctions at T_1 and T_2 is given by

$$E_{T_1}^{T_2} = a\,(T_2 - T_1) + b\,(T_2^2 - T_1^2) \quad \ldots\ldots\ldots\ldots (n)$$

(2) Again from (m) above we get by differentiating:—

$$dE/dT = a + 2bT; \text{ or } y = a + 2bT \dots\dots\dots\dots\dots\dots (o)$$

where we are again writing y for the thermo-electric power dE/dT. This is the equation of a *straight line* in which $2b$ represents, in the usual way, the tangent of the angle made by the line and the positive direction of the axis of temperature, and a is the intercept on the axis of thermo-electric power.

(3) Further if T_n be the neutral temperature then the thermo-electric power (y) at this temperature is zero and we have:—

$$a + 2bT_n = O. \quad \therefore\ T_n = -a/2b \quad \text{and} \quad a = -2bT_n \dots\dots\dots (p)$$

and substituting this for a in the total E.M.F. expression (n) for our couple with junction temperatures T_1 and T_2 we get:—

$$E_{T_1}^{T_2} = 2b\,(T_2 - T_1)\left(\frac{T_2 + T_1}{2} - T_n\right) \dots\dots\dots\dots\dots\dots (q)$$

which corresponds with the total E.M.F. formula on page 415 if $k = -2b$.

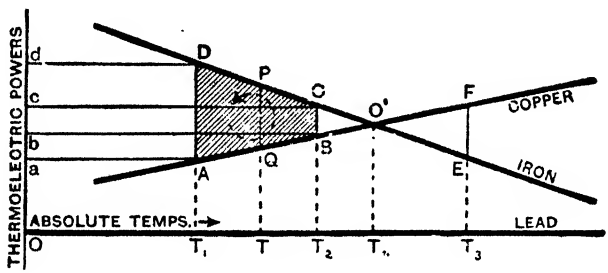


FIG. 386. The figure is *not* drawn to scale (see Appendix).

(4) The Peltier coefficient $P = TdE/dT$ and substituting (o) for dE/dT gives $P = aT + 2bT^2$. Further since $(\sigma_A - \sigma_B) = -Tdy/dT$ (see above), and form (o) $dy/dT = 2b$ we have $(\sigma_A - \sigma_B) = -2bT$. Now using these substitutions in the formula for the net gain of energy *per unit quantity* (by Peltier and Thomson) of Art. 5, page 421, we get for the net gain $a(T_2 - T_1) + b(T_2^2 - T_1^2)$ which is the expression obtained above (in (n) and therefore (q)) for the E.M.F.

7. The Thermo-Electric Diagram

We can now consider the construction and application of the thermo-electric diagram.

In the diagram *absolute temperatures* are taken as abscissae and thermo-electric powers as ordinates, and lead is taken as the standard and base line since the Thomson effect in lead is nil. The thermo-electric power lines are mainly straight, and the diagram is so drawn that for any pair of lines the E.M.F. acts one particular way round the circuit, viz. counterclockwise: when the current

goes from a lower to a higher point there is *gain* of energy (absorbed), when from a higher to a lower point there is *loss* of energy (evolved). Fig. 386 gives a *rough* diagram for copper, iron, and lead.

In Fig. 386 T_1A is the thermo-electric power for lead and copper at *absolute* temperature T_1 and T_1D is the thermo-electric power for lead and iron at T_1; hence AD is the thermo-electric power for copper and iron at absolute temperature T_1. Similarly BC is the thermo-electric power for copper and iron at absolute temperature T_2. At T_n (270° C.) the thermo-electric power for copper and iron is *nil.*

Consider now a copper iron couple with junctions at T_1 Absolute (say 0° C.) and T_2 Absolute (200° C.); a thermo-electric current will be flowing in the direction copper to iron through the hot junction (*counterclockwise* in the diagram). Since *the total E.M.F. is given by the product of the thermo-electric power at the mean temperature of the junctions and the difference in temperature of the junctions* it is evidently represented on the diagram by the product $\overline{PQ} \times \overline{T_1T_2}$ where PQ is the thermo-electric power at T (100° C.); but $\overline{PQ} \times \overline{T_1T_2}$ is the area (shaded) of the trapezium ABCD so that **the total E.M.F. is represented by the area ABCD on the diagram.**

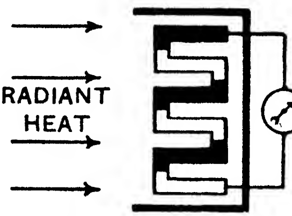


FIG. 387.

The current passes along the copper from cold to hot (low to high point) and therefore (Art. 4), *energy is absorbed.* Since *this Thomson effect* (*energy absorbed per unit quantity*) *is given by the product of the mean absolute temperature and the difference between the thermo-electric powers* (*copper lead couple*) *at the ends* it is evidently represented on the diagram by $ab \times OT$ where OT is the mean absolute temperature and $ab = Ob - Oa = T_2B - T_1A$; but $ab \times OT$ is the area $aABb$ so that **the Thomson Effect in the copper is represented by the area aABb on the diagram.**

The current passes from copper to iron through the hot junction (low to high point) and therefore (Art. 3), *energy is absorbed.* Since *this Peltier Effect* (*energy absorbed per unit quantity*) *is given by the product of the thermo-electric power of copper and iron at this temperature and the absolute temperature* it is evidently represented on the diagram by $BC \times OT_2$, *i.e.* by $BC \times Bb$; but $BC \times Bb$ is the area $bBCc$ so that **the Peltier Effect at the hot junction is represented by the area bBCc on the diagram.**

The current passes along the iron from hot to cold (low to high point) and in this case *energy is again absorbed* (Art. 4). As in the cases of the copper **the Thomson Effect in the iron is represented by the area cCDd on the diagram.**

The current passes from iron to copper through the cold junction (high to low point) and *energy is evolved* (Art. 3). **This Peltier Effect at the cold junction is represented by the area aADd on the diagram** as in the case of the hot junction.

Clearly the total energy absorbed per unit quantity is represented by

$$aABb + bBCc + cCDd - aADd = ABCD,$$

and this, as previously indicated, represents the E.M.F.

For reference, the diagram for a number of metals is given in the Appendix.

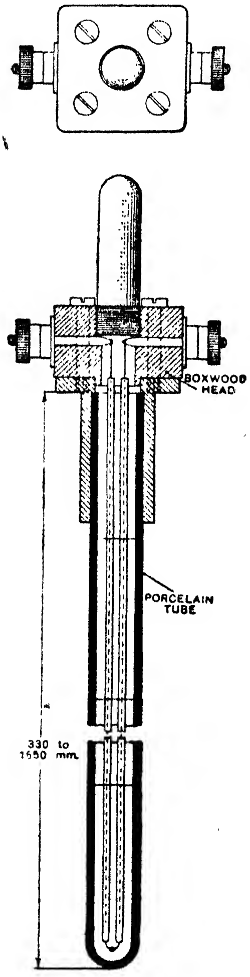


FIG. 388.

8. A Few Practical Applications

There are many useful practical applications of thermo-electric currents: a few are briefly indicated below to show the general idea.

The **thermopile** (Fig. 387) consists of a number of bismuth-antimony couples arranged in series so as to multiply the effect. If one set of junctions be protected as indicated, while radiant heat be allowed to fall on the other set, the galvanometer joined to the instrument will be deflected, and the deflection may be used as a measure of the heat falling on the exposed junctions.

The **thermo-electric pyrometer** is used for the measurement of high temperature, *e.g.* that of a furnace. The junction, with long, low resistance leads attached, is placed in the furnace, and the thermo-current produced gives a reading on a suitable measuring instrument in the circuit, such reading depending on the temperature: the instrument is graduated to read the temperature direct. A pyrometer for work up to 1400° C. is shown in Fig. 388. The couple consists of a platinum and a platinum-iridium alloy. Two long wires of these are threaded through fine porcelain tubes and their ends fused together at the bottom, and the whole is placed in a

wider porcelain tube provided with a boxwood head fitted with terminals to couple up the measuring instrument: the junction is placed in the furnace and, as stated, the measuring instrument can be arranged to read the temperature of the junction and furnace direct.

Incidentally, a *resistance pyrometer* is rather similar in appearance to Fig. 388. At the end of the tube is a coil of platinum which when placed in the furnace increases in resistance so that its resistance at any temperature can be used as a measure of the temperature.

Different couples, of course, give different thermo-E.M.F.'s, and the temperature of the hot junction which gives maximum current (neutral temperature) is different for different couples. Thus the alloy "Ferry" and the alloy "Brightray," if used as a couple, give an increasing E.M.F. as the temperature rises up to 1000° C. (max. E.M.F. = about 65 millivolts), a copper and constantin (eureka) couple give an increasing E.M.F. up to 500° C., a platinum and platinum-rhodium couple up to 1600° C., and an iron and nickel couple up to about 300° C.

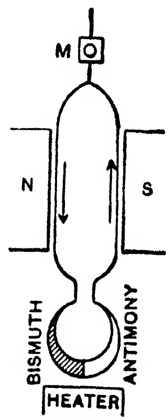


FIG. 389.

The **thermo-galvanometer** is largely used in measurements on small alternating or rapidly varying currents, *e.g.* in telephone circuits and in the receiving aerials of wireless. A loop of silver wire hangs by a quartz fibre in between the poles N, S of a magnet (Fig. 389). The lower ends of the loop are attached to pieces of bismuth and antimony respectively, joined at the bottom. Below this junction is situated the "heater," which is a filament of wire or a quartz fibre platinised. When the current to be measured passes through the heater, part of the heat developed by the current is radiated to the bismuth-antimony junction: a thermo-current is set up in the direction bismuth to antimony through the junction, and the loop is deflected just as in the case of the moving coil galvanometer. It can be used for alternating as well as continuous currents in the heater because the *heat produced does not depend on current direction.*

Professor Boys' **radio-micrometer** is similar to the above, the heater being omitted and the junction of the bismuth and antimony carrying a very thin blackened copper disc. When radiation falls on the disc a thermo-current flows and the coil is deflected. The instrument is extremely sensitive.

The **thermo-milliammeter** is a highly sensitive form of ammeter which works on the same principle and it is now extensively used for small high frequency current measurements (from about 500 cycles up to radio frequencies). The heater, in one type, is a fine constantin wire in a vacuum bulb and a tellurium-bismuth junction is soldered to it.

CHAPTER XV

ELECTRICAL MEASUREMENTS

THE *Wheatstone bridge* or *Wheatstone net* is an arrangement for the measurement of resistance. In it there are four resistances, three of which are known (or one is known and the *ratio* of the other two is known), and the fourth is the resistance to be determined.

1. The Principle of the Wheatstone Bridge

With the arrangement of the four resistances P, Q, R, S, battery and galvanometer as in Fig. 390 it is clear that taking a point D on the branch ADC, there must be on the branch AEC some point (say E) at the same potential as D, so that on joining D and E through the galvanometer there will be no deflection. Connexion between D and Y will result in a current *from* Y *to* D, for Y is above E and therefore above D in potential; connexion between D and Z will result in a current *from* D *to* Z. As D and E, then, are at the same potential we have:—

$$\frac{\text{P.D. between A and D}}{\text{P.D. between D and C}} = \frac{\text{P.D. between A and E}}{\text{P.D. between E and C}},$$

$$i.e. \quad \frac{\text{Current in AD} \times \text{Res AD}}{\text{Current in DC} \times \text{Res. DC}} = \frac{\text{Current in AE} \times \text{Res. AE}}{\text{Current in EC} \times \text{Res. EC}}.$$

But Current in AD = that in DC. Current in AE = that in EC;

$$\therefore \frac{\text{Resistance AD}}{\text{Resistance DC}} = \frac{\text{Resistance AE}}{\text{Resistance EC}} \quad or \quad \frac{P}{S} = \frac{R}{Q}.$$

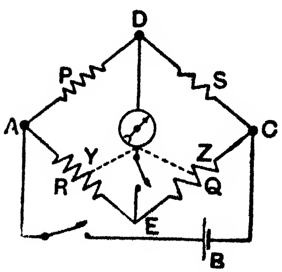


FIG. 390.

Thus an unknown resistance P can be found if (1) R, Q, and S are known, *or* (2) the resistance of one of the adjacent conductors, viz. R or S, is known and the *ratio* of the other two is also known. The *Metre Bridge* and the *Post Office Box* are two practical applications.

An examination of Fig. 390 will show that if the positions of the galvanometer and battery be interchanged the relation above will still

hold when adjusted for no deflection: the sensitiveness of the bridge may, however, be quite different in the two cases (the greater the galvanometer current due to a small lack of balance the more sensitive is the arrangement). It can be shown that the most sensitive arrangement is obtained by the following rule:—Whichever has the higher resistance—battery or galvanometer—should be put across from the junction of the two higher resistances to the junction of the two lower resistances (see *Advanced Textbook of Electricity and Magnetism*).

Fig. 390 is the same as Fig. 278 (page 298). As an exercise the student should apply the method there indicated (Kirchhoff's laws) to Fig. 390 and work out the expression for *the current in the galvanometer*: he will find that his result is clearly zero if RS = PQ, *i.e.* if P/S = R/Q.

2. The Metre or Slide-Wire Bridge

In its simplest form (Fig. 391) this consists of three thick bars of copper fixed to a board and provided with two gaps for the insertion of the resistance to be measured and a standard known resistance. A straight, hard uniform wire of fairly high resistance (platinoid or platinum iridium), usually 1 metre in length, joins the two copper end pieces and has behind it a graduated scale. By means of a slider, contact can be made at any point on the wire, the position being indicated by a pointer attached to the slider and moving over the scale. The battery circuit includes a key K, P is the unknown resistance and S a standard known resistance of somewhat similar magnitude. The experiment consists in (1) closing K and (2) finding a point of contact E at which the galvanometer is not deflected. We then have (compare the figure with Fig. 390)—

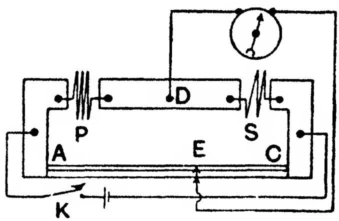


FIG. 391.

$$\frac{\text{Resistance P}}{\text{Resistance S}} = \frac{\text{Res. AE}}{\text{Res. EC}} = \frac{\text{Length AE}}{\text{Length EC}},$$

$$\therefore \text{ Res. of P} = \frac{\text{Length AE}}{\text{Length EC}} \times \text{Res. S},$$

but several corrections for accurate work are necessary.

ERRORS AND CORRECTIONS IN METRE BRIDGE WORK.—The main sources of error are as follows:—

(1) Lack of uniformity in the bridge wire. To eliminate, the wire must be *calibrated* so that the ratio Res. AE/Res. EC is known.

(2) The non-coincidence of the pointer with the edge of the tapper (which presses on the wire). To eliminate, interchange the coils and take the mean of the two results. (*Explain this.*)

(3) Errors due to thermo-electric effects. To eliminate, use a reversing key in the battery circuit and balance with current in opposite directions. (*Explain this.*)

(4) Variation of the resistance of the coils owing to temperature changes. To reduce this the current must be small, and flow for short intervals of time.

(5) Resistance of the end pieces too large and unequal to be neglected. These are determined experimentally as equivalent to so many divisions (say α and β) of bridge wire, and these are added to the respective sections AE and EC each time a test is made.

End Corrections for Metre Bridge. Join up as in Fig. 391, P and S being in this case *known* resistances, so that the ratio $P/S = r$ (say) is known. Let the resistance of the end A be equal to α divisions of the wire and that of C equal to β divisions of the wire. Secure a balance and let $AE = d_1$ divisions and $AC = L$ divisions; then

$$\frac{P}{S} = \frac{d_1 + \alpha}{(L - d_1) + \beta} = r \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Interchange P and S and balance; let d_2 = balancing distance from A:—

$$\frac{P}{S} = \frac{(L - d_2) + \beta}{d_2 + \alpha} = r \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From (1) $\qquad (d_1 + \alpha) = r(L - d_1) + r\beta \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$

From (2) $\qquad r(d_2 + \alpha) = (L - d_2) + \beta;$

$$\therefore\ r^2(d_2 + \alpha) = r(L - d_2) + r\beta \quad \ldots\ldots\ldots\ldots\ldots (4)$$

Eliminating β by subtracting (3) and (4) gives—

$$\alpha = \frac{d_1 + r(d_1 - d_2) - r^2 d_2}{r^2 - 1},$$

and since r, d_1, and d_2 are known, α is determined. β can similarly be found.

3. The Carey Foster Bridge

Fig. 392 will serve to explain the principle of the Carey Foster method of bridge measurement. In this case P and S are two nearly equal or equal resistances, whilst R_1 and R_2 are the two resistances under examination. Let x be the *resistance* of the end X, y the *resistance* of the end Y, and L the total length of bridge wire, and let ρ denote the resistance per unit length. Balancing as indicated we have:—

$$\frac{P}{S} = \frac{R_1 + x + \rho l_1}{R_2 + y + \rho(L - l_1)}.$$

Courtesy of the B B C.

HIGH DEFINITION TELEVISION

Scene from the opera, "Mr Pickwick," produced by D Bower at the Alexandra Palace. The televising was by the Marconi E M I Emitron Cameras—an all electric method, i e with no mechanically moving parts

Now let R_1 and R_2 be interchanged and let l_2 be the balancing distance from the end X; hence:—

$$\frac{P}{S} = \frac{R_2 + x + \rho l_2}{R_1 + y + \rho (L - l_2)}.$$

$$\therefore \frac{R_1 + x + \rho l_1}{R_2 + y + \rho (L - l_1)} = \frac{R_2 + x + \rho l_2}{R_1 + y + \rho (L - l_2)}.$$

Adding 1 to each side we get:—

$$\frac{R_2 + y + \rho L + R_1 + x}{R_2 + y + \rho L - \rho l_1} = \frac{R_1 + y + \rho L + R_2 + x}{R_1 + y + \rho L - \rho l_2}.$$

Here the numerators are the same, hence the denominators are equal; equating the denominators we get

$$R_1 - R_2 = \rho (l_2 - l_1) = \text{Res. of the part } (l_2 - l_1),$$

and thus *the difference between the two resistances* R_1 *and* R_2 *is equal to the resistance of the bridge wire between the two balancing points.* The result does not involve the end pieces or the values of P and S. The resistance of the length $(l_2 - l_1)$ may be taken from the calibration curve of the bridge wire, and if R_2 be known R_1 is determined.

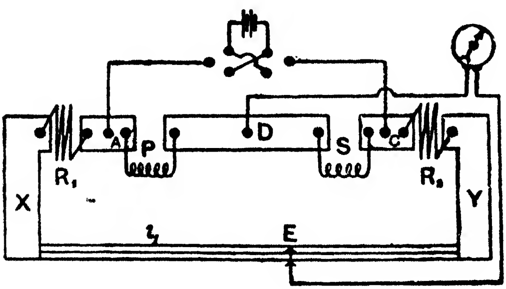


FIG. 392.

This method can be employed for "calibration of bridge wire" if $R_1 - R_2$ be accurately known, for $\rho = (R_1 - R_2)/(l_2 - l_1)$; thus if either P or S can be slightly altered at will, so as to bring the balancing points to various parts of the wire, ρ for the parts in question can be determined.

4. The Callendar and Griffith's Bridge

This bridge was devised for use with the platinum resistance thermometer (resistance pyrometer—page 426) for the measurement of temperature. In Fig. 393 the arms P and S of the bridge are equal. Leads join the thermometer *pt* to the gap in the arm Q, and a similar pair of dummy leads close to the main ones is connected to a gap in the arm R: thus the resistance of the leads to the

thermometer is eliminated. The wire *ab* is 50 cm. long and its resistance is ·25 ohm, so that 1 cm. of it = ·005 ohm. At r there are eight coils, the resistances being ·1, ·2, ·4, ·8, 1·6, 3·2, 6·4, and 12·8 ohms, and one or more of these can be brought into the circuit as desired: note that the first coil has a resistance equal to 20 cm. of *ab*, the next equal to 40 cm., and so on. The tapper consists of a slider E which connects *ab* and *cd*. The battery (not shown) joins the P.R. and S.Q. junctions as usual. The thermometer itself consists of a platinum coil in a tube of glazed porcelain (or glass according to its range).

In testing, the thermometer is subjected to the unknown temperature and the bridge balanced for no deflection, by altering r and the position of E: if this occurs with the slider x cm. *from the centre* of *ab*:—

$$\frac{P}{S} = \frac{R}{Q} = \frac{l + r + b + \rho x}{l + pt + b - \rho x},$$

where l is the *resistance* of the leads, b the resistance of half the wire *ab*, ρ the resistance of one cm. of it, and pt the resistance of the platinum coil of the thermometer: but P is equal to S, hence

$$l + pt + b - \rho x = l + r + b + \rho x;$$

$$\therefore\ pt = r + 2\rho x = r + \frac{x}{100}.$$

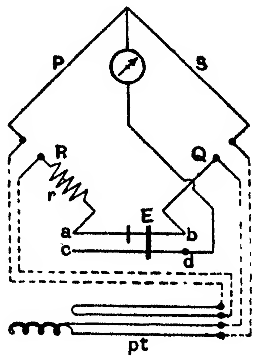


FIG. 393.

Having thus found the resistance pt of the thermometer, the temperature to which it is exposed is known since the variation of the resistance of platinum with temperature is known (page 237).

The above deals only with the electrical part—the " bridge " attachment to the resistance thermometer. For details of electrical thermometry and pyrometry and the method of making the apparatus " direct reading," some work on *Heat* must be consulted.

5. The Post Office Box

Another form of Wheatstone bridge is the *Post Office Box*; in dealing with this we shall for convenience write the relation of Art. 1, viz. P/S = R/Q, in the form R/P = Q/S, and shall assume S to be the unknown resistance. The Box consists of a number of coils of known resistance arranged so as to form three arms of a

Wheatstone bridge, the fourth being the resistance to be determined. The method in which the coils are arranged is shown in Fig. 394. Their ends are attached to brass blocks separated by conical gaps, into which conical brass plugs can be inserted. By inserting a plug that particular resistance is cut out of circuit, for the current will pass from one block to the next through the plug; removing a plug necessitates the current going through the coil, thus adding that resistance to the circuit. The coils are doubled upon themselves as shown so as to eliminate the effects of inductance (Chapter XVI.).

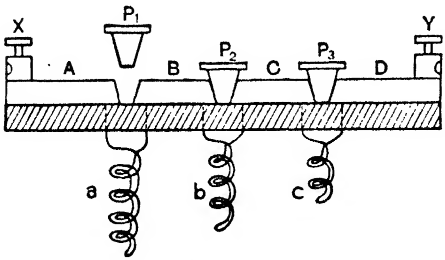


FIG. 394.

A plan of the box and its connexions is shown in Fig. 395. The arms P and R are known as the *ratio* arms; each consists of three coils of 10, 100, and 1000 ohms resistance. Q is called the *rheostat* arm, and consists of a series of coils whereby resistances ranging from 1 to 11,110 ohms can be obtained. S is the unknown resistance. The lettering of the box is identical with Fig. 390. The manipulations will be understood by considering the following experiment. Join up as indicated:—

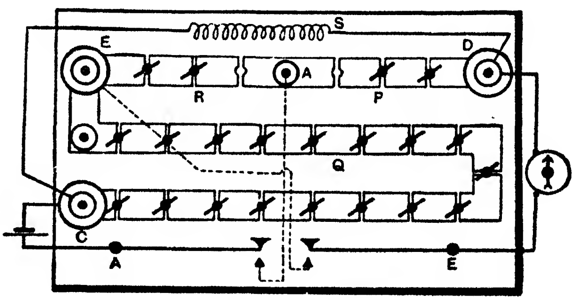


FIG. 395.

(*a*) Make R = 10 ohms and P = 10 ohms. By taking out plugs in Q endeavour to find a resistance such that on closing first the battery key and then the galvanometer key there is no deflection. Since R = P, Q must be equal to S to secure a balance. In an actual test 2 ohms in Q at this stage gave a deflection to the right and 3 ohms gave a deflection to the left; hence S *was between* 2 *and* 3 *ohms*.

(*b*) Make R = 100 ohms keeping P = 10 ohms. Since R is now equal to 10P, Q must be equal to 10S for a balance; hence in the test it was only necessary at this stage to work with resistances between 20 and 30 ohms in Q.

On taking 23 ohms the deflection was to the right and 24 ohms gave a deflection to the left; hence S *was between* 2·3 *and* 2·4 *ohms.*

(*c*) **Make R = 1000 ohms keeping P = 10 ohms.** Since R = 100P, Q must be equal to 100S for a balance; in the test it was therefore only necessary to work with between 230 and 240 ohms in Q. On taking 237 ohms the galvanometer was not deflected; hence S *was* 2·37 *ohms.*

(*d*) **A Step Further.** Should two consecutive resistances in the third step still produce deflections in opposite directions, the true value may be found by interpolation. If 237 ohms gave a deflection of 9 divisions to the right, and 238 gave 12 to the left, a balance would be obtained if Q could be made equal to $237 + \frac{9}{21}$, *i.e.* 237·43 ohms; in this case the value of S would be 2·3743 ohms.

If the unknown resistance is *very large*, R must retain its value, 10 ohms, throughout, and P must be made equal to (say) 1000. R being $\frac{1}{100}$ of P, Q must be $\frac{1}{100}$ of the unknown to secure a balance. Thus, if 1208 from Q gives no deflection, the value of S may be taken as 120,800 ohms.

6. Measurement of High Resistance

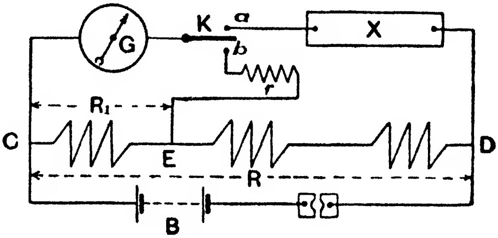


FIG. 396.

The Wheatstone bridge is unsuitable for the measurement of high resistances of the order of a megohm, since the conditions for sensitiveness would usually be violated. Of the many methods in use two will be briefly indicated.

(1) SUBSTITUTION METHOD.—This is a good laboratory method capable of giving accurate results. In Fig. 396 B is a convenient battery, R a high resistance of the order 100,000 ohms (adjustable), G a high resistance galvanometer (reflecting), K a key, and X the high resistance to be measured. The procedure is as follows:—

With the lever on stud *a* the deflection D_1 of the galvanometer is obtained. A known resistance r is now introduced, X being cut out of circuit by moving the lever to stud *b*. The resistance R_1 (also r if necessary) is adjusted, *the total* R *between C and D being kept constant*, until a deflection D_2 of somewhat similar magnitude to D_1 is obtained. If e be the P.D. between C and D, e_1 the P.D. between C and E, I_1 the current in the galvanometer in the first case, and I_2 the current in it in the second case,

$$I_1 = \frac{e}{G + X}, \qquad I_2 = \frac{e_1}{G + r};$$

$$\therefore \frac{I_1}{I_2} = \frac{e}{e_1} \cdot \frac{G + r}{G + X}, \qquad i.e. \ \frac{D_1}{D_2} = \frac{R}{R_1} \cdot \frac{G + r}{G + X};$$

$$\therefore X = \frac{D_2 R (G + r) - D_1 R_1 G}{D_1 R_1}.$$

The **resistance of the insulation of a cable** may be found by this method. The cable is coiled up in a metal tank containing water, the ends only being outside, and these are well insulated to prevent leakage. One end of the copper core is joined *via* the key (stud *a*) to the galvanometer, the other end of the core being left "free," and the metal tank, which through the water is in contact with the outside of the insulation, is joined to the end D of the high resistance R; thus the *insulation* takes the place of X in Fig. 396. A slight modification is indicated in Fig. 397. Both ends of the cable are brought together at A and joined to the key K, a metal plate A_1 being joined to D. To avoid surface leakage the potential of the cover just below the core A is raised to the same potential as the core: this is done by the wire *w* as indicated, and it is clear that no leakage current can pass through the galvanometer, the latter only indicating the current flowing through the insulation. It is usual to immerse the cable for 24 hours and then to measure the resistance "after one minute's electrification."

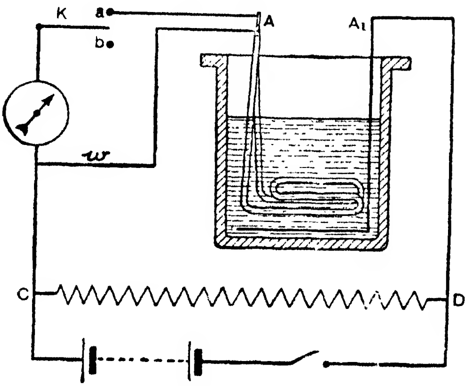


FIG. 397.

* The *resistivity* S of the insulation (in ohms per inch cube, say) may be found from the relation (13), page 305, if the above measured resistance be expressed in ohms and the length in inches.

(2) CONDENSER LOSS OF CHARGE METHOD.—This may be used to find (*a*) The dielectric resistance of the condenser itself. (*b*) The value of a high resistance through which the condenser is caused to discharge. (*c*) The capacitance of the condenser.

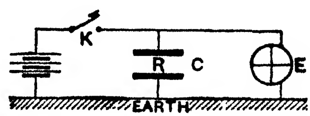


FIG. 398.

(*a*) In Fig. 398 C is the condenser of which the dielectric resistance R is required, E is a quadrant electrometer, and K a *well insulated* key. The key K is closed and when the deflection of E is steady K is opened, thus disconnecting the battery from E and R. The charge on the condenser

very gradually "leaks through" the dielectric and the P.D. falls: this is shown by the falling deflection of E. Time readings of the deflection are taken and a curve plotted with time as abscissae and deflections as ordinates. Taking any two points on the curve let V_1 denote the deflection corresponding to the first point and V_2 that to the second, and let t seconds be the time interval between the two; the dielectric resistance R of the condenser is found from the relation

$$R = \frac{t}{2{\cdot}3026\, C \log_{10} \frac{V_1}{V_2}} \text{ (see below)},$$

where C is the capacitance of the condenser. With C in microfarads R will be in megohms. It is assumed that the leakage only takes place at the condenser.

(*b*) Modify the arrangement as shown in Fig. 399 where R is the high resistance (several megohms) to be measured. The experiment and calculation are the same as above, the discharge now being considered to take place through the resistance R: thus R is determined. The discharge in this case is fairly rapid: the time taken for a condenser of a few microfarads to discharge through a resistance of a megohm is only of the order of seconds.

(*c*) From the expression for R we get $C = t/(2{\cdot}3026\, R \log_{10} V_1/V_2)$ so that the capacitance C is found if R be known.

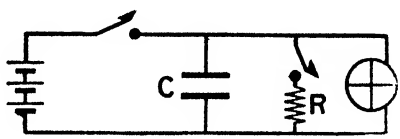


FIG. 399.

The expression for R used above may be established thus:—If Q = charge on the condenser at any instant, V = the P.D., and C = capacitance, we have: Q = CV, and therefore (neglecting sign) $dQ/dt = CdV/dt$. But dQ/dt is the leakage current and is equal to V/R: hence:—

$$C\frac{dV}{dt} = \frac{V}{R}; \quad \therefore \frac{1}{RC}dt = \frac{1}{V}dV,$$

and on integrating (limits V_1 and V_2) we arrive at the result:—

$$\frac{1}{RC}t = \log_e \frac{V_1}{V_2}; \quad \therefore R = t/C \log_e \frac{V_1}{V_2},$$

which is the formula used (on converting to common logarithms), the potentials V_1 and V_2 being proportional to deflections. Again:—

$$\log_e \frac{V_2}{V_1} = -\frac{1}{RC}t; \quad \therefore e^{-\frac{t}{RC}} = \frac{V_2}{V_1}; \quad \therefore V_2 = V_1 e^{-\frac{t}{RC}};$$

and if $t = RC$ we have $V_2 = V_1 e^{-1} = V_1/e$. Thus if a charged condenser has its coats connected by a wire of resistance R ohms,

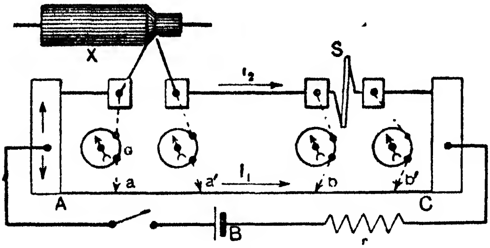


FIG. 400. Measurement of Low Resistance (Dynamo armature Section).

the potential (and charge) will fall to $1/e$, *i.e.* $1/2{\cdot}71828$ of its initial value, in a time RC seconds. RC seconds is called the **time constant** of a condenser of capacitance C discharging through a resistance R.

7. Measurement of Low Resistance

In Fig. 400 AC is a (calibrated) low resistance wire joined to an accumulator B and an adjustable resistance *r*. The unknown resistance X and a standard low resistance of somewhat similar magnitude S are connected in series, and the two put in parallel with the wire. G is a galvanometer. The test is as follows:—

Join one terminal of the galvanometer to one end of the unknown, and by means of the movable contact find a point *a* on the wire such that the galvanometer is not deflected. Repeat this at the other end of the unknown, and at both ends of S, as shown by the dotted lines in the figure. Clearly then since the P.D. at the ends of X = P.D. between *a* and *a'*, and the P.D. at the ends of S = P.D. between *b* and *b'*, we have:—

$$\frac{\text{P.D. on X}}{\text{P.D. on S}} = \frac{\text{P.D. between } a \text{ and } a'}{\text{P.D. between } b \text{ and } b'}; \quad \therefore \frac{\text{Res. X}}{\text{Res. S}} = \frac{\text{Res. } aa'}{\text{Res. } bb'},$$

and this may be written Length aa'/Length bb' if the wire be quite uniform; thus X is determined.

8. Measurement of Battery Resistance

Many ordinary laboratory tests are not a great success, for the resistance depends upon the current (decreasing as the current increases) and polarisation affects several determinations; the following methods are typical and illustrate important principles:—

(1) BY THE P.D. ON OPEN AND CLOSED CIRCUIT.—If a voltmeter V be joined to the poles of a battery, then if the resistance of V be very high so that it takes only a tiny current, we may assume the battery to be "on open circuit," so that the reading of V is the E.M.F. of the

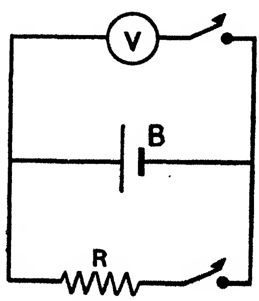


FIG. 401.

battery, say E volts. If a known standard resistance R ohms be now joined up as shown (Fig. 401) the battery supplies a current I equal to E/(B + R) where B is the battery resistance (we again neglect the tiny current in V). If e be the reading of V now, then e is the terminal potential difference, *i.e.* the P.D. at the ends of R, and $e = \mathrm{IR}$. Hence:—

$$\frac{\mathrm{E}}{e} = \frac{\mathrm{I}(\mathrm{B}+\mathrm{R})}{\mathrm{IR}} = \frac{\mathrm{B}+\mathrm{R}}{\mathrm{R}}; \quad \therefore \mathrm{B} = \frac{\mathrm{E}-e}{e} \times \mathrm{R}.$$

The electrometer method is similar. The cell or battery is connected to an electrometer and the deflection d_1 is observed; this is a measure of the E.M.F. (E), for the cell *is* on open circuit in this case. The poles are then joined by a wire of resistance R and the deflection d_2 is observed; this is a measure of the terminal potential difference (e). Here, as above,

$$\frac{\mathrm{B}+\mathrm{R}}{\mathrm{R}} = \frac{\mathrm{E}}{e} = \frac{d_1}{d_2}, \qquad i.e.\ \mathrm{B} = \frac{d_1 - d_2}{d_2} \times \mathrm{R}.$$

The same principle is utilised in the *potentiometer method*: this is a *good* laboratory method, and is described on page 446.

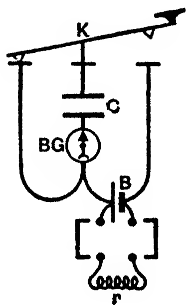


FIG. 402.

(2) BY A CONDENSER AND BALLISTIC GALVANOMETER.—In Fig. 402 B is the cell, the resistance of which (b) is required, C is a condenser, BG a ballistic galvanometer, K a charge and discharge key, and r a known resistance: the experiment is carried out as follows:—

With r disconnected from B, charge the condenser by depressing K, and note the *first swing* of BG; let this be d_1. Discharge the condenser. Connect the poles of B by the resistance r and again charge C, noting the first swing, d_2, of BG. Discharge the condenser.

If E be the E.M.F. of the cell and e its terminal P.D. when joined by r, the charge given to the condenser in the first case is CE and in the second case Ce, where C denotes the capacitance of the condenser; hence $d_1 : d_2 = \mathrm{CE} : \mathrm{C}e = \mathrm{E} : e$. Again, the current in r is given by $\mathrm{E}/(b + r)$, and also by e/r; hence $\mathrm{E} : e = (b + r) : r$. Clearly then

$$\frac{b+r}{r} = \frac{d_1}{d_2}; \quad \therefore b = \frac{d_1 - d_2}{d_2} \times r.$$

(3) BY MANCE'S METHOD.—In this method the cell is placed in the arm of the Wheatstone bridge ordinarily occupied by the resistance to be measured (S), and the usual battery branch contains a key K only (Fig. 403); the galvanometer will be deflected for a current will certainly flow through it. Resistances are then adjusted *until the galvanometer deflection remains the same whether*

the key K *is open or closed.* It can be shown that the usual calculation then applies, viz. Res. (S) of cell/Q = P/R, and since Q, P, and R are known, S can be determined.

The truth of this may be proved mathematically by an application of Kirchhoff's laws (see *Advanced Textbook of Electricity and Magnetism*), but it can be seen thus:—When K is closed a current goes through it just the same as if the branch CKA contained a battery in the usual way, and, in general, this would cause a deflection of G which, in this case, means an *alteration* in the deflection of G (for G already has a current in it). If, then, resistances are adjusted until there is no *alteration* of deflection, matters evidently correspond to the no deflection condition of the ordinary case, and the same relationship must exist between the four resistances as in the usual balanced bridge.

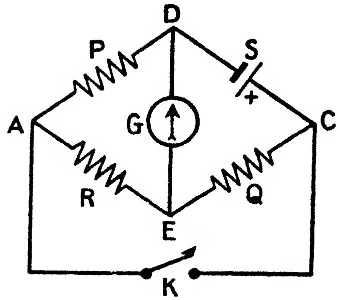


FIG. 403.

9. **Measurement of Galvanometer Resistance**

(1) The *best* method is to remove the needle and suspension in the case of a moving needle galvanometer, or clamp the coil of a moving coil galvanometer, and measure the resistance in the usual way by the Wheatstone bridge arrangement: this necessitates, of course, the use of a second galvanometer.

There are several more or less simple methods of finding the resistance of a galvanometer, but although they illustrate important points in theory, they are not of great accuracy: we briefly indicate two methods, the first suitable for one of low sensitiveness.

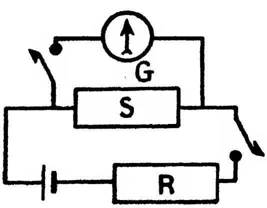


FIG. 404.

(2) In Fig. 404 G is the galvanometer under test, S and R are two resistance boxes, S being of moderately low value and R of higher value: both S and R can, of course, be varied. S and R are adjusted so that a fairly large deflection of G is obtained and the S and R values (say S_1 and R_1) are noted. S is shunting the galvanometer, and for the current in G we therefore have (it is S/(G + S) of total current—page 353):—

$$\frac{S_1}{G + S_1} \times \frac{E}{R_1 + \frac{S_1 G}{G + S_1}}$$

where E = E.M.F. of the cell and G = resistance of galvanometer (R_1 and R_2 are usually large compared with the resistance of the cell, so that the latter is neglected).

S and R are now both altered and adjusted until, with their new values (say S_2 and R_2), the galvanometer deflection is the same as before, so that the same current is flowing in it. Hence:—

$$\frac{S_1}{G + S_1} \cdot \frac{E}{R_1 + \dfrac{S_1G}{G + S_1}} = \frac{S_2}{G + S_2} \cdot \frac{E}{R_2 + \dfrac{S_2G}{G + S_2}},$$

and on solving this for G we get the result:—

$$G = \frac{S_1S_2 (R_1 - R_2)}{S_1R_2 - S_2R_1}.$$

(3) *Kelvin's method* resembles the Mance's method for the resistance of a cell. The galvanometer to be tested is placed in the arm of the Wheatstone bridge ordinarily occupied by the resistance to be measured (S in Fig. 405), and a key is placed in the the usual galvanometer branch DE: the galvanometer will be deflected, for current will be passing through it. Resistances are then adjusted *until the galvanometer deflection is the same whether the key referred to is open or closed.* It is clear that, when this is so, no current passes between D and E through the key when the latter is closed (if it did the galvanometer deflection would change) and therefore the points D and E (Fig. 405) are at the same potential: hence the usual calculation may be applied.

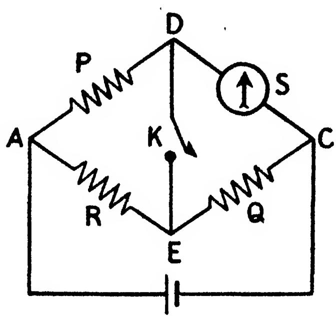


FIG. 405.

10. Measurement of the Resistance of Electrolytes

The main difficulty is that in most cases a back E.M.F. is set up, which, with ordinary methods of testing, would appear as a resistance, and, further, the back E.M.F. itself is not constant. In practice special electrolytic cells are used, and the electrodes are of platinum coated with "platinum black." The following methods, amongst others, have been resorted to:—

(1) KOHLRAUSCH'S A.C. METHOD.—This is a metre bridge method, the electrolyte being in one gap and a known standard resistance in the other. Alternating current is used instead of the

continuous current from a battery, so that the latter is replaced by the secondary of a small induction coil (page 481) or by a step-down transformer connected to the A.C. mains (page 485), or an oscillating (wireless) valve (Chap. XXI.): as the current is rapidly alternating in direction, the opposite polarising effects at the electrodes cancel each other. To detect the balancing point on the bridge wire the usual galvanometer must be replaced by one which can be employed with, and is sensitive to, small alternating currents, or, if the frequency of the A.C. supplied is within the range of audible frequencies, a telephone can be used, the balancing point being that for which the sound in the receiver is a minimum. The usual bridge calculation is applied.

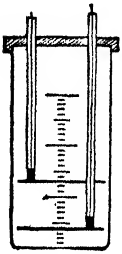


FIG. 406.

It should be noted that the induction coil does not give a *true* alternating current: in any case it must be used without its condenser (page 483) or the current from it will be more or less a pulsating direct current. The valve method is now mainly used.

Different forms of electrolytic cell are used according as the conductivity is high or low. Fig. 406 is a type suitable for solutions of low conductivity (high resistance). The horizontal electrodes, are made of stout platinum foil fitting as closely as possible into a cylindrical vessel. A stout platinum wire from each is sealed into a glass tube. The electrodes are held in position by an ebonite cover. Connecting wires pass down the glass tubes, contact with the electrodes being made by means of a little mercury introduced into each tube. The distance between the electrodes can be varied (Fig. 407 shows a type suitable for solutions of high conductivity).

The resistance determined (say R) is that of the column of electrolyte from one electrode to the other. If l be the distance between the electrodes of Fig. 406, a their cross-sectional area, and S the resistivity of the electrolyte, then $R = Sl/a$, and from this, having found R, S is determined: the specific conductivity $\kappa = 1/S$, and the equivalent conductivity $\gamma = \kappa V$, where V = volume (c.cm.) containing one gram-equivalent (page 384). Strictly, a is the cross-section of the effective liquid column, and there are " end

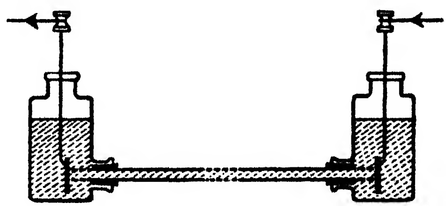


FIG. 407.

effects" (Fig. 408) when the electrodes do not touch the walls of the tube. It is therefore more usual, in practice, to determine the resistance of a standard solution (generally a potassium chloride solution) *in the same cell* and from the *known* conductivity of this solution to calculate the "cell constant," *i.e.* the factor by which any experimental result must be multiplied to give the actual resistivity, S, of the electrolyte.

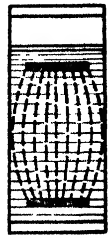


FIG. 408.

(2) STROUD-HENDERSON METHOD.—This again is a bridge method, but continuous current from a battery, and the usual galvanometer, are employed: it is indicated in Fig. 409:—

The resistances S and Q are made equal and large. The resistance P is that of a tube of the liquid under test. The resistance R is partly that of another tube of the liquid exactly like the first, except that it is shorter and partly made up by an adjustable resistance. This adjustable resistance, *r*, is altered until the galvanometer shows no deflection, in which case the resistance *r* equals that of a column of the liquid equal to the difference in length of the two tubes. Thus, although continuous current is employed, since the polarisation effects are the same in both arms of the bridge, their effect on the result is eliminated.

11. The Principle of the Potentiometer

One of the best methods for measuring E.M.F.'s and P.D.'s is that known as the "potentiometer" method. In Fig. 410 AB is a long uniform wire joined to a battery E, the end A to the positive pole: there is a fall of potential from A to B along the wire. Now imagine another circuit AGK joined up as shown, G being a galvanometer. A current will flow in this branch in the direction AGK, and G will be deflected. This current will depend on the P.D. between A and K produced by E: thus if K be moved to the left the P.D. will be less and the deflection less: if it be moved to the right the P.D. will be greater and the deflection greater.

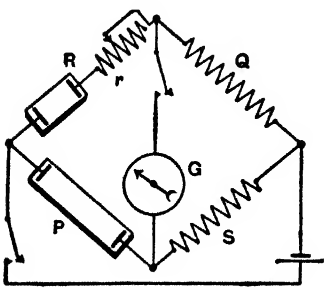


FIG. 409.

In Fig. 411 a cell S has been put in the lower branch, its positive pole also being joined to A. Its E.M.F. tends to send a current through the lower branch in the direction KSGA whilst the P.D.

between A and K produced by E tends to send a current in the opposite direction AGSK. Clearly then (assuming S to be of smaller E.M.F. than E) it will be possible to find a position for K

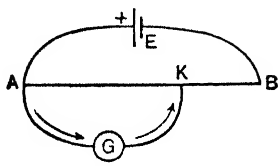


FIG. 410.

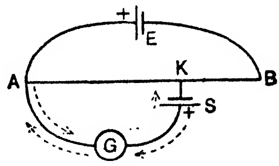


FIG. 411.

where these two opposing influences are equal, so that no current flows in the branch AGSK, and the galvanometer will not be deflected. If then we alter K until there is no deflection, we can say that *the E.M.F. of the cell* S *is equal to the P.D. between* A *and* K *produced by* E. This is the principle of the potentiometer.

Note that when a balance is obtained *no current is being taken from the cell* S: it is on "open circuit."

12. Measurements with the Potentiometer

A simple laboratory type of potentiometer is shown in Fig. 412. It consists of six uniform wires each 1 metre in length joined in series by thick copper bars: scales graduated in millimetres are placed alongside the wires. This wire has its ends connected to an accumulator E (sometimes two or three in series), the positive pole being joined to A.

The positive terminal of a standard cell S—a Weston of E.M.F.,

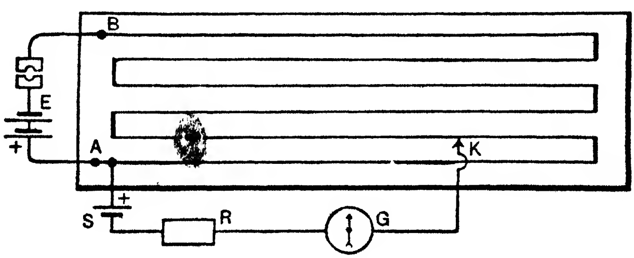


FIG. 412.

say, 1·019 volts—is joined to A, the negative being joined through a resistance R and a galvanometer G to the sliding contact K. The position of K is adjusted until G is not deflected. Then R is

made less for greater sensitiveness, and K again slightly changed if necessary: lastly R is removed and K finally adjusted until there is no movement of the galvanometer. If d_1 mm. is this final

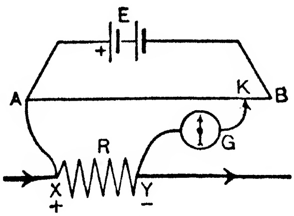


FIG. 413.

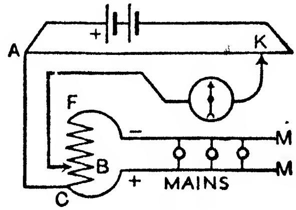


FIG. 414.

position of K from the end A, we know that the P.D. at the ends of this length d_1 of wire is equal to the E.M.F. of the standard cell S (viz. 1·019 volts).

TO MEASURE THE E.M.F. OF A CELL.—Remove S and place, say, a Leclanché cell (L) there, its positive pole being joined to A. Repeat the experiment. Suppose d_2 mm. is the distance of K from A when the galvanometer is not deflected: then the P.D. at the ends of length d_2 of wire is equal to the E.M.F. of the Leclanché.

$$\frac{\text{E.M.F. of L}}{\text{E.M.F. of S}} = \frac{\text{P.D. on } d_2}{\text{P.D. on } d_1} = \frac{\text{Curr. in } d_2 \times \text{Res. } d_2}{\text{Curr. in } d_1 \times \text{Res. } d_1} = \frac{\text{Res. } d_2}{\text{Res. } d_1};$$

and resistances d_2 and d_1 are proportional to lengths d_2 and d_1:—

$$\therefore \text{ E.M.F. of Leclanché} = \left\{1{\cdot}019 \times \frac{d_2}{d_1}\right\} \text{ volts.}$$

Below we indicate several tests with the potentiometer.

(1) **To Measure the P.D. on a Circuit.** To measure the P.D. at the ends of a resistance R (Fig. 413) which is part of a current circuit, we treat R in the same way as the Leclanché above, having first got a "balance" with the standard cell (d_1 mm.). Join the high potential end X to A of the potentiometer, and the low potential end Y through a galvanometer to the sliding contact K and find the position for no deflection: if it is d_2 mm. from A

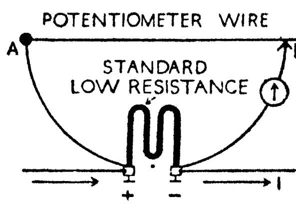


FIG. 415.

$$\text{P.D. between X and Y} = \left\{1{\cdot}019 \times \frac{d_2}{d_1}\right\} \text{ volts.}$$

If the P.D. is a high one, say that between the electric light mains (Fig. 414), a high resistance CF, say of 10,000 ohms, is put across them, and the P.D. for a part CB of, say, 50 ohms is measured as above: if this be 1·25 volts the total P.D. is 1·25 × 10,000/50 = 250 volts.

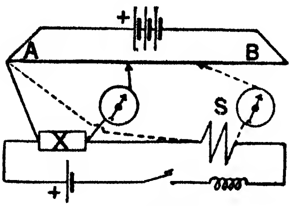


FIG. 416.

(2) To Measure the Current in a Circuit. The current to be measured is sent through a standard low resistance of ·1, ·01, or ·001 ohm, according to circumstances, and the P.D. at the ends of this standard is determined in the usual manner (Fig. 415); the P.D. thus found, divided by the resistance of the standard, gives the current required.

(3) To Measure a Resistance. The resistance X (Fig. 416) to be measured is put in series with a standard resistance S, an accumulator, and an adjustable resistance. The P.D. at the ends of the unknown resistance is balanced on the potentiometer, as in the preceding cases. This is repeated with the standard S, and—

$$\text{Res. of X} = \frac{\text{P.D. at ends of X}}{\text{P.D. at ends of S}} \times \text{Res. of S} = \frac{d_1}{d_2}\text{S},$$

where d_1 and d_2 are the balancing distances in mm. from A.

(4) To Measure the Resistance of a Cell. This is a better form of the first experiment of Art. 8, and is indicated in Fig. 417. With K_1 open find the position of K for no deflection of G: let d_1 be the balancing distance from A. Now close K_1 so that the cell gives a current through R and again find the position of K for no deflection: let d_2 be the balancing distance from A. In the first case the P.D. at the ends of d_1 is balancing the E.M.F. (E) of the cell for no current is being taken from it: in the second case the P.D. at the ends of d_2 is balancing the terminal P.D. (e) when current is flowing through R. Now the current in R is given by e/R and it is also given by $\text{E}/(\text{R} + b)$ where b is the resistance of the cell. Hence:—

$$\frac{\text{E}}{\text{R} + b} = \frac{e}{\text{R}}; \quad \therefore \frac{\text{R} + b}{\text{R}} = \frac{\text{E}}{e} = \frac{d_1}{d_2}; \quad \therefore b = \text{R}\frac{d_1 - d_2}{d_2}.$$

(5) To Measure the E.M.F. in a Thermo-Electric Circuit. In a thermo-electric circuit the E.M.F. may generally be measured by a suitable modification of the potentiometer method. As the E.M.F., usually expressed in *microvolts*, is very small, the difference of potential along the potentiometer must be small and so subdivided that a difference of one-millionth of a volt may easily be read. The principle of one method of arranging this is indicated in Fig. 418. If the wire be divided into 1000 divisions the P.D. for each division will be about $1/10^7$ of the E.M.F. of the battery, and may therefore be small enough to admit of sufficiently accurate measurement. Consider carefully this outline diagram.

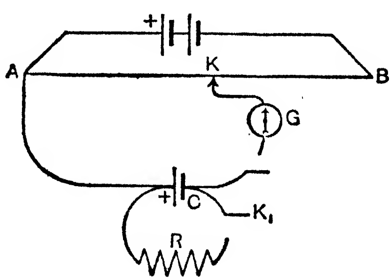


FIG. 417.

A simpler laboratory arrangement is shown in Fig. 419. The resistance S is fairly large—at least

about 500 ohms. SC is a standard cell, and E an accumulator. Let ρ = the resistance of 1 cm. of the potentiometer wire AB. Close K_1 and adjust R until there is no deflection of G: the P.D. on S is now equal to the E.M.F. of SC (say 1·018 volts). Now (P.D. on AB)/(P.D. on S) = (Res. of AB)/(Res. of S) = $(\rho \times \overline{AB})/S$ so that:—

$$\text{P.D. on 1 cm. of wire} = \frac{\rho \times 1{\cdot}018}{S}.$$

Open K_1 and close K_2, thus joining up the Cu-Fe couple and G to the wire, and adjust K for no deflection: if d cm. be the balancing distance from A the required E.M.F. is $(\rho \times 1{\cdot}018 \times d)/S$. (Hot junction on right in Fig. 419.)

13. A Modern Potentiometer

Modifications in the form of the potentiometer have been introduced (*a*) to increase its accuracy, (*b*) to simplify its use. As a simple example we will take *one type* of the **Crompton potentiometer.**

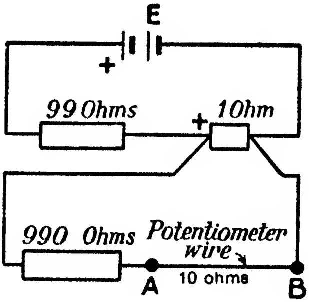


FIG. 418.

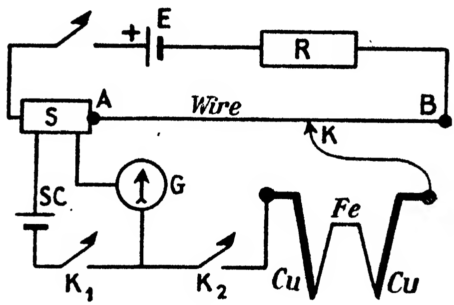


FIG. 419.

Fig. 420 shows the principle of the construction, and Fig. 421 an actual instrument. It consists of a wire divided into, say, fifteen segments of equal resistance; fourteen of these are formed into coils, their ends being connected to the numbered studs shown diagramatically at E on the right, the fifteenth being a wire lying over a scale about 25 inches in length divided into 1000 equal parts.

In series with the potentiometer coils and wire (*i.e.* in the accumulator circuit) are two adjustable resistances; these are shown at G and G_1 on the left, and are for the purpose of altering the P.D. at the ends of the fifteen segments, *i.e.* altering the P.D. applied to the potentiometer by the accumulator. C is the sliding contact which moves along the wire, and E is another contact by which one or more of the coils at E can be used with the wire for the galvanometer circuit. The accumulator is connected to the

terminals A, the galvanometer to terminals B, the standard cell to terminals 1, any unknown E.M.F.'s or P.D.'s to the terminals 2, 3, 4, 5, 6, and the double contact switch K at the centre enables

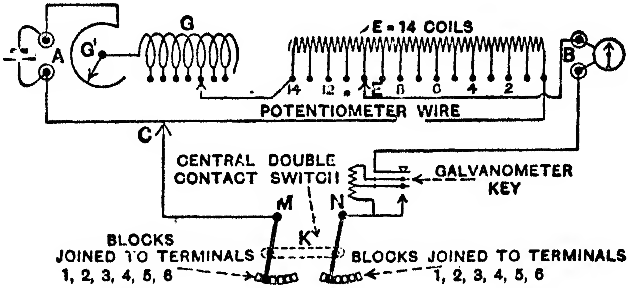


FIG. 420.

any of these to be put into the galvanometer circuit. The reader should examine these connexions and compare with Fig. 412.

Working with one accumulator (and using, say, the Clark standard cell—E.M.F. 1·433 volts) it is customary to standardise the instrument so that the total P.D. at the ends of the potentiometer coils E and wire is 1·5 volts. To secure this, the contact arm at E is placed on stud 14, and the sliding contact C at division 330 on the scale (the zero of the scale is on the right where the wire joins the coils E): the centre switch K is now set on studs 1, thus bringing the standard cell into the galvanometer circuit, and the resistances G and G[1]

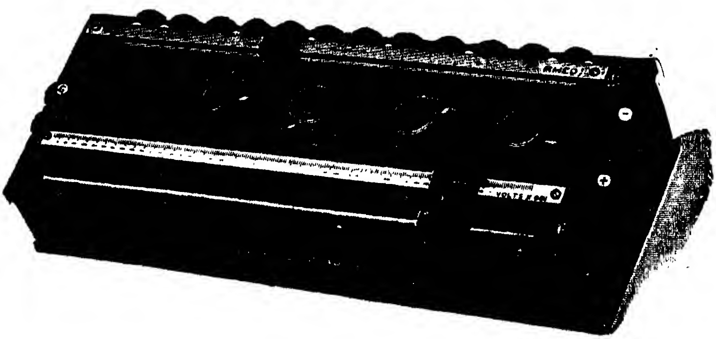


FIG. 421. The six pairs of terminals 1-6 are seen in the rear.

in series with the accumulator, E coils, and wire are adjusted till the galvanometer is not deflected. Clearly the P.D. for *each* coil at E and for the wire is now 1·433/14·33 or ·1 volt, and for *each* of the 1000 divisions ·0001 volt.

The E.M.F. to be measured is now brought into the galvanometer circuit by moving the centre switch K to the corresponding studs. With C on the

extreme right of the wire the revolving arm at E is adjusted till two successive contacts give deflections in opposite directions: E is now placed on the stud of lower value and C adjusted for no deflection. If this occurs with the arm E on stud 10 and the slider C on division 646, the E.M.F. is—

$\{(10 \times \cdot 1) + (646 \times \cdot 0001)\}$ volts,

i.e. 1·0646 volts.

The galvanometer key (Fig. 420) completes the circuit through two resistances which are short circuited in succession as the key is depressed.

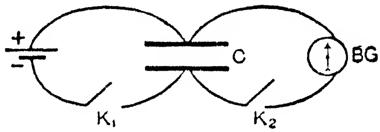


FIG. 422

14. Measurement of Capacitance

The student should first read again pages 227-231. The capacitance of a condenser may be found by using a *ballistic galvanometer* as follows (Fig. 422). By closing K_1 charge the condenser by the battery of constant E.M.F. (E): then open K_1 and close K_2 thus discharging through the ballistic galvanometer BG, and note the *first swing* (say d_1 mm.) of the spot of light along the scale. Repeat with a standard condenser of known capacitance and let d_2 mm. be the first swing. If C_1 and C_2 be the capacitances the *quantities* discharged are EC_1 and EC_2, and the quantities are proportional to the first swings: hence

$$\frac{EC_1}{EC_2} = \frac{C_1}{C_2} = \frac{d_1}{d_2}; \quad \therefore C_1 = \frac{d_1}{d_2}C_2,$$

and as C_2 is known, C_1 is determined.

More accurate results may be obtained by *null* methods, which depend upon adjusting for no deflection of the galvanometer. Two of the best known null methods are given below.

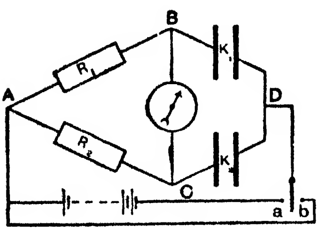


FIG. 423.

De Sauty's Bridge Method. In Fig. 423 K_1 is the condenser the capacitance C_1 of which is required, and K_2 a standard of capacitance C_2. The resistances R_1 and R_2 are adjusted until, on making contact at *a* and charging the condensers, the galvanometer is not deflected. When this is so, the potentials at B and C are equal, and since D is a common point, the P.D. on the condenser K_1 is the same as that on the condenser K_2. If V be this P.D., Q_1 the charge on K_1, and Q_2 the charge on K_2 we have $V = Q_1/C_1$ and $V = Q_2/C_2$, so that $C_1/C_2 = Q_1/Q_2$. But Q_1 has passed through R_1 and Q_2 through R_2 in the

same short interval of time, and as current divides inversely as resistances we can write $Q_1/Q_2 = R_2/R_1$: hence:—

$$\frac{C_1}{C_2} = \frac{Q_1}{Q_2} = \frac{R_2}{R_1}; \quad \therefore C_1 = \frac{R_2}{R_1}C_2.$$

The condensers are discharged by moving the key to contact *b*.

Kelvin's Mixture Method. The *principle* is shown in Fig. 424. The condenser C is charged to the P.D. (V_1) between A and B, and condenser S to the P.D. (V_2) between B and E. By adjusting the resistances P and Q these two P.D.'s can be made to have the *inverse ratio of the capacitances.* When this is so we have (calling the capacitances C and S) $V_1/V_2 = S/C$ or $V_1C = V_2S$, that is *the condensers have equal charges*: and further, since

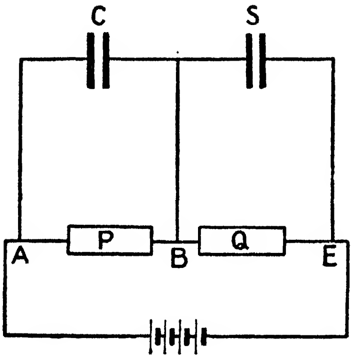


Fig. 424.

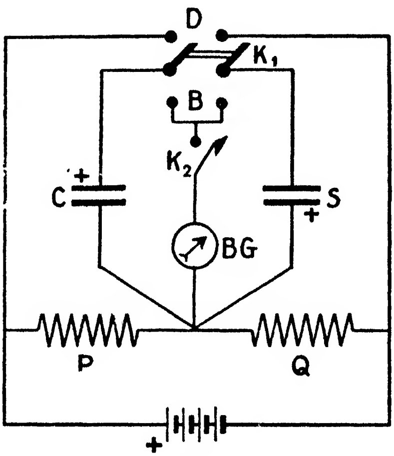


Fig. 425.

$V_1/V_2 = P/Q$ we have when this adjustment for equal charges is made:—

$$\frac{S}{C} = \frac{P}{Q}; \quad \therefore C = \frac{Q}{P}S,$$

and thus C is determined. To test the equality of the charges, the condensers after charging must first have their oppositely charged plates connected, so that the two charges are " mixed " and tend to neutralise: then the combined condensers are discharged through a galvanometer. If the charges are equal there will be no deflection. The operations therefore consist of charging, mixing, and discharging through the galvanometer, and P and Q are adjusted until there is no deflection on doing this.

Fig. 425 gives the actual connexions for readily performing the three operations. Turning K_1 to D charges, turning it to B mixes, and then closing K_2 discharges through the ballistic galvanometer.

15. Absolute Value of the Capacitance of a Condenser

A capacitance can be found without the use of a known standard condenser by (1) charging the condenser and measuring the P.D. (V) between its plates by an electrometer, and (2) discharging it through a ballistic galvanometer thus measuring its charge Q (page 351): its capacitance = Q/V. (Care must be taken with the units.) Another method is indicated in Fig. 426 in which only the ballistic galvanometer is employed.

R is a high resistance of about 10,000-12,000 ohms and S is a standard 1 ohm resistance. Place S_1 on *a* and S_2 on *d*. The condenser is charged to the P.D. (V) at the ends of R and the ballistic galvanometer will have a current (I) in it depending on the P.D. (v) at the ends of S: let θ be its *steady deflection*. Place S_1 on *b* and S_2 on *c*, and note the *first swing* α of BG as the condenser discharges.

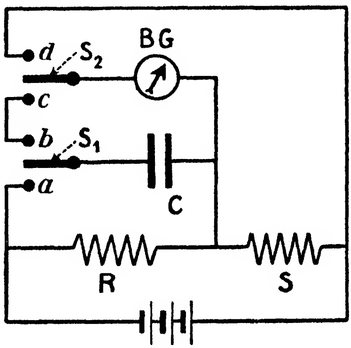


FIG. 426.

If the capacitance be C and the charge Q, C = Q/V. Further from page 351 (assuming BG to be a moving coil) we have:—

$$Q = \frac{kt}{2\pi HA}\alpha;$$

$$\therefore\ C = \frac{t}{2\pi}\cdot\frac{1}{V}\cdot\frac{k}{HA}\alpha.$$

Again, $I = k\theta/HA$ (page 343) and also $I = v/R_0$ where R_0 = galvanometer resistance: hence $k/HA = I/\theta = v/R_0\theta$. Substituting in the above:—

$$C = \frac{t}{2\pi}\cdot\frac{v}{V}\cdot\frac{1}{R_0}\cdot\frac{\alpha}{\theta} = \frac{t}{2\pi}\cdot\frac{S}{RR_0}\cdot\frac{\alpha}{\theta}$$

for v/V may be taken as equal to S/R if R_0 be very high compared with S. Linear deflections (say d_1 and d_2) on the galvanometer scale may be used for θ and α: d_2 should be corrected for damping.

Correction for Damping. If α, α_1, α_2, α_3, etc., be successive swings to right and left of zero it is found that $\alpha/\alpha_1 = \alpha_1/\alpha_2 = \alpha_2/\alpha_3 \ldots$ = a constant = d, say: d is called the **decrement.** The dying away for one half vibration is therefore d. The time for the *first swing* is a quarter vibration, and the amount of dying away is $\sqrt{d}$. Hence if the first swing is α then $\alpha\sqrt{d}$ is what it would be if there were no damping. Sometimes when d nearly equals unity, $\log_e d = \gamma$ is used: γ is called the **logarithmic decrement,** and the correction becomes $\alpha\ (1 + \gamma/2)$. See *Advanced Textbook of Electricity and Magnetism* for fuller treatment.

16. Lamp Tests. Determination of Candle-Power, and Lumens

Imagine a lamp A at distance a from a screen, the light falling normally: the illumination on the screen is given by E = (c.p. of A)/a^2. Suppose A replaced by another lamp B, and let its distance b be adjusted until it produces the *same* illumination: then E = (c.p. of B)/b^2. Hence—

$$\frac{\text{c.p. of A}}{a^2} = \frac{\text{c.p. of B}}{b^2}, \; i.e. \; \frac{\text{c.p. of A}}{\text{c.p. of B}} = \frac{a^2}{b^2} = \frac{(\text{Distance of A})^2}{(\text{Distance of B})^2},$$

i.e. *the candle-powers of two lamps (in the direction of the screen) are directly proportional to the squares of the distances they must be placed from the screen in order to produce the same illumination.* This is the principle used in *photometry*, and an early type of apparatus was **Bunsen's photometer** which in its simplest form consists of a grease spot at the centre of a sheet of white paper.

If such a sheet be held up to the light, the grease spot looks brighter than the rest of the paper, but darker when seen from the same side as the light. Light passes through the spot, and hence when seen from the side remote from the light it looks brighter than the rest of the screen; when looked at on the other side the spot looks darker, because the light which is passing through is not illuminating that surface. Hence if the paper be placed between two lights, and its position adjusted until the spot cannot be seen on either side (or, at any rate until the two sides look alike), then the screen must be equally illuminated on both sides, for the light which is lost for illuminating purposes at the spot by passing through is made up by an equal light which comes through from the other side.

In Bunsen's photometer, then, the two lamps, say L_1 and L_2, are placed at opposite sides of a grease spot screen S, and distances adjusted until the spot at each side cannot be readily distinguished from the rest of the screen. When this is so—

$$\frac{\text{Candle-power of } L_1}{\text{Candle-power of } L_2} = \frac{(\text{Distance of } L_1)^2}{(\text{Distance of } L_2)^2} = \frac{(L_1S)^2}{(L_2S)^2},$$

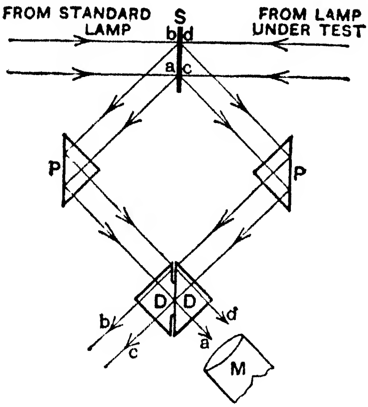
and if L_2 is a standard lamp of known c.p. the c.p. of L_1 is found.

A much improved form is the **Lummer-Brodhun photometer** which consists of a milk-white screen S [Fig. 427 (*a*)], two totally reflecting prisms PP and two prisms DD placed base to base, the one on the left being ground down round the edge so that the two are really in contact only over a circular area round about the centre. Any light from the left, which, after reflection at S and P falls on the central parts of DD in contact, passes through to the telescope M (e.g. *a*) whilst any light from the left which falls on the outer ground parts of D is reflected again and does not enter M (e.g. *b*).

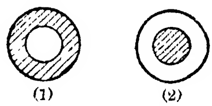
Again, any light from the right which after reflection at S and P (on the right) falls on the central parts of DD in contact passes straight on and does not enter M (e.g. *c*), whilst light from the right which falls on the other part of D is reflected and enters M (e.g. *d*).

Thus, on looking through M the central part of the field of vision is illuminated by light from the left of S and the outer part by light from the right of S and the appearance may be as indicated in Fig. 427 (*b*). The distances of the two lights from S are adjusted until the two parts appear the same, in which case the two lights are producing equal illuminations at S and

$$\frac{\text{C.p. of lamp}}{\text{C.p. of standard}} = \frac{(\text{Distance of lamp from S})^2}{(\text{Distance of standard from S})^2}.$$



(*a*)



(1) Left side of S brighter.
(2) Right side of S brighter.

FIG. 427. (*b*)

Mean spherical candle-power (M.S.C.P.) and *lumens output* are measured by arranging that the lamp can be rotated and tilted at various angles, finding the c.p. in various planes and at various angles in each plane, and taking the mean of the results. (A graphical method of calculation of this mean value is usually employed—see *Technical Electricity*.)

A new type of apparatus known as the **integrating photometer** enables the M.S.C.P. (and lumens output) of a lamp to be determined without having to carry out numerous c.p. determinations in various directions. Imagine a lamp hanging in a large sphere (Fig. 428), the inner surface of which is painted a dead white. Any pencil of rays from the source falling upon the matt surface will be partly absorbed, and the remainder irregularly reflected, the reflected rays falling on other portions of the spherical surface

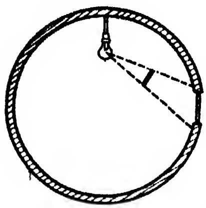


FIG. 428.

where they in turn are partly absorbed and partly reflected, and so on, until absorption finally becomes complete. Thus *every* pencil of *reflected* rays serves to illuminate *the whole surface.* The brightness of any small portion of the surface *shielded from direct light* is thus proportional to the M.S.C.P. of the lamp. In practice the sphere has a small translucent window, a small shield being fixed between it and the light so that no *direct* light from the lamp can fall on the window, and the latter is treated, as it were, as the light. One method of test is then as follows:—

First a standard lamp of known M.S.C.P. is put inside, then the globe is set at one side of a photometer screen (say a Lummer Brodhun) and a lamp L at the other side, and a balance is obtained: let the distance of the window from the screen be d_1. The standard lamp is removed from the globe, the lamp (X) to be tested is inserted, and (keeping the distance of L from the screen the same) the position of the globe is adjusted until balance is again obtained: if d_2 be its distance:—

$$\frac{\text{M.S.C.P. of X}}{\text{M.S.C.P. of standard}} = \frac{d_2^2}{d_1^2} \quad \text{or} \quad \frac{\text{Lumens from X}}{\text{Lumens from standard}} = \frac{d_2^2}{d_1^2}.$$

Latest methods employ a photo-electric cell (page 553) in conjunction with the integrating sphere (or an equivalent enclosure). This cell, as will be seen later, gives a current when suitably exposed to light, and the more intense the light the greater the current. Thus the window of Fig. 428 might be presented to the cell, first with a standard lamp inside, then with the lamp X inside, and the currents (given by, say, a low reading ammeter) would be proportional to their M.S.C.P.'s, and no balancing distances for equal illumination on a screen would be necessary. In some forms, however, the whole apparatus is a compact, self-contained appliance, and the scale of the ammeter (now spoken of as a "lumen-meter") gives the lumens output direct for any lamp placed inside, and from this the M.S.C.P. is found if required. (M.S.C.P. = Lumens/4π. Page 368.)

The *watts absorbed by the lamp* under test can be obtained by noting the reading (I) of an ammeter in series with the lamp, and the reading (E) of a voltmeter joined across the lamp terminals (watts = EI): the *luminous efficiency* of the lamps is then found from the relations (8) or (9), page 369.

CHAPTER XVI

ELECTROMAGNETIC INDUCTION

REFERRING back to Fig. 304 we saw that with currents flowing as indicated there was a force of attraction between them, as shown by F. We come now to another force which would come into action if the current say in AB were increased or decreased or stopped or started: this force acts *at right angles to the direction of the magnetic force* referred to above, it only lasts while the change is taking place, *i.e.* while the current in AB is increasing, decreasing, etc., and it is known as an **inductive force.**

For simplicity, suppose no current is flowing in CD and that current (conventional) flows in AB from B to A, in which case the electron movement in the wire is from A to B. If the current in AB be *increased* inductive force comes into action, the result being that the electrons of CD are accelerated in the *opposite* direction to that in which the electrons of AB are being accelerated, *i.e.* in the direction D to C, which means (if CD is part of a closed complete circuit) that a conventional current is set up in CD in the direction C to D or *opposite* to the *increasing* current in AB. Similarly, if the current in AB is *decreased* inductive force again acts but in the opposite direction, and a conventional current flows in CD from D to C, *i.e.* in the *same* direction as the *decreasing* current in AB. These inductive effects on CD only last while the change is taking place in the AB current.

It is simpler, however, to refer these inductive actions to the changes occurring in the magnetic fields when current circuits are being changed, and this is the method usually adopted.

In 1831 Faraday concluded from experiment that whenever the *magnetic flux,* or *flow of magnetic induction* or *number of tubes of magnetic induction* through a conducting circuit is *changed* an E.M.F. is set up in the circuit (and a current flows if the circuit is closed), such E.M.F. (and current) *lasting only while the change is taking place,* and, moreover, *being greater the greater the flux changed, and the more rapid the change.* E.M.F.'s and currents produced in this way are spoken of as **induced E.M.F.'s and currents.**

The primary effect is the production of the induced E.M.F., but current flows if the circuit is closed (see later). The magnetic flux through a circuit may be changed (1) by moving a magnet near it, or by moving the circuit near a magnet, *i.e.* suitably moving the circuit in a magnetic field, (2) by starting, stopping, or changing the current in a neighbouring circuit, or by

moving the two circuits relative to each other (this is referred to as *mutual induction*), and (3) by starting, stopping, or changing a current in the circuit itself (referred to as *self-induction*).

1. Relative Motion of a Magnet and a Conducting Circuit

The following very simple experiment is important for it illustrates points which will frequently be required.

Connect a coil (Fig. 429) to a galvanometer. Include a cell and a resistance in the circuit and note the direction in which the galvanometer is deflected; let it be, say, *to the right*. While this current is flowing, test the "polarity" of the face of the coil towards the left; let it be, say, *a north face*. Hence, with the present connexions, a deflection to the right indicates that the current in the coil is making the face towards the left a north, and is therefore, counter-clockwise viewed from this side. A deflection *to the left* will indicate that this face is *a south* and the current clockwise. Remove the cell and the resistance.

Quickly insert the north pole of a magnet (Fig. 429); the galvanometer will be deflected, showing that an E.M.F. and current are induced in the coil; this deflection will be found to be to the right. Hold the magnet in the coil and the galvanometer will come to rest, *showing that the induced E.M.F. and current are momentary, lasting only while the magnet is moving, i.e.* during the time the flux is *changing*. Quickly withdraw the north pole, and a momentary deflection to the left will be produced. Insert the south pole, and the momentary deflection will be to the left; withdraw it, and the momentary deflection will be to the right. Tabulate thus:—

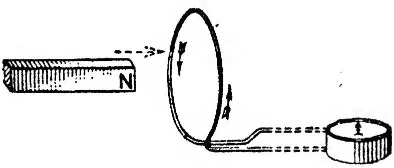


FIG. 429. The coil really consists of *several* turns and the galvanometer is of the reflecting type.

MOTION OF MAGNET	DIRECTION OF DEFLECTION	INDUCED CURRENT MAKES NEAR FACE A
N. pole inserted	Right	North
N. pole withdrawn	Left	South
S. pole inserted	Left	South
S. pole withdrawn	Right	North

When the north pole is inserted the induced E.M.F. and current direction is such that the face of the coil approached is a north face, *which, therefore, tends to oppose the motion of the magnet*—to force the N. pole back. When the north pole is withdrawn the induced E.M.F. and current direction is such that this face is a south, which, therefore, *tends to draw the magnet up again*, and so on.

Fig. 430 will also make this "opposing" effect of the induced E.M.F. and current clear. At (*a*) the magnet is putting magnetic lines into the solenoid in the direction of the full line arrow. The induced current is as shown, and its lines are in the direction of the dotted arrows: they therefore *oppose* the growth of the others. At (*b*) the N. pole is moving away and its lines are being taken out. The induced current causes lines in the same direction (dotted arrows) and therefore *opposes* the decay of the others. By such results as the above we verify the important law known as **Lenz's law**, viz. **the direction of the induced E.M.F. (and current) is such that it tends to oppose the motion or change which produces it,** *i.e.* the induced current sets up a flux which tends to neutralise the *change in flux* which causes it.

Again, by moving the magnet (1) quickly, (2) slowly, it will be found that the deflection is more pronounced in the first case; thus *the induced E.M.F. and current depend upon the rate of change of the flux, being greater the more rapid the change.* Both induced E.M.F. and current also *depend upon the actual number of magnetic lines "changed," being greater the greater the change in the flux*: two magnets pushed into the coil will result in a bigger deflection than one alone. The induced *current* also depends, of course, upon the resistance of the coil and galvanometer.

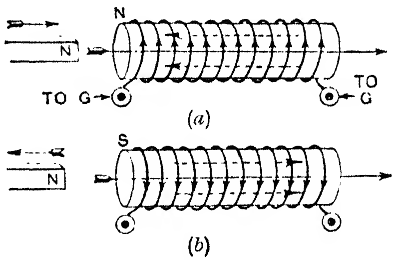


FIG. 430.

In the above experiments, if the magnet be fixed and the coil be moved similar results will be obtained. Moving the coil towards the magnet pole gives the same result as moving the magnet pole towards the coil, and so on. Again, consider for simplicity a rectangular coil facing the N. pole of a magnet (Fig. 431), and imagine it to be moved vertically upwards. As the magnetic flux through it is changing there will be an induced E.M.F. and current in it, and by Lenz's law the direction will be round the coil *clockwise* viewed from the face towards the magnet, *i.e.* along BA, for example, from B to A. This is clear because the induced current opposes the motion and tend to bring the coil down again: there must therefore be attraction between the coil face and N, *i.e.* the coil face must be a S face, and the current must therefore be *clockwise* or from B to A along BA. In a case like this, where a wire such as

BA moves at right angles to itself and the field, and " cuts " the magnetic lines of the field, **Fleming's right-hand rule** is a convenient one for finding the direction of the induced E.M.F. in the wire, viz.:—*Place the thumb and the first two fingers of the right hand mutually at right angles: let the forefinger point in the direction of the magnetic lines of the field and the thumb in the direction of motion of the wire: the second finger will point in the direction of the induced E.M.F.*

Another and *important* case of the relative motion of a conducting circuit and a magnetic field is that of a coil *rotating* in a field: this is dealt with on page 508.

Note the difference between the two Fleming rules. The *left-hand rule* (page 322) is used in finding the direction in which a wire carrying a current

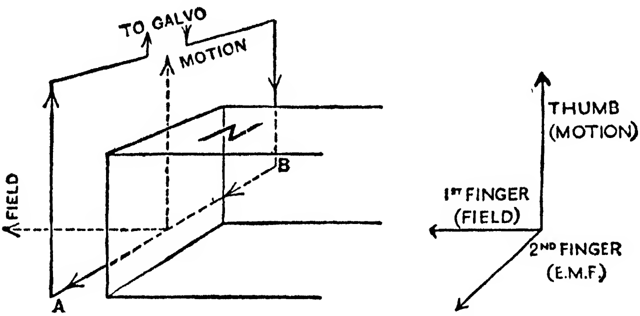


FIG. 431. Illustrating the Fleming Right-hand Rule.

moves in a magnetic field: the *right-hand rule* is used in finding the direction of the induced E.M.F. in a wire when it moves in a magnetic field.

It has been stated that the primary effect is the production of the induced E.M.F. In the preceding cases the path has been closed and a pulse of current circulated through it. If the conductor is a simple straight wire, say in a magnetic field, and the field be changed or the wire be moved so *as to " cut " the tubes of induction of the field,* the inductive forces tend to accelerate the conductor's electrons in one direction and its positive ions in the opposite direction. But the ions in a metallic conductor are more or less fixed so that only its electrons move: the result is an induced E.M.F.—a momentary piling up of electrons at one end of the wire and a deficiency of electrons at the other. Thus if an *electrometer* (which has no way through) be joined to the ends of the wire it will give a momentary deflection indicating this momentary displacement of electrons and momentary P.D. between the ends of the wire (Fig. 432).

Note that inductive effects are often described as taking place whenever tubes of induction are "cut" by a conductor. When the magnet (Fig. 429) approaches the coil tubes of induction of the magnet "cut" through the coil. When a conductor moves across a field so as to "cut" the tubes of induction an E.M.F. is set up even if the conductor does not form a closed circuit.

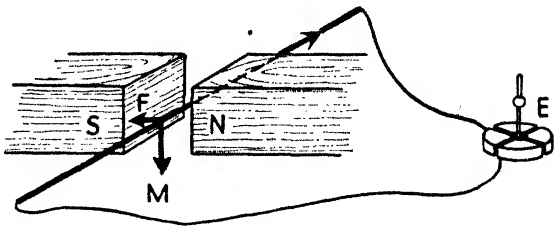


FIG. 432.

2. Mutual Induction

In Fig. 433 CD is a coil of wire (several turns) joined to a battery and a key, and AB is another *coil* joined to a galvanometer. The coils are standing *vertically* facing each other and the reader is looking down on the top of them in the figure. Perhaps Fig. 434 will make the general idea clearer. CD is referred to as the **primary** and AB as the **secondary** circuit.

Start a current in the primary in the direction C to D and a momentary deflection of the galvanometer will follow, showing that an E.M.F. and current are induced in the secondary in the direction B to A, *i.e.* opposite or inverse to the primary current; note that starting a current in the primary means **increasing** the magnetic flux in the secondary, for the flux in the primary reaches over to the secondary, *i.e.* some tubes of induction of the primary "cut" through the secondary.

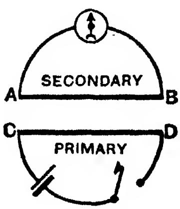


FIG. 433.

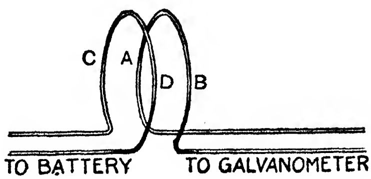


FIG. 434. The coils each consist of several turns.

Switch off the primary current, and a momentary deflection will follow, indicating a secondary E.M.F. and current in the direction A to B, *i.e.* in the same direction as, or direct to, the primary current; stopping the primary current means decreasing the flux in the secondary.

Increasing the primary current or *moving it nearer* to the secondary circuit results in an *inverse* E.M.F. and current; *decreasing* the primary or *moving it away from* the secondary results in a *direct* E.M.F. and current.

From the above we get another law which is sometimes convenient, viz. *an increase in the magnetic flux results in an inverse induced E.M.F. and a decrease in the flux results in a direct induced*

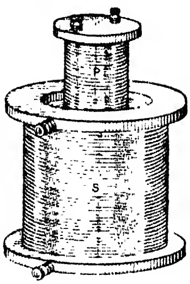


FIG. 435.

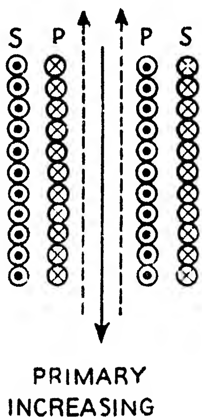


FIG. 436 (*a*).

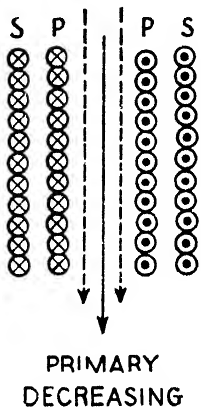


FIG. 436 (*b*).

E.M.F. Note:—Increase—Inverse: Decrease—Direct. Further, a little consideration will show that this is in agreement with Lenz's law. Finally (Fig. 432), if the galvanometer be replaced by an electrometer (no way through) similar momentary deflections will be obtained indicating induced E.M.F.'s and electron displacements in the secondary in accordance with the rules.

Better results are obtained if the secondary is a closely wound solenoid, and the primary is another which can be inserted in it (Fig. 435).

As a further help, Fig. 436 (*a*) shows the primary P inside the secondary S: the current in P is *increasing* and the direction of the induced current in S is shown: it is opposite to the primary. Further, the full-line arrow shows the direction of the increasing flux which P is putting there, and the dotted arrows the direction of the flux of S which is *opposing* the growth of that of P. In Fig. 436 (*b*) the primary current is *decreasing*, the induced current in S is in the same direction as the current in P, and S is *opposing* the cutting off of the field by creating lines (dotted arrows) in the same direction as the P lines which are decreasing.

Not only does the primary in these cases act inductively on the secondary, but the secondary changes react upon the primary: hence these inductive effects between two circuits are referred to as **mutual induction,** and we speak of the **mutual inductance** of two neighbouring circuits.

3. Self-Induction

When a current starts in a wire the growth of the current means also the growth of a magnetic field round about it, and whilst the magnetic flux about the wire is *altering* there is an induced E.M.F. in it which has an "opposition effect." Hence when we start a current in a wire, the growing field reacts upon the current itself by setting up in the wire an *opposing or back E.M.F.* which delays the growth of the current—chokes it back—the result being that it takes time for a current to reach its full strength in a circuit. Similarly when the current is switched off, the field is destroyed and an induced E.M.F. is set up in the wire in the *same* direction as the one cut off which opposes and delays the cut-off: in fact this induced *direct E.M.F.* at break (called the "after E.M.F.") often gives rise to a current (the "after or extra current") which shows itself as a spark when a circuit is broken. Similar results happen if the current in a wire increases or decreases. These effects are known as **self-induction,** and we speak of the **self-inductance** of a circuit.

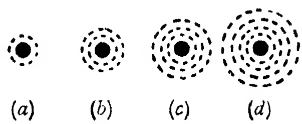


FIG. 437.

We might look at the preceding as follows:—When a current is started the magnetic field with its tubes of induction does not come into complete existence *immediately*. It seems to begin at the wire and gradually extend outwards (Fig. 437): at (*a*) the current has just started and the field is small, at (*b*) it has grown, at (*c*) it has grown still more, and so on, until finally it extends to a great distance. Now the tubes of induction are assumed to be in a state of tension and energy is therefore expended in overcoming this tension and establishing and expanding them in the field. This energy is taken from the current so that the latter grows gradually, not reaching its final value until the field is fully established, and this is equivalent to an *opposing E.M.F.* being induced in the wire. Similarly, when the current is switched off the magnetic field gradually collapses into the wire, the tubes giving up their energy to the charged "particles" of the wire as they contract: thus the current does not immediately drop to zero, and the effect is equivalent to an *induced direct E.M.F.* (and current), *i.e.* one in the same direction as that cut off.

From the preceding it will be seen that self-induction in a circuit behaves like *inertia*: when we start a current in a wire this self-induction tends to choke the current back, and when we stop the current self-induction tends to make it keep on.

We saw (Fig. 394) that resistance coils were doubled back on themselves to reduce the effect of self-induction. The field and any inductive effect of the current going down are opposite to and cancel those of the current coming up.

In a straight wire the self-inductance is only small, but it is still *smaller* if the wire is bent into a *single* loop. This is clear because if a current is started in the loop the inductive action of every small part of the loop *on the part diametrically opposite* to it *assists* the growth of the current: thus a current starting in *ab* in the direction indicated (Fig. 438) tends to induce a current in *dc* as shown, *i.e.* in the same direction as the current growing in the loop. It is therefore easier to start a current in the loop than it is if the same wire be straight, *i.e.* the self-inductance of the loop is less. On the other hand, if the wire be formed into a *coil of several turns*, the turns being close together, the self-inductance will be *much greater*, for each *turn* acts on the *adjoining turns* just as a primary does on a secondary, and a little consideration (and a sketch) will indicate that this will increase the opposition effect to the growth, say, of a current in the coil.

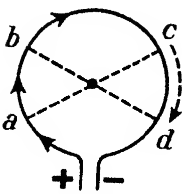


FIG 438.

Inductive effects, both self and mutual, are considerably increased if the coils are wound on iron cores: if iron is there it is magnetised and the magnetic flux "changed" is much greater. In fact inductive effects depend on the permeability (μ) of the medium and thus it is the change in the magnetic induction and number of tubes of induction which is involved in any calculations (Art. 4). Of course, *in air* (strictly vacuo) magnetic induction and field intensity, tubes of induction and tubes of force, are identical.

(1) As simple illustrations of self-inductance effects, consider first a D.C. motor (nowadays everyone knows the general principle of the motor) just coupled up to D.C. mains. The "field coils" of the motor (wound on iron) have a big self-inductance, and the self-induced (opposing) E.M.F. at "start" retards the growth of the current in the "fields," so that a fairly considerable time may elapse before the field current reaches its full strength. When the motor is "shut off" the self-induced direct "after-E.M.F." *may be very*

great. The result is that an arc may be formed at the switch which will, in time, burn the contacts away. For this reason these switches are often designed so that the parts breaking circuit can be renewed. For the same reason switches are usually arranged to have a "quick break" and a "long gap" so that the induced E.M.F. has not time to build up sufficiently to cause an arc to bridge the gap: and power station switches (cut-outs) are often "oil-immersed" to avoid this arcing.

(2) A **choke coil** (or choke) is simply a coil of several turns (but often of low resistance) wound on an iron core so as to have a *large self-inductance*. First arrange a non-inductive resistance R, a lamp L, and a battery B in series, and adjust R until the battery produces only a dull glow of L. Then keeping R at this value join up as shown (Fig. 439) where R_2 is a choke. Turn the switch to X and start a current in R_2. After a short time quickly turn the switch to Y. The lamp L *suddenly glows very brightly* for a moment—brighter than when it was directly joined to B. This shows that the self-induced E.M.F. in R_2 at the break is greater than the original E.M.F. impressed on the circuit by the battery.

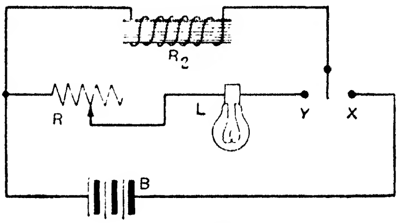


FIG. 439.

4. Magnitude of the Induced E.M.F.

We have seen that the induced E.M.F. is greater the greater the flux changed and the more rapid the change. Let PQ (Fig. 440) be a copper bar capable of sliding along the parallel rails X and Y, and B a battery of constant E.M.F. (E). Suppose this arrangement is in a medium of permeability μ and that a uniform magnetic field of intensity H is applied in a direction perpendicular to the plane of the circuit (and, say, downwards in the figure as shown). There will be a force F on PQ in the direction indicated (Fleming left-hand rule) the magnitude of which is μHIl dynes (page 323) where I = steady current round the circuit and l = distance between the rails. Suppose the rod moves a small distance δx a small time δt. The current will change slightly owing to the induced E.M.F. opposing it when the rod moves: let its value be i

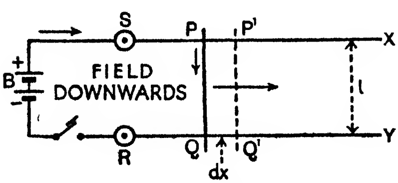


FIG. 440.

while the rod moves. The work done in moving the rod (= force $\times$ distance) is $\mu H i l \delta x$ ergs (page 324), and the *rate of doing work* in moving it is $\mu H i l \delta x / \delta t$ or, in the limiting case:—

$$\text{Rate of doing work in moving PQ} = \mu H i l \frac{dx}{dt}.$$

Now μH = number of **unit tubes of induction** *per unit area*, and ldx = *area* swept out, so $\mu H l dx$ is the *change* in the total tubes through the circuit, and therefore $\mu H l dx/dt$ is the "*rate of change*" *in the tubes of induction through the circuit:* hence:—

$$\text{Rate of doing work in moving PQ} = i\frac{dN}{dt},$$

where N is used to denote total tubes of induction and dN/dt the rate of change of the tubes through the circuit.

The energy both for the heating of the circuit and for doing this work is supplied by the battery. The *rate* of supply by the battery is Ei, and i^2R is the *rate* of dissipation as heat where R = circuit resistance: hence

$$Ei = i^2R + i\frac{dN}{dt}; \quad \therefore\ i = \frac{E - dN/dt}{R}$$

and the E.M.F. (E) of the circuit is therefore opposed by an E.M.F. equal to dN/dt, *i.e.* if e denote this induced E.M.F.—

$$e = -\frac{dN}{dt};$$

thus **the induced E.M.F. is equal to the rate of change of the number of tubes of induction threading the circuit.** The negative sign indicates that the induced E.M.F. opposes the change in the flux.

Note that μH (= B) was used in the above investigation, *i.e.* it is "change in the *magnetic induction*" which is involved: in air $\mu = 1$ and $B = H$.

For calculation purposes we may, neglecting sign, write—

$$\text{Induced E.M.F.} = \frac{\text{Change in flux}}{\text{Time in seconds}} = \frac{\text{Change in flux}}{10^8 \times \text{Time in seconds}},$$

the first expression giving *e.m. units* and the second *volts*.

Further, if there are n turns in a coil *each* acquires the above E.M.F., and since the turns are in series the total E.M.F. is n (Change in flux)/Time in seconds. The product of the flux

through a coil and the number of turns is called the *effective flux* or the *linkages* (sometimes the " cuts "); hence:—

$$e = \frac{\text{Change in effective flux}}{\text{Time in seconds}} = \frac{\text{Change in linkages}}{\text{Time in seconds}}.$$

Professor Cramp considered that " cuts " is a more correct expression than " linkage " (*Jour. I.E.E.*, 1936).

Example.—*A coil of wire is connected to a galvanometer, the resistance of the coil and galvanometer being* 200 *and* 400 *ohms respectively. The coil is moved in the field, and at a given instant there are* 20,000 *unit tubes through it, whilst* $\frac{3}{4}$ *of a second later the number is* 2000. *If there are* 100 *turns in the coil find the average E.M.F. and current during this period.*

Effective flux or linkages at the beginning of the time in question $= 100 \times 20{,}000 = 2{,}000{,}000$, and at the end of the time $= 100 \times 2000 = 200{,}000$; hence change in effective flux $= 1{,}800{,}000$, and

$$\cdot\ \text{Average induced E.M.F.} = \frac{18 \times 10^5}{10^8 \times \frac{3}{4}} = \cdot 024 \text{ volt},$$

$$\text{Induced current} = \cdot 024/(200 + 400) = \cdot 00004 \text{ ampere}.$$

It is useful to find an expression for the induced E.M.F. (e) between the ends of a *straight* conductor moving with velocity v across a magnetic field. Taking the simplest case of Fig. 440 and the moving bar PQ we have $dN = \mu Hl dx = Bl dx$; hence $dN/dt = Bl dx/dt = Blv$: $\therefore\ e = Blv$ e.m. units $= Blv/10^8$ volts.

For simplicity, the field H in Fig. 440 was taken perpendicular to the plane of the circuit, but the same result is obtained if this is not the case. As a further help we indicate the " proof ":—

Suppose H makes an angle θ with the bar PQ and that the plane containing H and PQ makes an angle α with the plane of the circuit. The force on PQ is due to the component of H which is perpendicular to PQ, viz. H sin θ: this force is perpendicular to both H and PQ, so that the force on PQ $= \mu H \sin \theta l i$. The component of this force on PQ which is parallel to the rails $= \mu H \sin \theta l i \sin \alpha$. Hence:—

$$\text{Rate of doing work in moving PQ} = \mu H \sin \theta . li . \sin \alpha . \frac{dx}{dt}$$

and, reasoning as above, we get for the induced E.M.F. (e):—

$$e = -\mu Hl \sin \theta . \sin \alpha . \frac{dx}{dt}.$$

Now the component of the field perpendicular to the plane of the circuit is H $\sin \theta \sin \alpha$, the change of the flux is $\mu H \sin \theta \sin \alpha\, l dx$, and the *rate of change of the flux* is therefore $\mu Hl \sin \theta \sin \alpha\, dx/dt$. Thus the induced E.M.F. (e) = rate of change of flux, or denoting flux by N we have $e = -dN/dt$ as before.

5. Total Charge in an Induced Current

If R = the resistance of the circuit and e = induced E.M.F. (both in e.m. units) then i the current in e.m. units $= e/R = -(1/R)(dN/dt)$, and the *quantity* of electricity passing in time $dt = idt = -dN/R$ e.m. units.

It follows that for any finite change, say an *increase*, in the magnetic flux from N_1 to N_2 we have (by integrating) that the total quantity is given by

$$Q = -\frac{N_2 - N_1}{R} \text{ e.m. units} = -\frac{N_2 - N_1}{10^8 R} \text{ coulombs (if R = ohms)}$$

from which it follows that the quantity does not depend on rate of change. If the flux *decreases* from N_2 to N_1 the total charge $= +(N_2 - N_1)/R$. The sign indicates the direction in which the charge passes round the circuit: the *magnitude* is in each case $(N_2 - N_1)/R$.

The change of units indicated above is readily seen since 1 ohm = 10^9 e.m. units and 1 coulomb = $\frac{1}{10}$ e.m. unit: thus if R = ohms = R × 10^9 e.m. units, then neglecting sign:—

$$Q = \frac{N_2 - N_1}{10^9 R} \text{ e.m. units} = \frac{N_2 - N_1}{10^9 R} \times 10 = \frac{N_2 - N_1}{10^8 R} \text{ coulombs,}$$

and of course $N_2 - N_1$ means again change in *linkages*. We therefore have:—

$$\text{Quantity (coulombs)} = \frac{\text{Change in linkages}}{10^8 \times \text{Resistance (ohms)}}.$$

6. The Self-Inductance (L) of a Circuit

(1) Consider first, for simplicity, a circuit consisting of a single turn coil of wire. When a current flows through the coil it produces a magnetic flux or flow of magnetic induction through it. This flux is proportional to the current when the permeability of the medium is constant. In this case if N denotes the flux, and I the absolute current:—

$$N \propto I \quad \text{or} \quad N = LI \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where L is a constant called the *coefficient of self-induction or the* **self-inductance** *of the coil.* If I be unity, N is numerically equal to L; thus **the coefficient of self-induction or the self-inductance of the circuit in absolute units is numerically equal to the magnetic flux through it when absolute unit current passes.** Clearly *the circuit has a coefficient of self-induction of one C.G.S. unit if the magnetic flux be unity when the unit e.m. current passes.* The practical unit is the **henry,** which is equal to 10^9 C.G.S. units; thus *the circuit*

has a self-inductance of one henry if the magnetic flux be 10^9 *when the unit e.m. current passes, and therefore* $1/10$ *of* 10^9, *i.e.* 10^8, *when a current of one ampere passes.*

If the coil consists of several turns, then it will be clear that N above refers to *effective* flux or "linkages" in practice. Equation (1) would therefore be written $SN = LI$ if $S =$ number of turns on the coil, and if I be unity $L = SN =$ linkages, and if the linkages be unity L is unity. Thus a more convenient definition of L for practical purposes is as follows: *the self-inductance of a circuit in absolute units is numerically equal to the linkages of the circuit when absolute unit current is flowing through it: this divided by* 10^9 *will give the self-inductance in henries.* Further, we can say *the self-inductance is one henry if the linkages be* 10^8 *when one ampere passes.*

It will make the above clearer if we work out the value of the self-inductance L of a *long and thin* closely wound solenoid:—

Self-Inductance of a Long Solenoid.—Let $l =$ length, $S =$ total number of turns on it, and $a =$ cross-sectional area. *First assume an air core.* Imagine a current I e.m. units to be flowing in the solenoid.

Field inside $= 4\pi SI/l$ gauss $=$ Tubes of force per sq. cm., and these are also, in this case, tubes of induction;

$$\therefore \text{ Total flux through } \textit{each} \text{ turn} = N = (4\pi SI/l) \times a;$$

$$\therefore \text{ Linkages} = NS = \frac{4\pi S^2 Ia}{l}; \quad \text{But } NS = LI;$$

$$\therefore LI = \frac{4\pi S^2 Ia}{l}; \quad \therefore L = \frac{4\pi S^2 a}{l} \text{ e.m. units} = \frac{4\pi S^2 a}{l \times 10^9} \text{ henries.}$$

Example.—*A solenoid (air core) has* 400 *turns, is* 20 *cm. long, and has a cross-sectional area of* 4 *sq. cm. Find the coefficient of self-induction of the solenoid.*

$$\text{Linkages} = \text{Flux} \times \text{Turns} = \frac{4\pi SIa}{l} \times S = \frac{4\pi S^2 Ia}{l}.$$

If I be unity this expression gives the self-inductance L;

$$\therefore L = \frac{4\pi S^2 a}{l} = \frac{4 \times 3{\cdot}14 \times 400 \times 400 \times 4}{20} = 401{,}920 \text{ e.m. units,}$$

$$\textit{i.e. } L = 401920/10^9 = {\cdot}0004 \text{ henry.}$$

In the above we assumed the solenoid to have an *air core* ($\mu = 1$). If the core consists of a rod of iron of cross-section a_1 and permeability μ_1, the remainder being a medium of cross-section a_2 and permeability μ_2, then

$$L = \frac{4\pi S^2}{l}(\mu_1 a_1 + \mu_2 a_2) \times 10^{-9} \text{ henries,}$$

whilst in the case of a very long solenoid closely wound throughout

its entire length on an iron rod of permeability μ then L may be taken as given by $\mu \times (4\pi S^2 a/l \times 10^9)$ henries.

(2) If the magnetic flux is changing owing to the current changing then the *rate of change* of flux depends upon the *rate of change* of current. In fact from (1) above, viz. $N = LI$, we can write $dN/dt = L dI/dt$, where dN/dt is the rate of change of flux, and dI/dt rate of change of current. But the induced E.M.F. (e) is given by $- dN/dt$;

$$\therefore \text{ Induced E.M.F.} = e = -L\frac{dI}{dt} \dots\dots\dots\dots (2)$$

that is, $e = -$ L (Rate of change of Current). The negative sign means it is an *opposing* E.M.F. when the current is growing, that is, when the rate of change of current is positive, and is a *direct* E.M.F. when the current is decreasing. Clearly, if dI/dt is unity, L is numerically equal to e: thus the coefficient of self-induction or the self-inductance of a circuit is numerically equal to the induced E.M.F. round the circuit due to unit rate of change of the current in it. Hence: *A circuit has a self-inductance of one C.G.S. unit when current increasing at the rate of one e.m. unit per second produces an opposing E.M.F. of one e.m. unit. A circuit has a self-inductance of one henry when current increasing at the rate of one ampere per second produces an opposing E.M.F. of one volt.* From (2) it is convenient for practical purposes to remember for example that:—

$$\text{L (henries)} = \frac{\text{Self-induced E.M.F. (volts)}}{\text{Rate of change of current (amperes per sec.)}}.$$

Some idea of the magnitude of the self-inductance effects may be gathered from the relation above, viz. $e = -$ L (Rate of change of Current). (1) Using practical units, in a circuit for which $L = 0{\cdot}5$ henry and in which current is growing uniformly at the rate of 2 amperes in $\frac{1}{100}$ of a second, then $e = -0{\cdot}5 \times 2/{\cdot}01 = -100$ volts. (2) If the rate of dying away of current be taken, and if it be very rapid, then the forward E.M.F. will be very large: for example, if the current were dying away at the rate of 2 amperes in $\frac{1}{1000}$ second, then—

$$e = -0{\cdot}5 \times (-2/{\cdot}001) = 1000 \text{ volts},$$

and hence the necessity—as mentioned earlier—for exercising ***great care when cutting off the current in highly inductive circuits***, *e.g.* circuits with electromagnets.

(3) It will be seen later (Art. 15) that the work done in establishing a current I in a circuit of self-induction L is given by the expression

$$\text{Work (W)} = \tfrac{1}{2}LI^2 \dots\dots\dots\dots\dots\dots (3)$$

and this supplies another definition of L, for if I be unity L is numerically equal to 2W; thus **the coefficient of self-induction or the self-inductance of a circuit is numerically equal to twice the work done in establishing the magnetic induction accompanying unit current in the circuit.** The corresponding definitions of the C.G.S. unit and the henry may be readily derived.

7. Measurement of Self-Inductance (L)

In the Wheatstone bridge method of measuring resistance *the battery key is closed first and then the galvanometer key*, thus ensuring that the currents in the various branches have become steady (and any induction effects due to any inductance in the branches have ceased) before the galvanometer is joined in the circuit. If the keys be worked in the reverse order, i.e. *the galvanometer key closed first and then the battery key*, the galvanometer will be in circuit during the variable state (growth) of the current, and (unless the inductive effects in the arms balance) there will be a momentary kick of the galvanometer even if resistances happen to be adjusted for perfect balance with steady currents. This fact is involved in the test which follows.

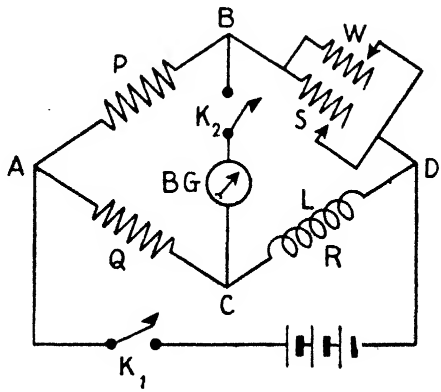


FIG. 441.

The experiment below indicates one method of measuring self-inductance (for other methods and fuller details of inductance measurement see *Advanced Textbook of Electricity and Magnetism*).

The coil the value of L for which is to be measured is at R in Fig. 441, in one arm of a Wheatstone Bridge arrangement of non-inductive resistances. If the resistance of the coil is very small it is advisable to insert a non-inductive resistance in the same arm with it. BG is a ballistic galvanometer. The simplest arrangement is to make P = Q, and to facilitate exact balancing for steady currents the resistance in the arm BD should be two resistance boxes S and W arranged in parallel. The test is carried out thus:—

(1) With W disconnected, adjust S until on *closing first the battery key* K_1 *and then the galvanometer key* K_2 (in the usual way) there is no deflection. Now make S slightly larger, then couple up W and adjust W until *exact*

balance is obtained with a resistance SW/(S + W) equal to R (the resistance of the inductance coil) in the arm BD.

(2) Now work the keys in the reverse order, i.e. *close first the galvanometer key* K_2 *and then the battery key* K_1. The inductance in CD will cause a sudden throw of the galvanometer. This first throw is noted; let the scale deflection be d_1, indicating a first angular deflection, α.

(3) Finally increase W so that the value of SW/(S + W) is increased by a small amount ρ. On repeating (1), i.e. *closing first* K_1 *and then* K_2 there will be a *permanent* deflection for the balance for steady currents has been upset by increasing W. Let the steady scale deflection be d_2 indicating a steady angular deflection θ. Then it can be shown that (with a moving magnet ballistic galvanometer):—

$$L = \frac{T}{\pi} \times \rho \times \frac{\sin \alpha/2}{\tan \theta} \text{ (see below).}$$

If L denote the self-inductance of the coil in CD and I the current established in CD on testing for transient current by closing K_1 after K_2 ((2) above), then Q, the quantity discharged through BG, is proportional to LI, that is $Q = k\text{LI}$, where k is a constant. But (page 351):—

$$Q = \frac{HT}{\pi G} \sin \frac{\alpha}{2}; \quad \therefore k\text{LI} = \frac{HT}{\pi G} \sin \frac{\alpha}{2} \quad \ldots\ldots\ldots (1)$$

Also, when the resistance in BD is increased by ρ ((3) above) the PD is increased by an amount $I'\rho$, where I' is the current in BD after the increase is effected and the balance for steady currents disturbed. Hence the permanent current through BG in (3) above may be said to be due to this increment of potential difference in BD and is therefore proportional to $I'\rho$. That is i, the current through BG, is given by $i = kI'\rho$, where k is, on account of the symmetry of the bridge, the same constant as for Q above. But:—

$$i = \frac{H}{G} \tan \theta; \quad \therefore kI'\rho = \frac{H}{G} \tan \theta \quad \ldots\ldots\ldots (2)$$

and from these two results (1) and (2) we get:—

$$\frac{k\text{LI}}{k\rho I'} = \frac{T}{\pi} \frac{\sin (\alpha/2)}{\tan\theta}; \quad \therefore L = \frac{T}{\pi}\rho\frac{I'}{I}\frac{\sin (\alpha/2)}{\tan \theta}.$$

Now if ρ be very small I and I' are approximately equal, and we obtain the expression given for L in the experiment above. Further, since α and θ are small we can write $\sin (\alpha/2)/\tan \theta$ as $(d_1/2)/d_2 = d_1/2d_2$. Hence:—

$$L = \frac{T}{\pi} \rho \frac{d_1}{2d_2}.$$

Since 1 ohm = 10^9 e.m. units, and 1 henry = 10^9 e.m. units this gives L in e.m. units if ρ is in e.m. units and L in henries if ρ is in ohms.

8. The Mutual Inductance (M) of Two Circuits

The details of the **coefficient of mutual induction** or the **mutual inductance** (denoted by M) of two circuits are similar to those of self-induction, so we need only summarise the main points.

(1) Consider two adjacent circuits A and B. Let N be the magnetic flux through one of them, say B, when a current I flows in the other (A): then

$$N \propto I \quad \text{or} \quad N = MI \quad \dots\dots\dots\dots\dots(1)$$

where M is a constant called the *coefficient of mutual induction* or the *mutual inductance* of the two circuits. If I be unity M is numerically equal to N: thus **the coefficient of mutual induction or the mutual inductance of two circuits is numerically equal to the magnetic flux or flow of induction through one when unit current passes in the other;** clearly *the coefficient of mutual induction of two circuits is one C.G.S. unit if the magnetic flux through one is unity when the unit e.m. current passes in the other;* clearly also *the mutual inductance is one henry if the magnetic flux through one is* 10^8 *when one ampere passes in the other.*

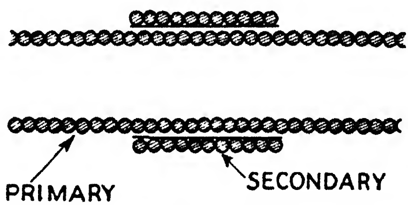


FIG 442.

Here again, as in Art. 5, the above really refers to *effective flux* or *linkages* in one circuit due to unit current in the other: thus more convenient definitions of M in practice are as follows:—*The mutual inductance of two circuits is one C.G.S. unit if the linkages of one circuit be unity when unit e.m. current flows in the other* (this divided by 10^9 gives the mutual inductance in henries). Further, *the mutual inductance is one henry if the linkages of one circuit be* 10^8 *when one ampere flows in the other.* To make the idea clear we will work out the value of M for two solenoids.

Mutual Inductance of Two Solenoids.—For simplicity take the case of two concentric solenoids, a long primary and a secondary wound round the middle of it (Fig. 442): we shall assume therefore that there is no magnetic leakage, *i.e.* all the flux passes through both coils and that the medium is air.

Let S_1, l_1, and a_1 apply with their usual meaning to the primary and suppose a current I e.m. units is flowing in it. Let S_2 = turns on the secondary.

Total tubes in primary $= \frac{4\pi S_1 I a_1}{l_1}$; Linkages with secondary $= \frac{4\pi S_1 I a_1}{l_1} \times S_2$.

If I = unity this gives M in e.m. units;

$$\therefore \; M = \frac{4\pi S_1 S_2 a_1}{l_1} \text{ e.m. units} = \frac{4\pi S_1 S_2 a_1}{10^9 \times l_1} \text{ henries.}$$

Example.—*A solenoid is* 16 *cm. long, has* 1280 *turns, and its cross-section is* 10 *sq. cm. Closely wound on its central part is another coil of* 1000 *turns. Find the mutual inductance of the coils.*

Imagine a current I e.m. units in the solenoid.

$$\text{Tubes per sq. cm. in solenoid} = \frac{4\pi S I}{l} = \frac{4 \times 3{\cdot}14 \times 1280 \times I}{16} = 1004{\cdot}8\ I;$$

$$\therefore \text{ Total tubes in solenoid} = 1004{\cdot}8\ I \times 10 = 10048\ I.$$

$$\text{Linkages with other coil} = \text{Flux} \times \text{Turns in other coil}$$
$$= 10048\ I \times 1000 = 10048000\ I.$$

If I is unity this gives M is electromagnetic units;

$$\therefore \; M = 10048000 \text{ e.m. units} = 10048000/10^9 = {\cdot}01 \text{ henry approx.}$$

If the primary is closely wound on an iron core the flux and therefore the mutual inductance is increased μ times. In general if the iron has cross-section a_1 and permeability μ_1, and the remainder is a medium of cross-section a_2 and permeability μ_2,

$$M = \frac{4\pi S_1 S_2}{l}(\mu_1 a_1 + \mu_2 a_2) \times 10^{-9} \text{ henries.}$$

(2) If the flux in A is changing owing to the current in it changing, then there is an induced E.M.F. (e) in B. From (1) above, viz. N = MI, we get

$$-\frac{dN}{dt} = -M\frac{dI}{dt}, \; i.e. \; e = -M\frac{dI}{dt} \quad \ldots\ldots\ldots\ldots (2)$$

that is, e (in one coil B) = — M × Rate of change of current (in other coil A), and if the rate of change of current is unity e numerically equals M. Thus **the coefficient of mutual induction or the mutual inductance of two circuits is numerically equal to the induced E.M.F. round one circuit due to unit rate of change of the current in the other.** Hence: *Two circuits have a mutual inductance of one C.G.S. unit when current changing in one at the rate of one c.m. unit per second results in an induced E.M.F. of one e.m. unit in the other. Two circuits have a mutual inductance of one henry when current changing in one at the rate of one ampere per second results in an induced E.M.F. of one volt in the other.*

That very high voltages may be obtained by devices suitably employing the effects of mutual inductance is readily seen. Suppose, for example, that for two coils M is 4 henries, and the primary current changes at the rate of one ampere in $\frac{1}{5000}$ of a second, then the induced E.M.F. in the secondary circuit is given by $e = -$ M (rate of change of primary current) $= -4 \times 1/\cdot 0002 = -20{,}000$ volts.

9. Measurement of Mutual Inductance (M)

Here again, as in the case of self-inductance, there are several methods available. Below we give Carey Foster's method which involves the use of a standard condenser of known capacitance.

The two coils P and S (Fig. 443), P the primary and S the secondary are, connected in circuit with a condenser of capacitance C, the non-inductive resistances R and Q, and the galvanometer G. By means of a key at K the current in P can be made, broken, or reversed, and an inductive impulse equal to MI (if made or broken) or 2MI (if reversed) thereby set up in S, where M is the mutual inductance of P and S and I is the current in R and P. The condenser also, with its terminals connected to the points A and B, becomes charged or discharged or has its charge reversed according as the current is made, broken, or reversed. There are thus two simultaneous "rushes" through G, one due to the condenser, the other due to the charge induced in S. It will be noted, however, that by the arrangement of the circuit these two discharges through G are in opposite directions, and may be adjusted to equality by adjusting R or Q until the galvanometer shows no deflection on working the key at K. When this adjustment is made we have:—

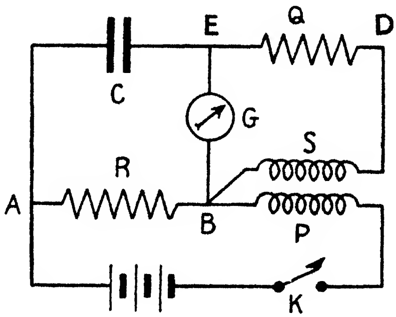


FIG. 443.

$$M = CR(Q + S).$$

PROOF.—The charge in the condenser is CIR (for IR is the P.D, between A and B), and the portion of this which passes through the galvanometer is:—

$$\frac{Q+S}{Q+S+G}.CIR,$$

since (Q + S) is shunting G. The electromotive impulse set up in S is MI (for N = MI), and the quantity of electricity set in motion by it is (Art. 5):—

$$\frac{MI}{G+S+Q}.$$

This quantity all passes through the galvanometer. Hence we have when G is not deflected:—

$$\frac{Q+S}{Q+S+G}.CIR = \frac{MI}{Q+S+G}; \quad \therefore M = CR(Q+S),$$

from which M is determined.

When the student first begins his experimental work on inductance measurements he usually comes to the conclusion either that he is a poor experimenter, or that "the apparatus is wrong"—probably the latter. Both conclusions may possibly be correct, but the experiments are nevertheless troublesome at the best. Methods depending on the employment of alternating currents in conjunction with a type of galvanometer known as the **vibration galvanometer** have been developed which are much more satisfactory for these measurements (see page 529). In fact by these methods and with circuit arrangements as in Fig. 443 tests can readily be made on self- and mutual inductances and capacitance.

10. Measurement of Magnetic Fields

In Art. 5 it was shown that the *quantity* of electricity (Q) in an induced current is given by the expression $Q = (N_2 - N_1)/R$ e.m. units if R is in e.m. units, or by $Q = (N_2 - N_1)/10^8R$ coulombs if R is in ohms: thus:—

$$Q = \frac{\text{Change in linkages}}{10^8 \times \text{Resistance (ohms)}} \text{ coulombs},$$

and this fact is used in the measurement of magnetic fields. The principle will be gathered from the following preliminary experiment:—

(1) **To Determine the Angle of Dip by Induced Currents.**—The apparatus consists of a coil of wire, say of S turns and cross-sectional area a, joined to a ballistic galvanometer some distance away. Set up the coil with its plane at right angles to the meridian, in which case the linkages = HSa where H = horizontal component of the earth's field = tubes per sq. cm. Quickly rotate the coil *through* 180° about a vertical axis, and note the throw θ_1 of the galvanometer. The change in the effective flux is $2HSa$, for it is reduced from HSa to zero in the first 90° of rotation, then put it in the opposite direction so far as the coil is concerned, and increased from zero to HSa. The quantity induced is therefore $2HSa/R$ where R = resistance of coil and galvanometer:

$$\therefore \frac{2HSa}{R} \propto \theta_1.$$

Now arrange the coil in a horizontal position with the axis of rotation in the meridian, again turn through 180° and let θ_2 be the throw of the galvano-

meter; if V be the vertical component, it is clear that the quantity induced in this case is $2VSa/R$; hence

$$\frac{2VSa}{R} \propto \theta_2;$$

$$\therefore \tan D = \frac{V}{H} = \frac{\theta_2}{\theta_1},$$

where D = angle of dip which is thus determined.

The experiment could be used to actually determine either H or V if the constant (k) of the galvanometer, the resistance (R) of coil and galvanometer, and the turns (S) and face area (a) of the coil be known: thus for the first part of the experiment $2HSa/R = k\theta_1$; $\therefore$ $H = k\theta_1 R/2Sa$, and all terms on the right are supposed known.

A coil of known dimensions properly mounted and arranged for rotation for the above and similar work is called an **earth inductor.** A simple type is shown in Fig. 444: the figure is self-explanatory.

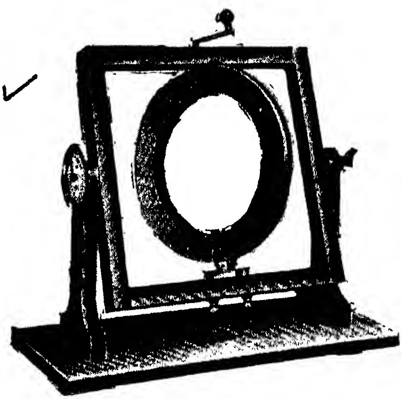


FIG. 444.

When we wish to measure more intense fields, *e.g.* between the poles of an electromagnet, we generally use a small coil known as a **search coil** (or **test coil**) of, say, 50 or 60 turns of fine wire, the face area being about 1 sq. cm. This is placed in the field and either rotated through 180° as above or completely removed from the field, the latter being the usual method. A ballistic galvanometer is in series with the search coil and if its constant (k) be known the field is determined. Usually, however, the galvanometer is "calibrated" as it were at the time of the experiment by means of a "standard inductor" included in the circuit. An earth inductor rotated in the earth's field can be used as this standard if the value of the earth's field be accurately known.

A more usual type of standard for experimental work is that known as the **standard solenoidal inductor,** which makes use of the uniform field in the interior of a long solenoid. If I denote the current in absolute units, the field in the interior of the solenoid is given by $4\pi nI$, where n is the number of turns *per unit length*

($H = 4\pi SI/l$ and $n = S/l$). The inductor for use with this field usually consists of a few turns of thin, well insulated wire wound round the outside of the solenoid near its middle point. The induction throw is obtained by reversing the current in the solenoid. If S, A denote the *total* number of turns and area of the inductor respectively, then, on reversing the current in the long solenoid the change in the magnetic flux (linkages) in the inductor coil is $2\,(4\pi n I)\,SA$, *i.e.* $8\pi n ISA$, and the quantity of electricity (absolute units) set in motion is $8\pi n ISA/R$, where R is the resistance (absolute units) of the circuit in which the inductor is placed. If I is measured accurately by means of a galvanometer or ammeter in the solenoid circuit, this inductor may be used for standardising the observations of a search coil placed in the same circuit with it. The method of using this standard is indicated in the next experiment.

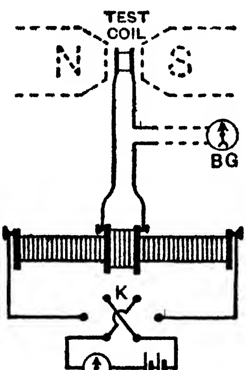


FIG. 445.

To Determine the Strength of the Field between the Poles of an Electromagnet, using a Standard Solenoidal Inductor.—The arrangement is indicated in Fig. 445, where K is a reversing key in the long solenoid circuit and G a galvanometer or ammeter to measure the current (I) in the solenoid. Start the current in the long solenoid, and when the ballistic galvanometer (BG) is quite steady *reverse* the current by means of the key K and note the throw of BG. If this be d_1, then:—

$$\frac{8\pi n ISA}{R} \propto d_1,$$

where R = resistance of inductor coil + test coil + BG.

With no current in the long solenoid place the test coil in the field to be measured, withdraw it, and let d_2 be the galvanometer throw; then if *s*, *a* be the number of turns and face area of the test coil, and F the strength of the field:—

$$\frac{saF}{R} \propto d_2;$$

$$\therefore \frac{saF}{R} \Big/ \frac{8\pi n ISA}{R} = \frac{d_2}{d_1}; \quad \therefore F = \frac{8\pi n ISA}{sa} \cdot \frac{d_2}{d_1}.$$

An excellent standard for laboratory purposes is that known as **Hibbert's magnetic flux standard**. It consists (Fig. 446) of a block of hard steel provided with a cylindrical groove, and magnetised as indicated. A brass cylinder B carries a coil C; it can be lowered into the groove, the coil thereby cutting the tubes due to the magnet, in consequence of which an induced charge circulates in

the coil. The flux is determined at the outset by comparison with (say) a solenoidal standard.

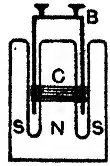


FIG. 446.

11. Determination of the Constant of a Ballistic Galvanometer

A standard inductor—say of the solenoidal type (Art. 10)—may conveniently be used in this test. The arrangement is indicated in Fig. 447, where BG is the ballistic galvanometer under test. The current is started in the solenoid, and when BG is quite steady again at zero, the current is reversed by means of the key K: let α be the first angular swing of BG. Assuming a galvanometer of the moving needle type, if Q be the quantity discharged, then (pages 351, 451):—

$$Q = \frac{HT}{\pi G} \sin \tfrac{1}{2} \alpha \left(1 + \frac{\gamma}{2}\right) = k \sin \tfrac{1}{2} \alpha \left(1 + \frac{\gamma}{2}\right),$$

where k is the constant required. But (Art. 10) $Q = 8\pi n \mathrm{IS}a/\mathrm{R}$, where n = turns per cm. on solenoid, I = current in solenoid (absolute units), S = total turns on, and a = sectional area of, inductor coil, and R = resistance of inductor and galvanometer branch (absolute units). Hence:—

$$k \sin \tfrac{1}{2} \alpha \left(1 + \frac{\gamma}{2}\right) = \frac{8\pi n \mathrm{IS}a}{\mathrm{R}};$$

$$\therefore\ k = \frac{8\pi n \mathrm{IS}a}{\mathrm{R} \sin \tfrac{1}{2}\alpha\left(1 + \frac{\gamma}{2}\right)}.$$

If BG be of the reflecting type and α be small, we can assume that $\sin (\alpha/2) = \delta/4\mathrm{D}$ where δ = first deflection along the scale and D = distance between scale and mirror: then if we neglect damping:—

$$k = \frac{32\pi n \mathrm{IS}a\mathrm{D}}{\mathrm{R}\delta}.$$

which gives k in absolute units per scale division of throw.

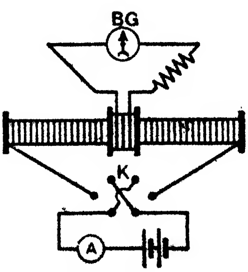


FIG. 447.

If the galvanometer be of the moving coil type then $Q = k\alpha$, or correcting for damping $Q = k\alpha\sqrt{d}$ say, where d = decrement (page 451). Equating as above, $k = 8\pi n \mathrm{IS}a/\mathrm{R}\alpha\sqrt{d}$.

In the above we have assumed absolute units for I and R: if the current is in

amperes and the resistance in ohms, then taking the last case in illustration:—

$$k = \frac{8\pi n I S a}{10^{10} \times R a \sqrt{d}},$$

where k numerically equals coulombs per unit angular of throw.

Another method of finding the constant of a ballistic galvanometer is to charge a condenser of known capacitance C by a cell of known E.M.F. (E) and then to discharge the condenser through the galvanometer. Taking the moving coil type, if Q be the charge $Q = CE = ka\sqrt{d}$; $\therefore\ k = CE/a\sqrt{d}$. Care again must be taken with the units: if E is in volts and C is microfarads, Q is in microcoulombs, and k gives microcoulombs per unit of throw.

12. Growth of Current in Circuit with Resistance and Inductance

We have seen that owing to the induced opposing E.M.F. which is set up when a circuit containing a source of constant E.M.F. (E) is closed, the current does not instantly reach its final steady value, say I_m—it takes time to do so. Let i be the value of the current at any instant during the variable stage when the current is growing (say at time t from the start), and R the total resistance of the circuit. Then (since $i = (E - e)/R$ or $E - e = Ri$):—

$$E - L\frac{di}{dt} = Ri \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\therefore\ \frac{E}{R} - i = \frac{L}{R}\frac{di}{dt}, \quad i.e.\quad \frac{di}{(E/R - i)} = \frac{R}{L}dt.$$

But $d\,(E/R - i) = -\,di$; $\quad \therefore\ \dfrac{d\,(E/R - i)}{(E/R - i)} = -\dfrac{R}{L}dt.$

Integrating this from the lower limits where t and $i = 0$ gives:—

$$\log_e \frac{E/R - i}{E/R} = -\frac{R}{L}t;$$

$$\therefore\ e^{-\frac{R}{L}t} = \frac{E/R - i}{E/R} \quad\text{or}\quad i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right),$$

where e, the base of the Napierian logarithms, is, of course, equal to 2·7182818.

Now i is the current at the end of time t from the instant of closing the circuit, and E/R is the final steady value of the current: denoting these by I_t and I_m respectively we have:—

$$I_t = I_m\,(1 - e^{-\frac{R}{L}t}) \quad\text{or}\quad I_t/I_m = 1 - e^{-\frac{R}{L}t} \quad \ldots\ldots\ldots (2)$$

From this it is evident that, when t is equal to L/R, 2L/R, 3L/R, etc., the ratio of the actual current to the maximum value attainable is given by $1 - 1/e$, $1 - 1/e^2$, $1 - 1/e^3$, etc. That is, at the ends of the time L/R, 2L/R, 3L/R, etc., the current value is ·6321, ·8647, ·9502, etc., of the final attainable value. The quantity L/R is called the **time constant** of the circuit.

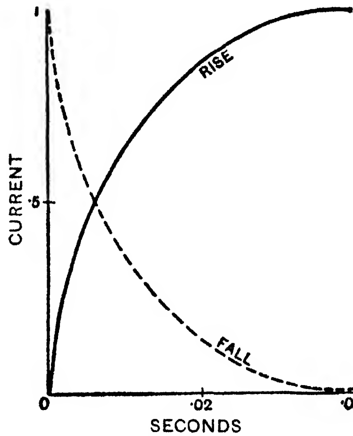


FIG. 448.
R = 2 ohms. L = ·02 henry.

Further, it follows from the above that the current will actually require an infinite time to attain its full value, but that, as L/R is usually very small, it rapidly attains a value very nearly equal to its final value. This is shown in Fig. 448, which gives the curve (full line) of rise of a current in a circuit, where R = 2 ohms and L = ·02 henry, and for which, therefore, the time constant L/R is $(\cdot 02 \times 10^9)/(2 \times 10^9) = \cdot 01$. In this circuit the current rises to ·6321 of its full value in ·01 sec., to ·8647 of its full value in ·02 sec., to ·9502 of its full value in ·03 sec., and so on, evidently attaining practically its full value in a small fraction of a second. In practice we often assume that a current reaches its full strength after a period of 5 times its time constant.

13. Decay of Current in Circuit with Resistance and Inductance

When the circuit is broken the E.M.F. of the source (E) disappears from the first equation of Art. 12, and we get:—

$$iR = -L\frac{di}{dt}; \quad \therefore \frac{di}{i} = -\frac{R}{L}dt.$$

Hence if I_m be the steady maximum value of the current at the instant of cutting out the E.M.F. (E), and I_t the value at time t afterwards, we get on integrating as before:—

$$\log_e \frac{I_t}{I_m} = -\frac{R}{L}t;$$

$$\therefore I_t/I_m = e^{-\frac{R}{L}t} \text{ or } I_t = I_m e^{-\frac{Rt}{L}} \quad \ldots\ldots\ldots\ldots\ldots (3)$$

Here it is evident that in times L/R, 2L/R, 3L/R, etc., from the break of the circuit, the current falls to $1/e$, $1/e^2$, $1/e^3$, etc., of its

initial value, that is, to ·3679, ·1353, ·0498, etc., of the initial value. The dotted curve of Fig. 448 shows the fall of the current in the circuit for which the full line gives the curve of rise of the current.

Note that in this section the current has a maximum value at start and falls to zero, whilst in Art. 12 it started at zero and rose to a maximum.

The investigation assumes that the E.M.F. of the source (E) is cut out without changing the resistance R of the circuit, *e.g.* by the use of a special cut-out.

14. Comparison with the Charging and Discharging of a Condenser

The charging and discharging of a condenser through a *non-inductive* resistance is merely *stated* here for comparison with Arts. 12, 13.

When the condenser of Fig. 400 is *being charged* the *P.D. on it starts at zero and rises to its maximum value* E (the E.M.F. of the cell) and *in this sense* it resembles the current growth in Art. 12: on the other hand the *charging current starts at its maximum value* (*say* I_m) *and falls to zero* when the condenser is fully charged, and in this sense resembles the current decay in Art. 13. On *discharging* the condenser, *both the P.D. on it and the discharge current start at maximum value and fall to zero* (this discharge case was dealt with on page 437). It can be shown that with the usual notation (V_t and I_t for P.D. on condenser and current at any instant t seconds from " start ") the results listed in column (2) below are obtained: the subject is dealt with fully in Chapter XVIII.

COMPARISON

CIRCUIT WITH RESISTANCE R AND INDUCTANCE L	CIRCUIT WITH RESISTANCE R AND CAPACITANCE C
Growth:—$I_t = I_m\left(1 - e^{-\frac{R}{L}t}\right)$	Charge:—$V_t = E\left(1 - e^{-\frac{1}{CR}t}\right)$ $I_t = I_m e^{-\frac{1}{CR}t}$
Decay:—$I_t = I_m e^{-\frac{R}{L}t}$	Discharge:—$V_t = Ee^{-\frac{1}{CR}t}$ $I_t = I_m e^{-\frac{1}{CR}t}$
Time Constant $= \frac{L}{R}$ seconds	Time Constant = CR seconds

Note that in this section we have assumed the circuit does *not* possess inductance (L)—only resistance and capacitance. Note also that in Arts. 12, 13 the circuit did *not* possess capacitance (C)—only resistance and inductance. The case of practical circuits possessing all three properties *resistance, inductance*, and *capacitance* is referred to later (pages 530-4).

15. Work in Establishing a Current: Energy in Magnetic Field

Consider again a circuit of self-inductance L, and imagine a current started in the circuit, the current growing from zero to its maximum value I_m in a time, say t seconds. Let i denote current at any instant. Now work must be done against the induced opposing E.M.F. (e), and this work (page 306) is given by: Work done = E.M.F. × Current × Time: in this case then of a growing current:—

$$\text{Work} = \int_0^t eidt = \int_0^t L\frac{di}{dt}idt;$$

$$\therefore \text{ Work} = \int_0^{I_m} Lidi = \tfrac{1}{2}LI_m^2 \quad \ldots\ldots\ldots\ldots\ldots (1)$$

and this work must be stored as energy in the magnetic field.

Take now a *long thin* solenoid of length l and cross-section A carrying a current. With the usual notation:—

$$H = \frac{4\pi SI}{l}; \quad \therefore I = \frac{Hl}{4\pi S}; \quad B = \mu H = \frac{4\pi SI\mu}{l}.$$

$$\text{Flux per turn} = BA = 4\pi SI\mu A/l; \quad \therefore \text{Linkages} = 4\pi S^2 I\mu A/l;$$

$$\therefore \text{(putting I = unity)} \ L = \frac{4\pi S^2\mu A}{l}.$$

$$\text{Energy stored in field} = \tfrac{1}{2}LI^2 = \frac{1}{2}\cdot\frac{4\pi S^2\mu A}{l}\left(\frac{Hl}{4\pi S}\right)^2 = \frac{\mu H^2}{8\pi}Al;$$

$$\therefore \text{Energy in field} = \frac{\mu H^2}{8\pi} = \frac{BH}{8\pi} = \frac{B^2}{8\pi\mu} \text{ ergs per c.cm.} \ldots (2)$$

since Al = volume of field in this case: for air $\mu = 1$ and B = H.

We can obtain two further expressions for the work in (1) above:—

$$\text{Work} = \tfrac{1}{2}LI^2 = \tfrac{1}{2}(N/I)I^2 = \tfrac{1}{2}NI,$$

$$\text{and Work} = \tfrac{1}{2}LI^2 = \tfrac{1}{2}L(N/L)^2 = \tfrac{1}{2}N^2/L.$$

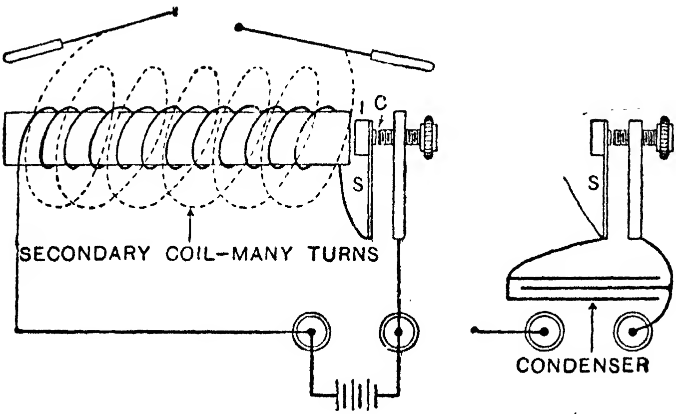
These should be compared with the expressions for the energy of a condenser and of a charged body, viz. Energy = $\tfrac{1}{2}CV^2 = \tfrac{1}{2}QV = Q^2/2C$. There is a real analogy between the two: one measures the electromagnetic energy in the magnetic field set up in the medium surrounding a circuit carrying a current, the other measures the electrostatic energy in the electric field in the dielectric of a condenser, or in the medium surrounding a charged body.

16. The Induction Coil

This is a practical application of the principles of mutual induction: it is an apparatus for transforming a low P.D. between the terminals of a primary coil into a high P.D. between the terminals

of a secondary coil. The two essential points are (1) A given current in the primary coil shall make the linkages—flux × number of turns—in the secondary coil as large as possible. (2) The current in the primary coil must be made and broken very rapidly. The construction is shown in Fig. 449 (*a*).

Round an inner core of a bundle of soft-iron wire is wound a coil of a few turns of stout wire (the *primary*): this is in series with a break consisting of a spring touching a contact screw (both tipped with platinum at the contact), the spring carrying at its upper end a small piece of soft iron, which is attracted by the iron core when current flows through the primary, and falls back again when the primary current ceases and the iron becomes demagnetised.



(*a*) FIG. 449. (*b*)

Hence, on attaching the battery, an intermittent current passes through the primary, the frequency depending upon the strength and inertia of the spring. Surrounding the primary, but insulated from it with great care, is a coil of a large number of turns of very fine wire (the *secondary*). The ends of this wire are attached to terminals on the top of the coil, and usually when the coil is working the secondary circuit is complete, except for a gap, across which it is often desired to cause sparks to pass.

Up to this stage in the construction the action is as follows. When the primary current passes (1) *an induced inverse E.M.F. is developed in the secondary*, and (2) the core is magnetised, the soft iron head and spring are attracted and the primary circuit is broken; when the primary current ceases (1) *an induced direct*

E.M.F. is developed in the secondary, and (2) the core is demagnetised, the spring falls back against the screw, the primary current again starts and the actions are repeated. It follows from this that so long as the primary current is made and broken so long will E.M.F.'s alternating in opposite directions be produced in the secondary.

Of course, if the secondary coil is part of a closed circuit an oscillatory current flows: if the ends of the coil are separated by a *small air gap* an oscillatory spark discharge occurs. It will be seen presently that in a practical induction coil the induced secondary E.M.F. at " break " is much greater than at " make," so that if a *long air gap* were in the secondary, the induced E.M.F. at " make " would not be sufficient to break down the insulation of the air, but that at " break " might be able to do so: in this case the discharge spark would be intermittent and unidirectional.

In practice matters are not so simple as indicated above, owing to the self-inductance (L) of the primary itself. When the primary is " made " the resistance is small and the time constant L/R is therefore great; when the primary is " broken " the resistance is great (due to the air gap at the interrupter) and the time constant relatively smaller. Hence the decay of the primary current is quicker than the growth, and therefore the inductive effect on the secondary at " break " is more pronounced than that at " make."

On the other hand, when the primary is broken the " extra current " developed in it in the *same* direction as the primary current just cut off makes the break less sudden and definite than it would otherwise be (thereby reducing the inductive effect on the secondary), and, sparking across the interrupter, damages the surfaces of contact. To prevent these effects a condenser is connected across the break (Fig. 449 (*b*)). The action of this condenser may be briefly explained thus. When the primary current is broken the self-induced current in the primary coil, having to charge the condenser, is not able to spark across the break, and thus the current is very suddenly broken. Again, the charged condenser discharges round the primary, but in the opposite direction to that of the primary current just cut off, and this tends to produce an induced E.M.F. in the secondary in the *same* direction as that due to the actual break: in fact, in practice the flux in the core is practically *reversed* at each break, and the induced quantity in the secondary is almost twice that without a condenser. (To be exact, the condenser current is really oscillatory, but these oscillations are quickly damped, dying out before the circuit is closed again, so that only the first discharge is important.)

Summarising, then, we may say that the secondary induced E.M.F. at make is comparatively small, while that at break is intensified, so that in an ordinary induction coil with a condenser the secondary induced E.M.F.'s are practically those due to the breaks of the primary. Fig. 450 shows approximately the variations in the primary current and the induced E.M.F.'s in the secondary.

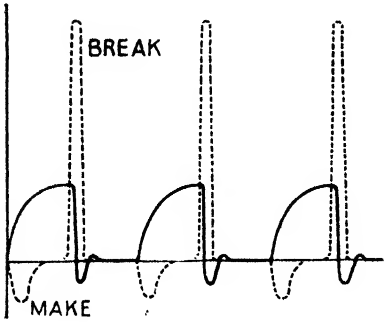


FIG. 450.

For a full mathematical treatment of the action of the condenser see *Advanced Textbook of Electricity and Magnetism*.

The induction coil converts a comparatively large current at a low voltage (in the primary) into a small current at a very high voltage (in the secondary). The secondary has a large number of turns and the "linkages" is large, whilst the "time of change" is small, so that the induced E.M.F. is great although only a comparatively small voltage even from a battery is applied to the primary. With some modern large induction coils sparks a metre in length can be obtained at the secondary terminals which means an induced E.M.F. of some half-million volts. It must be remembered, however, that we cannot *gain* energy by the transformation: in fact the *power* output is less than the power input, so that any secondary current will be much less than the primary current.

If a solid iron cylinder were used as the core of the coil the changing primary current would induce currents in it (known as **eddy currents**). These would circulate round the axis of the core in the same way as the currents induced in the secondary, and their fields would oppose the change of flux due to the changing primary: moreover they would cause loss of energy as heat in the core. To prevent these defects the core is built up of parallel iron wires each covered with shellac or some form of insulating varnish. The iron is thus continuous in the direction of the flux but discontinuous (and high resistance) in the direction of any eddy currents.

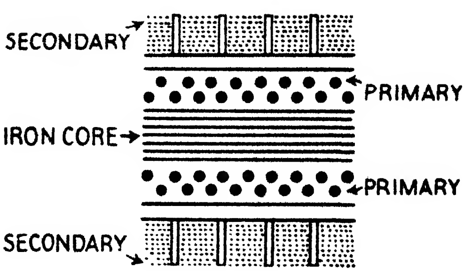


FIG. 451.

To reduce the risk of the insulation of the secondary breaking down

under the strain of the large E.M.F.'s induced in it, the coil is wound in sections which are separated from each other by discs of insulating material (Fig. 451).

In modern coils the vibrating hammer as a "make" and "break" is frequently replaced by other devices giving much more rapid interruptions. A form much used is the **motor mercury interrupter** diagrammatically shown in Fig. 452. In this a jet of mercury is discharged from a tube and caused to strike a toothed wheel which is rotated by a small motor at a high speed. The circuit of the primary coil is completed through the jet. Hence when the jet strikes a tooth the primary is made, but when the jet misses the tooth the primary is broken, and the rate of this depends, obviously, upon the speed of rotation of the toothed wheel, which is under complete control. Also the teeth are tapered, and thus by either moving the jet or the wheel up or down relatively to each other the proportion of time during which the circuit is made and broken is under control too. This device gives excellent results. The mercury is pumped up by the action of the motor which rotates the wheel.

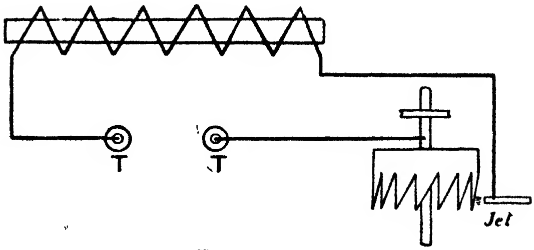


FIG. 452.

Interrupters based on electrolytic action are also employed.

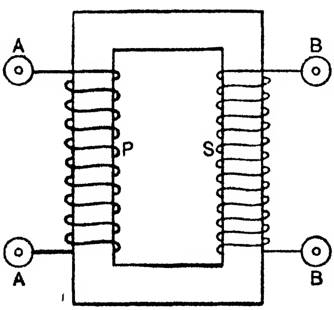


FIG. 453.

17. A Few Elementary Facts about Transformers

A **transformer** is another device in which the inductive action of a primary coil on a secondary is employed in practice: in this section only a few *general ideas* about its construction and action will be considered.

The transformer consists of two coils, a primary AA (Fig. 453) and a secondary BB wound upon a laminated iron core usually in the form of a "closed" iron circuit. **It is used with alternating currents,**

so that as the primary current is constantly varying—rising, falling, reversing—inductive effects on the secondary are always occurring. The alternating current in the primary produces a varying magnetisation of the iron core and varying magnetic flux in the secondary, which results in an induced alternating E.M.F. in the secondary of the same frequency as the primary E.M.F.

An alternating current may be represented, as we have seen (page 22), by a curve such as the firm line curve in Fig. 454. At points such as O, Q, S, and V the primary current is *changing most rapidly*: it has been flowing in one direction and is just being reversed and commencing to flow in the opposite direction. Hence the magnetic flux through the secondary is changing most rapidly at these points, and therefore the induced E.M.F. in the secondary is very great. At P, R and T the primary current is for a moment *constant* at its maximum value, the *rate of change* is for a moment zero, and the secondary induced E.M.F. is zero: the dotted curve represents the secondary E.M.F. Further details on this are given in Chapter XVIII.

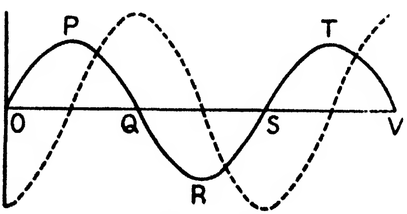


FIG. 454.

Note that the two cycles depicted in Fig. 454 are of the same frequency, but they are not "in step" with each other—there is, as we say, a "phase difference" between them (page 507). The curves are not drawn to scale.

The induced E.M.F. in the secondary depends, of course, on the number of turns of wire in it, and in fact it can be shown that we practically have the relation:—

$$\frac{\text{Secondary terminal voltage}}{\text{Primary applied voltage}} = \frac{\text{Number of turns in secondary}}{\text{Number of turns in primary}};$$

but this statement for the "ratio of transformation" assumes that there are no losses of energy in the transformer (see below). Further, since we cannot gain energy by the transformation, it follows that if the secondary voltage is *greater* than the primary, the secondary current is *less*: the transformer is, however, a very efficient instrument, so that we can say *approximately*:—

$$\frac{\text{Current in secondary}}{\text{Current in primary}} = \frac{\text{Number of turns in primary}}{\text{Number of turns in secondary}}.$$

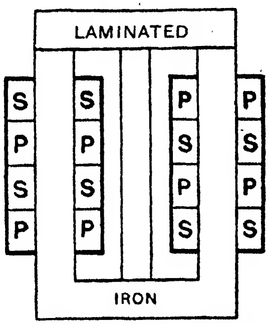
The method of measuring alternating E.M.F.'s and currents is explained on page 511.

Transformers are used commercially in two ways, viz.:—

(1) *Step-up Transformers.*—In these the alternating current in the primary induces alternating current in the secondary at a higher voltage, i.e. *they raise the voltage, and for this the secondary has more turns than the primary.*

(2) *Step-down Transformers.*—In these the alternating current in the primary induces alternating current in the secondary at a less voltage, i.e. *they lower the voltage, and for this the secondary has fewer turns than the primary.*

Some of the losses which must be taken into account in an exact treatment of transformer action are (1) Losses in both coils due to the heating effect of the currents in them. (2) Losses due to *magnetic leakage, i.e.* due to the fact that all the magnetic flux produced by the primary does not pass through the secondary: these losses are kept down by suitably arranging the coils, *e.g.* by interlacing them as shown in Fig. 455, and by making the iron core in the form of a closed magnetic circuit, *i.e.* the core is "endless." (3) Losses due to *hysteresis* in the iron core, which, of course, is constantly going through complete cycles of magnetisation (pages 109,): to reduce these special attention must be paid to the quality of the iron (or iron alloy) used. (4) Losses due to *eddy currents* in the core: these are cut down by using laminated cores, *i.e.* cores built up of separate sheets of iron which are continuous in the direction of the magnetic lines and do not oppose them, but discontinuous in the direction of the eddy currents and therefore oppose them.



P=PRIMARY S=SECONDARY

FIG. 455.

As transformers become hot in action proper means of ventilation must be provided. Some are equipped with an air blast to keep them cool. Further, large transformers are generally immersed in tanks of oil which gives more efficient cooling and improves insulation, and as a further aid to cooling, water is sometimes circulated through pipes placed in the top of the tank thus cooling the heated oil as it rises from the transformer.

Modern transformers in practice have, however, despite the above losses, a high efficiency, so that they can be used on A.C. circuits for changing the voltage with little loss of energy. Many large transformers have efficiencies as high as 98 and 99 per cent., *i.e.* the watts output is very nearly equal to the watts input. Fig. 456 shows a typical transformer for an ordinary (or, as it is called, *single phase*) A.C. circuit.

Brief reference may be made to one or two uses of the transformer in practice. Taking first a simple D.C. case, suppose a power station has to deliver 10,000 watts to a suburb 4 miles away. It could do it by transmitting 100 amperes at 100 volts (watts = EI), or it could transmit 2 amperes at 5000 volts. In the first case the current in the cables between the station and the suburb is 50 times what it is in the second case, and from this point of view the power lost as heat in them would be 50^2, *i.e.* 2500 times as great. To reduce this I^2R loss thick cable would have to be used to lessen R and safely carry the big current. Cable is *very expensive*, so that from the station economy point of view sending at high pressure (and therefore small current and thinner cable) is much the better plan—in practice it is essential—and A.C. generators in conjunction with transformers are generally employed for this high pressure transmission. One method, used in a Midland station, generates A.C. at 6600 volts, raises this at the station to 33,000 volts by a step-up transformer, transmits at this pressure (through cables known as **feeders**) to a sub-station at the district to be supplied, and there transforms down to 230 volts by a step-down transformer: from the sub-station comparatively short cables (known as **distributors**) deliver the power at 230 volts over the district, and from these **service mains** lead to individual consumers' premises. A general outline scheme of the principle on which a fairly large area would be arranged is indicated in Fig. 457.

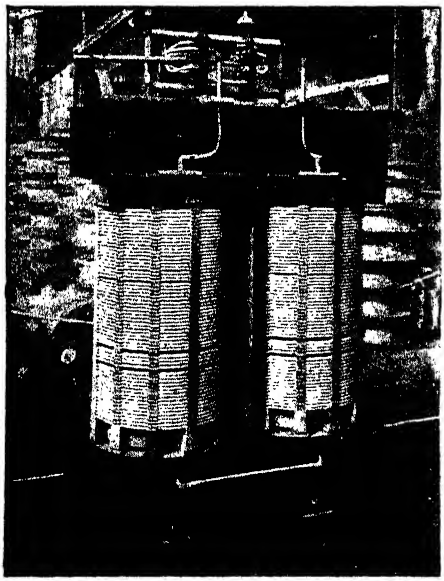


FIG. 456. Single Phase Transformer (removed from tank). 750 k.V.A; 50 cycles; 50,000/6360 volts (Metropolitan Vickers).

Over the *main* transmission lines of the "Grid" in this country the voltage is 132,000, whilst secondary transmission lines are at 66,000 and 33,000 volts.

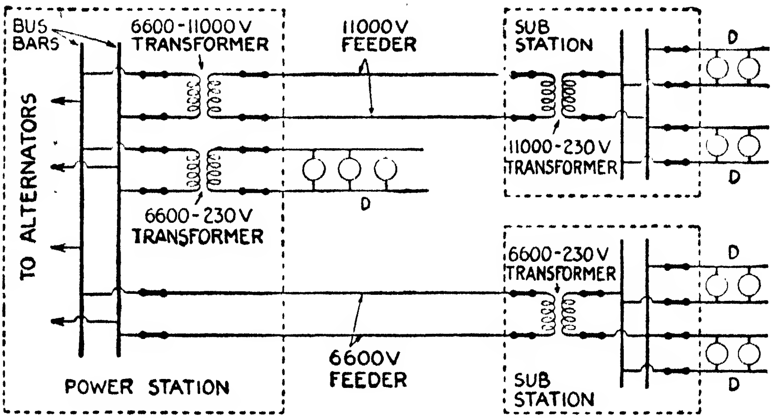


FIG. 457. "Bus-bars" are two conductors which are really the main terminals of the station. The top portion shows diagrammatically the arrangement for a distant district, the bottom portion that for a less remote district, and the middle portion that for a district near the station.

Again transformers are often used with measuring instruments on high voltage systems (they are known as instrument transformers). Thus suppose we wish to measure the voltage (of the order, say, 33,000 volts) between the mains of Fig. 458. Instead of putting a voltmeter straight across this big P.D., a step-down transformer is used. If the transformer steps-down 300 : 1, then a voltmeter with a maximum reading of only 110 volts can be used across the secondary, and the full voltage on the primary can be calculated from the instrument reading and the known ratio of transformation. Generally, however, the scale of the voltmeter is graduated to read the full voltage on the primary direct. Similarly, if a very large current is to be measured a step-up transformer is often used. The big current goes through the primary (which often consists of only one turn) and the ammeter is joined across the secondary. Remember a step-up transformer "steps down" the *current*, so that a smaller current flows in the instrument (Fig. 458).

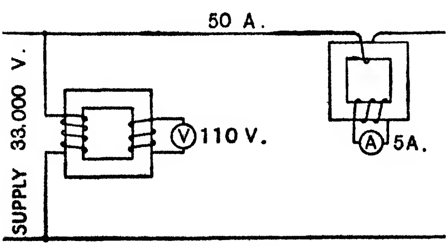


FIG. 458.

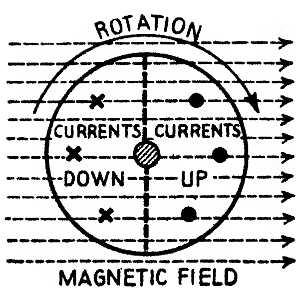


FIG. 459.

Transformers are also used in telephone circuits, in radio transmission and reception circuits, and in other appliances.

18. Foucault or Eddy Currents

In 1895 Foucault discovered that when metal objects moved in magnetic fields there was a dissipation of energy as heat due to the setting up in them of induced currents: these were referred to as **Foucault** or **eddy currents**.

(*a*) If a magnetic needle be caused to oscillate and a sheet of copper be then placed beneath it, the needle quickly comes to rest; eddy currents are developed in the copper which oppose the motion. For the same reason if a block of metal hanging between the poles of an electromagnet be started swinging, and then the current be switched on, the swinging soon stops. The damping in galvanometers " by induced currents " is explained in the same way.

(*b*) If a disc of copper be made to rotate in a horizontal plane immediately below a delicately balanced magnetic needle, the needle is gradually deflected in the same direction, and, if the rate of rotation is sufficiently high, the needle rotates in the same direction as the disc: in this case the eddy currents reduce the *relative motion* and the needle follows the disc.

(*c*) Fig. 459 represents the end view of a copper cylinder with its axis at right angles to the magnetic field (*i.e.* the axis is at right angles to the plane of the paper). If rotated about its axis as indicated (clockwise) eddy currents will flow in the cylinder as shown (apply Fleming's right-hand rule), *i.e.* parallel to the axis of rotation. To reduce these and the resulting energy loss the cylinder should be built up of flat circular discs insulated from each other (*i.e.* it must be *laminated*).

CHAPTER XVII

INDUCED MAGNETISM. THE MAGNETIC CIRCUIT

BEFORE proceeding with this chapter it is important that clear ideas should be obtained on magnetising force (H), magnetic induction (B), and total flux (N); tubes of force and tubes of induction; permeability (μ) and susceptibility (κ); and hysteresis. These points have been dealt with in preceding pages, more particularly in pages 35-38, 47-48, 85-95, and 101-111.

1. Energy Dissipation due to Hysteresis

In Chapter IV., Art. 5, it is stated that the loss of energy (in ergs) per unit volume of, say, iron, for a cycle of magnetisation is represented by the area of the IH hysteresis loop, and also by the area of the BH hysteresis loop divided by 4π; these facts can now be established.

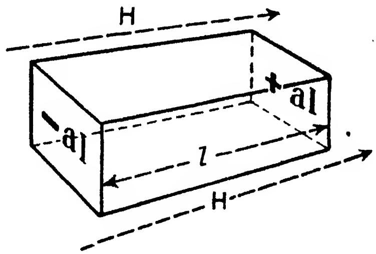


FIG. 460.

Suppose a rod of iron, length l and cross-section a, is placed along a field of intensity H (Fig. 460): it is magnetised to an intensity of magnetisation I, a + or north pole ($+aI$) being at one end, and a — or south pole ($-aI$) at the other. If H be increased to $H + \delta H$ the intensity increases to $I + \delta I$ and the pole strengths to $+a(I + \delta I)$ and $-a(I + \delta I)$.

Now the above operation is really equivalent to (1) increasing H to $H + \delta H$ without changing the intensity I, and then (2) moving an amount of pole $a\delta I$ from one end to the other. Step (1) involves no work, but step (2) does, this latter being given by:—

Work done = force on pole × distance = (pole × field) × distance;

$$\therefore \text{ Work} = a.\delta I \times (H + \delta H) \times l = (H.\delta I \times a.l) \text{ ergs,}$$

$$\textit{i.e. } \text{Work} = H.\delta I \text{ ergs per c.cm. (since } a \times l = \text{volume);}$$

so that when the intensity changes from I_1 to I_2 we have:—

$$\text{Work done} = \int_{I_1}^{I_2} H.dI \text{ ergs per c.cm.}$$

We might look at this in an even simpler way. Consider a *cube* 1 cm. side in the field H, the intensity being I: its poles are therefore + I and − I. Let the intensity change to I + dI owing to a *very small* change in the field. This is equivalent to a pole dI being carried from one face to the opposite face through a distance 1 cm. The force may be taken as H.dI, and the work done (force × distance) for the 1 cm. path, or the *change of energy for the unit volume* is given by:—

Work per unit volume = H.dI ergs.

For a more detailed proof of this relation H.dI, however, see *Advanced Textbook of Electricity and Magnetism.*

Now let OA (Fig. 461) be the magnetisation curve (IH) of a specimen of iron, and consider a *small* step in the process represented by PQ. The step we will suppose to be so short that the magnetising force may be assumed almost constant during the step and of magnitude Pp or Qq which for a very small step are almost equal. Let this be denoted by H. Let the intensity Op be I and let Oq be I + dI, so that pq represents dI. Since the work done *on* the material per unit volume in magnetising is HdI, this work for this short step is represented by the shaded area PQqp. Clearly then *the work done on the material per unit volume* for the whole magnetisation represented by the path OA will be the sum of all the small areas such as PQqp for all the *short* steps into which the magnetisation may be supposed to be divided, *i.e.* it will be represented by the area OPQAaO. Similarly if the field be reduced to zero (in which case the curve AC is obtained) the work *restored*, i.e. *the work done by the material per unit volume* will be represented by the area AaCA. Hence:—

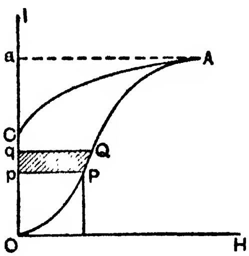


FIG. 461.

Excess of work done *on* unit vol. over work done *by* unit vol. } = area OPQACO.

Consider now the complete hysteresis loop shown in Fig. 462. From what has been said, the horizontally shaded areas (aABxa + bCDyb) represent work done *on* the material per unit volume, and the vertically shaded areas (BxbB + DyaD) represent work done *by* the material per unit volume, and, as appears from the figure, *the excess of the work done* on *the material per unit volume over that done* by *it is represented by the area of the* (IH) *hysteresis loop* DABCD, *i.e.* the net work done on unit volume is represented by the area of the IH loop.

If in the hysteresis diagram the ordinates represent B and not I, then, since $B = 4\pi I + H$, it is evident that the length of each ordinate *intercepted by the loop* will be 4π times its length on a diagram where the ordinates represent I. Hence, if we suppose the loop to be divided into an infinite number of vertical strips, the *area* of each strip in the B,H diagram will be 4π times the area of the same strip in the I,H diagram. But we have shown that the area of the loop on the I,H diagram represents the energy (ergs) dissipated as heat in unit volume of the material; therefore on a B,H diagram the area of the loop divided by 4π will represent this energy.

The latter is also seen thus:—From $B = 4\pi I + H$ we have:—

$$HdB = 4\pi HdI + HdH;$$

$$\therefore \int_o HdB = 4\pi \int_o HdI + \int_o HdH,$$

where $\int_o$ is the integral round the whole path. The term $\int_o HdH$ is zero, for if H be plotted against H the result is a straight line, for which the enclosed area is zero; hence the BH area $\int_o HdB$ is 4π times the IH area $\int_o HdI$.

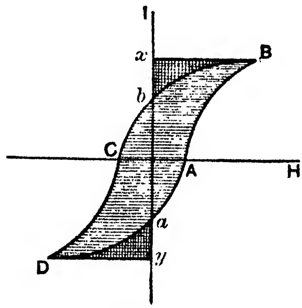


FIG. 462.

2. Ballistic Method of Determining Magnetic Properties

In the magnetometer method (page 101) the specimen was in the form of a wire, and for accurate work a correction had to be made (owing to the demagnetising effect of the end poles) in order to determine the *effective* or *true* magnetising force in the specimen (page 105). In the first method which follows the specimen is in the form of a ring, in which case, as there are no free poles, the effective magnetising force in the specimen may be taken as identical with that calculated from the dimensions of the solenoid and the strength of the current. The magnetising solenoid is *closely wound* over the ring (Fig. 463) and is connected to a key, preferably a reversing key S, a battery F, an adjustable resistance R, and an ammeter or galvanometer A. An additional coil D of a few turns is wound over a part of the ring and connected to a ballistic galvanometer BG, and a standard inductor: a standard solenoidal inductor is best, but for simplicity in writing down the mathematics we will imagine an earth inductor (EC).

The earth coil or inductor is first placed with its plane horizontal, and rapidly rotated through 180° about a horizontal axis in the magnetic meridian, and the throw θ of BG is noted. If V be the vertical component of the earth's field, n the number of turns and a the face area of EC, and R the resistance (say absolute) of the circuit made up of D, BG, and EC, the change in linkages is $2Vna$, and the quantity induced is $2Vna/R$ (absolute): but the quantity is also given by $k\theta$, where k is a constant for the galvanometer;

$$\therefore\ k\theta = \frac{2Vna}{R}; \quad \therefore\ k = \frac{2Vna}{R\theta}.$$

A small current i_1 (absolute) is now started in the magnetising solenoid; this produces a flux density b_1, say, in the specimen, and a momentary induced quantity circulates through BG, producing a throw θ_1. If a_1 be the cross-sectional area of the ring the total flow of induction or the flux is b_1a_1, and if there are n_1 turns in the coil D the change in the linkages is $b_1a_1n_1$, and the quantity induced

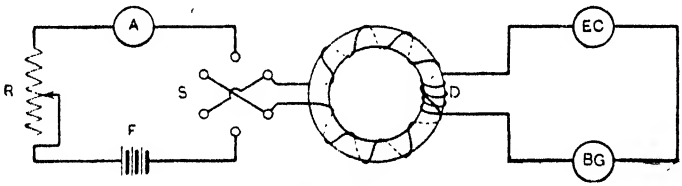


FIG. 463.

is $b_1a_1n_1/R$; but the quantity is also $k\theta_1$, hence

$$\frac{b_1a_1n_1}{R} = k\theta_1; \quad \therefore\ b_1 = \frac{kR\theta_1}{a_1n_1} = \frac{2Vna\theta_1}{n_1a_1\theta}.$$

The current is now increased by a *further* step i_2 producing an *additional* flux density b_2; if θ_2 be the galvanometer throw—

$$b_2 = \frac{2Vna\theta_2}{n_1a_1\theta}.$$

This step by step process is repeated. If B be the final flux density and i the *final total current* (absolute):—

$$B = b_1 + b_2 + b_3 + \quad = \Sigma b \text{ and } H = 4\pi Si/l,$$

where S is the number of turns in the magnetising solenoid and l the mean circumference of the ring. Knowing H and B for each step (*e.g.* B at the end of the first step is b_1, at the end of the second $(b_1 + b_2)$, and so on), the corresponding values of μ, I, and κ can

be obtained from the relationship previously established (Chapter III.), and the magnetisation curves can be plotted (Chapter IV.).

Since merely starting a current does not always immediately produce the full magnetisation due to that current a much better method is to start a current in the solenoid and then reverse it by means of the key S: the ballistic throw obtained is then proportional to $2b$ instead of to b as above. This, of course, does not give the cycle of magnetisation dealt with on page 109 but only the curve passing through the tips of the cycles for various magnetising fields. To obtain the hysteresis cycle slight modification of the apparatus and method is necessary, but this need not be dealt with in this book.

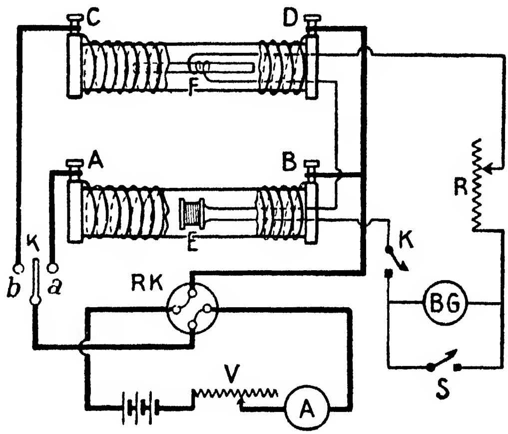


FIG. 464.

The ballistic method can, of course, be used for rod specimens, but in this case corrections must be applied to the calculated values of H owing to the end poles: the rod merely takes the place of the ring in Fig. 463. Fig. 464 shows a practical arrangement using a solenoidal inductor (AB and E) for calibration instead of an earth inductor. The switch K is turned to a for calibration and to b for test. The diagram is self-explanatory if the student traces out the current circuits. Sometimes the rod specimen has its magnetic circuit completed by being fixed in a massive yoke of soft iron (Fig. 465) so that the demagnetising effect is almost entirely eliminated. (In this case, however, a correction has to be made for the "ampere-turns" required to send the flux through the yoke—Art. 6.)

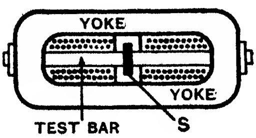


FIG. 465.

3. Commercial Tests

The importance of tests on the magnetic properties of iron, etc., in electrical engineering has led to the introduction of several commercial instruments and methods for the purpose, but space will permit only a brief reference to two here.

(1) **Ewing's Hysteresis Tester.**—This is shown diagrammatically in Fig. 466. M is a permanent magnet pivoted at Y so that it can move in a plane

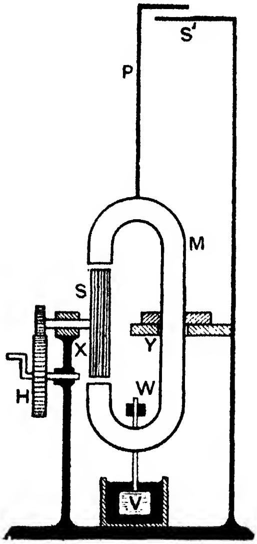


FIG. 466.

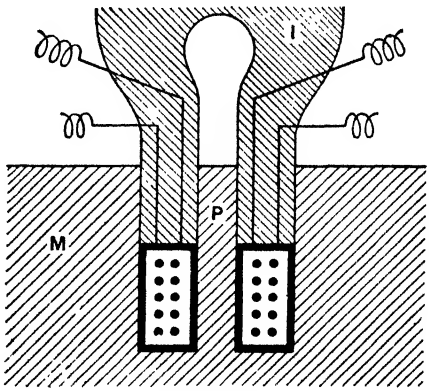


FIG. 467.

at right angles to the plane of the paper; it carries a pointer P which moves over the scale S′, a control weight W, and a vane V which, moving in oil, renders the instrument dead beat. The specimen S is pivoted at X and can be rotated between the poles of M by means of the hand wheel H. As the specimen rotates it is magnetised by the field of M, the lag in magnetisation causing it to drag the magnet M after it, thereby producing a certain deflection of P over the scale S′; the greater the lag the greater the deflection, so that the latter is proportional to the hysteresis loss in the specimen. The instrument is calibrated by means of a specimen of known hysteresis supplied with it.

(2) **Drysdale's Permeameter.**—This enables tests to be carried out on castings and forgings in bulk so that the material has not to be made into rings or bars. By a special drill a hole is bored in the material M leaving a central projection P (Fig. 467). An iron plug I is fitted into this hole. It carries a bobbin which fits on P and the bobbin carries a magnetising coil and a secondary coil, the latter being joined to a ballistic galvanometer. The experiment can be carried out as in the preceding ballistic methods.

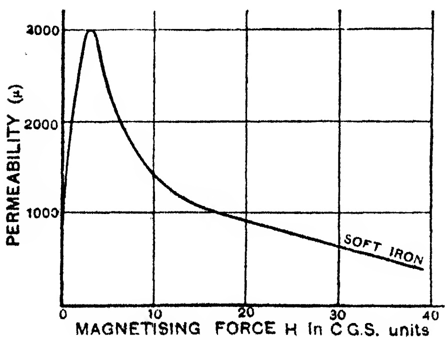


FIG. 468.

There are several others—Ewing's Double Bar and Yoke, Ewing's Permeability Bridge, the Bismuth Spiral, etc.

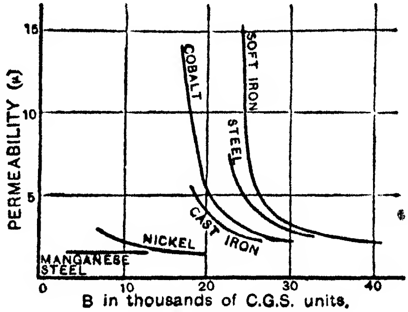


FIG. 469.

4. "Variations" in Connexion with Magnetisation

(1) VARIATION OF PERMEABILITY WITH B AND H.—It is instructive to calculate μ at various points on a BH curve and then to plot a curve showing how permeability varies with the magnetising force (H) or the flux density (B). It will be found that with comparatively small magnetising forces μ rises rapidly in the case of soft iron to a maximum and then falls (Fig. 468—see also Fig. 120). With strong magnetising forces (Fig. 469) μ for all the magnetic bodies falls with the flux: for manganese steel μ is almost constant (note the level line).

(2) VARIATION OF PERMEABILITY WITH TEMPERATURE.—The variation of magnetisation with temperature has been mentioned (Chapter III.). So far as μ is concerned, Fig. 470 indicates the result. With a small magnetising force (H) μ increases with the temperature at first slowly, then suddenly, and finally drops suddenly to unity at about 800° C., *i.e.* at the *critical temperature* (page 45) for the specimen. Under strong magnetising forces μ steadily falls as the temperature rises, and then drops rather suddenly at about 800° C. as before.

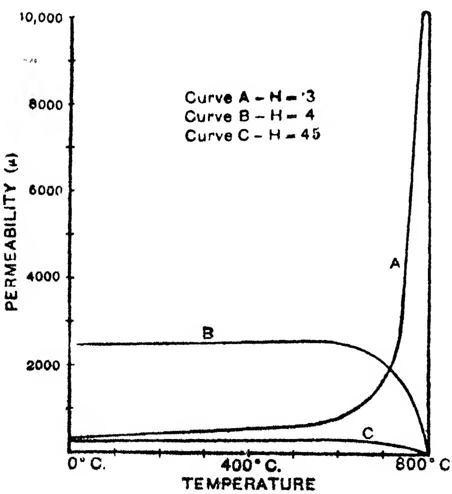


FIG. 470.

(3) VARIATION IN DIMENSIONS ON MAGNETISATION.—Bidwell showed that a bar of iron subjected to a gradually increasing

magnetising force (1) increased in length, reaching its maximum elongation with a magnetising force of from 60 to 120, (2) decreased in length, reaching its normal value with a magnetising force of from 200 to 400, (3) became actually shorter than originally with still higher magnetising forces, and (4) ceased to contract with a magnetising force of from 1000 to 1100. In the case of cobalt no change takes place until the magnetising force reaches about 50, and then the rod (1) decreases, reaching its minimum length with a magnetising force of about 400, (2) increases, reaching its normal value with a magnetising force of about 750, and (3) becomes actually longer than originally with still higher magnetising forces. Nickel decreases in length throughout, the decrease being much more pronounced than in iron. Fig. 471 represents these results graphically, and Fig. 472 indicates the apparatus used.

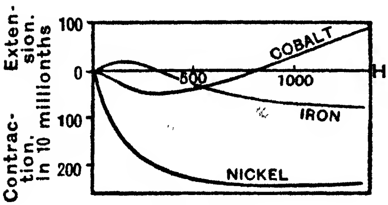


FIG. 471.

5. Diamagnetism

Before proceeding with this section the student should again read Art. 10 of Chapter II.

Weber's magnetised elements or "molecular" magnets afforded some explanation of *ferromagnetism* and *paramagnetism*, but it did not account for *diamagnetism*. Weber, however, was the first to attempt to frame a theory of the latter, and Maxwell gives the theory somewhat as follows: Imagine that the "molecular" particle is also a *perfect* (note this) conductor of electricity. When placed in the magnetic field due to (say) an inducing magnet, induced currents are formed in each molecule, and these tend to *oppose* the magnetic flux which causes them, *i.e.* the inducing field (Lenz's law). These molecular currents, in fact,

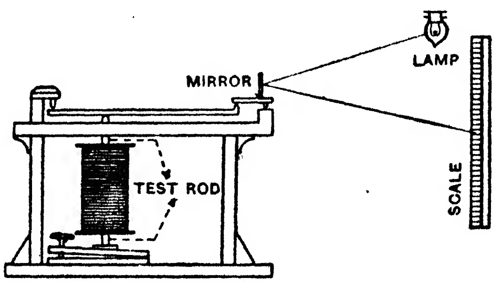


FIG. 472.

act like small magnets whose poles are, therefore, turned *towards* the *like* pole of the inducing magnet, thus *lessening the induction*, and since they flow in a *perfect* conductor they will continue to do so until they are wiped out by equal and opposite induced currents due to the destruction of the field. These induced currents thus provide the necessary explanation of diamagnetism. Up to this point, then, a body is paramagnetic or diamagnetic according as to whether the effects of the orientation of the molecular magnets into the direction of the field which increases the induction are greater or less than the opposing effects due to the induced molecular currents.

We have seen that Ampère postulated currents flowing round *perfectly* conducting circuits in the " molecules," and, further, that in a magnetic field these circuits set themselves so that, for example, their " clockwise " directions or " south faces " were towards the north pole of the inducing magnet. At the same time these perfectly conducting circuits provide the necessary paths for the opposition induced currents mentioned above which would have their " north faces " towards the inducing north pole. A body is, therefore, paramagnetic or diamagnetic according as to whether the effect of the original or the induced currents is the greater.

Now modern theory, as already emphasised (pages 52-55), brings the explanation of all magnetism down to atoms and their spinning and revolving electrons. A moving electron is, of course, equivalent to a current: an electron moving, for example, round a circle in a clockwise direction would be equivalent to a counter-clockwise (conventional) current and one moving counter-clockwise would be equivalent to a clockwise current.

The movements of the various electrons in an atom may therefore be such that they help each other, so that the atom has a resultant magnetic moment—a residual field. When a material with such atoms is placed in a magnetic field the latter will tend to orientate the atoms into the direction of the field so as to increase the induction, just as with the Weber elements, and we have the phenomenon associated with *paramagnetism*. At the same time the field has its inductive effect on the moving electrons, and this is an *opposite effect* tending to weaken the above, *i.e.* tending to produce in each atom a magnetic field which opposes the applied field—in other words, tending to produce the *diamagnetic* property. In the case we are now considering, however—the case where the atoms have a residual field—the orientation and alignment with the field in which the material is placed does more to reinforce it than the inductive effect does to neutralise it: the inductive effect

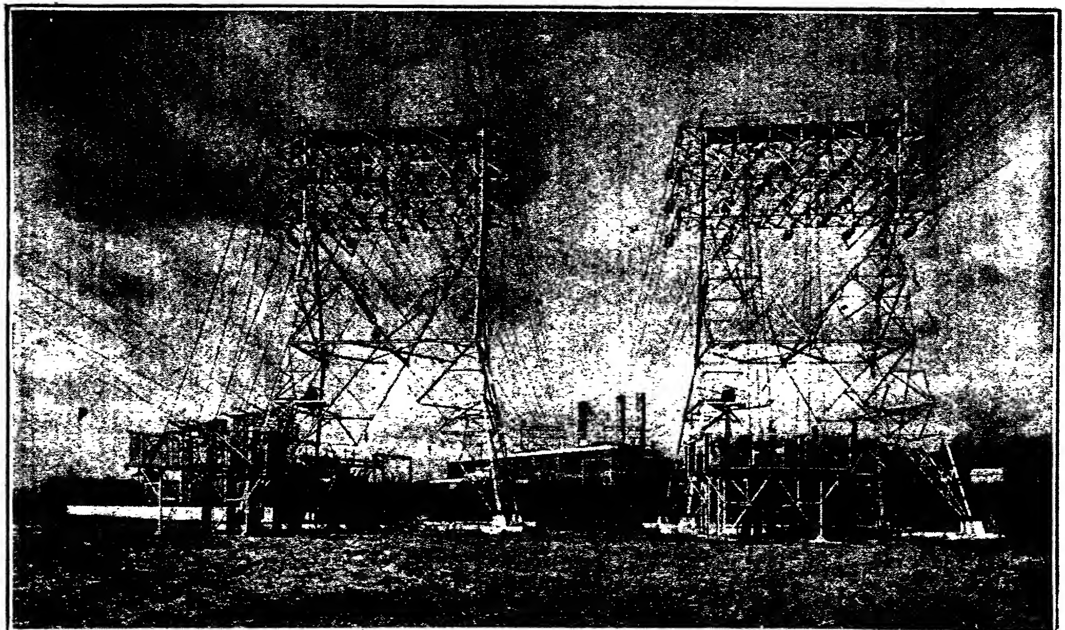
is weak in comparison and is masked, and the material is paramagnetic (or ferromagnetic—page 55).

In some atoms, however, the directions of the movements of the electrons are such that they cancel each other's effect so that the atom has no resultant magnetic moment—no residual field. When a material with such atoms is placed in a field the latter has no tendency to produce alignment with itself, only the second or inductive effect is present, *i.e.* the field produces in each atom a magnetic field which opposes it, and the material shows ***diamagnetic*** properties. In fact, those electrons in the atom whose rotations or spins are such that they already set up fields which *oppose* the applied field are *accelerated* by the inductive forces brought into play by the applied field, whilst those whose rotations and spins cause fields which *assist* the applied field are *retarded*: the result of these changes in motion—these alterations in the "orbits"—is to produce in each atom a magnetic field which *opposes the applied field*, *i.e.* it produces in the material the phenomenon of diamagnetism. By considering mathematically the accelerations and retardations referred to above and the resulting changes in orbit dimensions and "frequencies" an expression can be obtained for the *susceptibility* (κ) of the material: this quantity comes out to be *negative*, which is the case with a *diamagnetic* body. (For this mathematical treatment in full see *Advanced Textbook of Electricity and Magnetism*.)

Zeeman applied a magnetic field to the atoms of a substance emitting light and measured the changes in the wave-lengths and frequencies of the characteristic lines in the resultant spectrum. In some of his experiments on gases, when the light was examined in the direction of the field, what was originally a single line became replaced by two lines, one indicating a frequency slightly above, and the other a frequency slightly below, the original (in other experiments a line in the original position also appeared). This (known as the **Zeeman Effect**) not only indicated the magnetic properties of *single atoms*, but it also indicated the accelerations and retardations referred to above, and the consequent changes in the "orbits" and frequencies by the field.

6. The Magnetic Circuit

The present-day treatment of the magnetic circuit arises out of the fact that *magnetic induction* or *magnetic flux* proceeds in a closed path just as does the electric current. Thus the flux of a bar magnet comes out of the N. pole, passes through the air field, and enters the S. pole, completing its circuit through the magnet from S. to N. A *tube of induction*, as already explained, is never discontinuous, but will, if traced far enough, double back to form a closed tube.



The National Electric Power Schemes—"The Grid."
Multiple circuit towers at the power station at Dunstan. (North-East England "Scheme.")

Consider the case of an electromagnet. The **magnetic flux** (N) produced needs an agent to produce it, just as current needs E.M.F., and the name given to this agent is **magnetomotive force** (M.M.F.). The amount of flux produced by a certain magnetomotive force depends upon a property of the magnetic circuit somewhat analogous to resistance in the electric circuit, and the name given to this property is **reluctance** (**R**)—sometimes referred to as *magnetic resistance.* The law used for magnetic circuits, analogous to Ohm's Law, is due originally to Rowland, and may be stated thus—

$$\text{Magnetic flux} = \frac{\text{Magnetomotive force}}{\text{Reluctance}},$$

$$i.e.\ \mathrm{N} = \frac{\text{M.M.F.}}{\mathbf{R}} \text{ corresponding to } \mathrm{I} = \frac{\text{E.M.F.}}{\mathrm{R}}.$$

Now we saw (pages 286-290) that the electromotive force in an electric circuit is represented numerically by the work done (ergs) in moving unit quantity of electricity once round the complete electric circuit. Similarly, *the magnetomotive force in any magnetic circuit is defined as numerically equal to the work done (ergs) on unit magnetic pole moved once round the magnetic circuit.* Consider the simplest case of a long closely wound solenoid bent round so that its ends meet to form a ring. If a current passes the magnetic lines or the magnetic flux produced will be entirely in the ring. The field intensity inside is $4\pi SI/l$, where I = current in absolute units, S = total number of turns, and l = mean circumference in cm., and this is the force on unit pole inside in dynes. If a unit pole be moved once round this mean circumference thus looping through all the coils once, the work done will be $(4\pi SI/l) \times l$, *i.e.* $4\pi SI$ ergs, and this work is a measure of the *magnetomotive force.*

$$\text{Magnetomotive force} = 4\pi SI \text{ units} = 4\pi SI/10 \text{ units} \quad \ldots.(1)$$

the current in the first case being absolute units and in the second case amperes. Further, since $4\pi/10 = 1{\cdot}257$ and SI = total number of turns on the coil × current in amperes = total *ampere-turns,* we can write:—

$$\text{Magnetomotive force (M.M.F.)} = 1{\cdot}257 \times \textit{total} \text{ ampere-turns} \quad ..(2)$$

The unit of magnetomotive force is called the **gilbert** (formerly the *weber*), but the name is not often used.

Distinguish between the *magnetic force* and the *magnetomotive force* of the above solenoid: the former $= 4\pi SI/10l = 1{\cdot}257 \times$ ampere turns *per cm.* which measures the *force on unit pole* (in dynes), and the field intensity in

gauss: the magnetomotive force $= 4\pi SI/10 = 1{\cdot}257 \times$ *total* ampere turns, which measures the ***work in moving unit pole*** once round the magnetic circuit (in ergs) and the M.M.F. in gilberts.

Note that it was proved in Chapter XI. that the work done in moving unit pole once round a current circuit $= 4\pi \times$ current linked $= 4\pi SI$ for current I in S turns.

Imagine now our closely wound solenoid to be closely wrapped on, say, a closed iron ring (Fig. 473), and that again a current I absolute units is passed. As above, the intensity H of the field inside is $4\pi SI/l$, and the magnetic induction or flux density B $(= \mu H) = 4\pi\mu SI/l$, where $\mu =$ the permeability of the iron. If $a =$ cross-section of the iron the *total magnetic induction* or the *total magnetic flux* (N) across any section of the ring $=$ Ba: hence:—

$$N = Ba = \frac{4\pi\mu a SI}{l} = \frac{4\pi SI}{\dfrac{l}{\mu a}} \quad \ldots\ldots\ldots\ldots\ldots (3)$$

The unit of magnetic flux is called the **maxwell**, but the name is not often used: it is represented by one *unit tube of induction per square centimetre.*

Now $4\pi SI$ denotes the M.M.F. (in gilberts), for it is $4\pi SI/l$ multiplied by l, *i.e.* it measures the work done in ergs in moving unit pole round the magnetic circuit, and N denotes the total magnetic flux (in maxwells) through the circuit. Comparing this with Ohm's law for a current circuit (viz. I = E.M.F./R) we can say that the flux (N) corresponds to current, the magnetomotive force ($4\pi SI$) to electromotive force, and the quantity $l/\mu a$ to resistance. The quantity $l/\mu a$ is, in fact, referred to as the **reluctance** (or magnetic resistance) **R** of the magnetic circuit, and may be defined *in a general way* as the opposition offered by the substance to the passage of magnetic flux: the unit of reluctance is called the **oersted**, but here again the name is not often used. Comparing the expression $\mathbf{R} = l/\mu a$ with the expression for the resistance of a current conductor, viz. $R = \rho l/a$ (where $\rho =$ *resistivity* or specific resistance), and assuming the analogy to resistance correct, we might say that $1/\mu$ corresponds to ρ, and call it the *reluctivity* or the magnetic resistance offered to the passage of magnetic flux between two opposite faces of a centimetre-cube of the substance; and just as $1/\rho$ measures the specific electrical conductivity, so we might say that μ measures the specific magnetic conductivity (but see below).

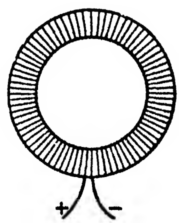


FIG. 473.

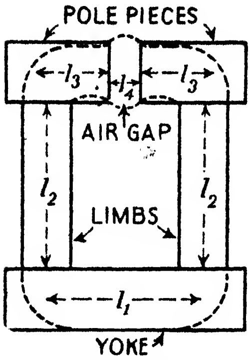


FIG. 474.

The resemblance between the magnetic circuit and the electrical circuit is, however, mainly in the form of the expressions for, of course, there is no such thing as a magnetic *current* in the sense that there is an electric current or *continuous drift of electrons* in a wire. Further, the name *reluctance* should be used in preference to magnetic resistance for $l/\mu a$ is *really* not the true analogue of electrical resistance. Thus, as stated above, μ should correspond to specific conductivity: but μ really corresponds to dielectric constant, and *the better analogue of a piece of magnetised material is a polarised dielectric.* Again, whilst the resistance of a conductor at a given temperature is independent of the current strength, the reluctance in the case of the magnetic circuit varies considerably with the induction or flux. Further, the electric circuit involves the expenditure of energy *as long as the current lasts*; in the magnetic circuit energy is only required to establish the flux.

Just as the electric circuit may be made up of a number of parts of various resistances, so the magnetic circuit may be made up of a number of parts having different cross-sections and permeabilities and therefore different reluctances. Consider the core of an electromagnet (Fig. 474) in which the magnetic circuit will be practically that indicated by the dotted lines. Let l_1 be the effective length and a_1 the cross-section of the yoke, and let it be made of iron of permeability μ_1: then $l_1/\mu_1 a_1$ is the reluctance of the yoke. Similarly $l_2/\mu_2 a_2$ is the reluctance of *each* limb, $l_3/\mu_3 a_3$ that of *each* pole-piece, and l_4/a_4 that of the air gap (μ for air $= 1$). Hence for the total reluctance ($\mathbf{R}$) of the magnetic circuit we have:—

$$\mathbf{R} = \frac{l_1}{\mu_1 a_1} + \frac{2l_2}{\mu_2 a_2} + \frac{2l_3}{\mu_3 a_3} + \frac{l_4}{a_4} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Summarising, then, our formulae for magnetic circuit calculations and expressing the magnetising current in amperes we have:—

$$\text{Flux} = \frac{\text{M.M.F.}}{\mathbf{R}},\ i.e.\ \text{N} = \frac{4\pi \text{SI}}{10\,\dfrac{l}{\mu a}} = 1{\cdot}257\,\frac{\text{SI}}{\dfrac{l}{\mu a}};$$

$$\therefore\ \text{Ampere-turns} = \text{SI} = \frac{10}{4\pi}\cdot\frac{\text{N}l}{\mu a} = {\cdot}8\frac{\text{N}l}{\mu a}$$

$$\text{And generally } \mathbf{R} = \left(\frac{l_1}{\mu_1 a_1} + \frac{l_2}{\mu_2 a_2} + \ldots\right).$$

An example or two will make the method of application clear.

Example.—*Find the number of ampere turns needed to send a flux of 40,000 through a closed soft iron ring formed of square iron of 2 cm. side bent into a circle of outside diameter 22 cm. Find also the ampere turns for the same flux if the ring be cut into two halves, the two halves being separated by two air gaps each of 1 mm.*

Case 1. $a = 4$ sq. cm.: $l = \pi \times$ mean diameter $= 20\pi$ cm.

$$B = \text{Total flux}/a = 40{,}000/4 = 10{,}000.$$

Referring to permeability tables or curves for soft iron, when $B = 10{,}000$ we find $\mu = 1900$ nearly;

$$\therefore \text{ Amp. turns} = \cdot 8\frac{Nl}{\mu a} = 0\cdot 8 \times \frac{40000 \times \pi \times 20}{1900 \times 4} = 264.$$

Case 2. $a = 4,\ l_1 = \pi \times 20,\ l_2 = \cdot 2,\ N = 40{,}000$ as before;

$$\therefore \text{ Amp. turns} = \cdot 8\left[\frac{40000\ (20\pi)}{4 \times 1900} + \frac{40000 \times \cdot 2}{4 \times 1}\right] = 1864,$$

μ for the air gap being 1.

To cause the flux in the above iron alone, 264 ampere turns only will suffice, but when we make an air gap even so small as 2 mm., then the additional ampere turns to cause the flux to pass it equal 1600, making a new total of 1864. This effect of air gaps must be carefully remembered. The following example will further illustrate this and bring in an additional point *in practice*, viz. the question of leakage of lines.

Example.—*An iron ring, 20 cm. mean circumference and 4 sq. cm. cross-section, has an air gap cut in it ·5 cm. wide. The flux for the gap is required to be 25,000. If the permeability of the iron be 100, find the ampere turns required if the leakage coefficient be 1·3.*

Owing to leakage of lines in practice, the maximum flux to be generated must be bigger than the flux necessary for the gap. In this example *the maximum flux to be generated in the iron* must be 25,000 × 1·3, *i.e.* 32,500, since the definition of the coefficient of leakage (u) is—

$$u = \frac{\text{Maximum flux generated}}{\text{Useful flux}},$$

i.e. maximum flux $= u$ (useful flux), and useful flux in practice, means the flux in the gap. The point is that in this problem we want a maximum flux of 32,500 in the iron to get a flux of 25,000 in the gap. Of course, this leakage occurs all along the iron (Fig. 475), but to simplify the problem we assume that the flux *for the whole*

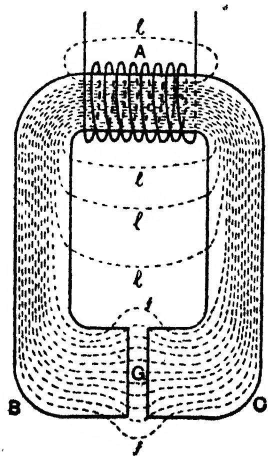


FIG. 475.

iron circuit is the maximum, viz. 32,500. Now—

(1) *Ampere turns for the iron.*

$$\text{Ampere turns} = \cdot 8\text{N}\frac{l_1}{a_1\mu_1} = \cdot 8 \times 32500 \times \frac{20}{4 \times 100} \doteq 1300.$$

(2) *Ampere turns for the air gap.*

$$\text{Ampere turns} = \cdot 8\text{N}\frac{l_2}{a_2\mu_2} = \cdot 8 \times 25000 \times \frac{\cdot 5}{4 \times 1} = 2500;$$

$$\therefore \text{ Total ampere turns} = 3800.$$

There is still another point, viz. "fringing," *i.e.* the spreading out of the lines at the air gap so that the cross-section of the gap is really more than 4 sq. cm. A method of dealing with this is to add to the pole face area a strip all round of width equal to half the length of the gap, and to take this bigger area as the effective area (a_2) of the gap. The electromagnet of Fig. 475 will make these points clear: *lll* = leakage lines, *ff* = fringing.

7. Pull between Two Magnet Faces

An expression for the theoretical lifting power of a magnet may be developed thus:—Let I be the intensity of magnetisation of the magnet. Then, assuming the magnetisation to be uniform in the magnet and, say, its attracted iron or keeper, we may take the magnetism per unit area on the opposing surfaces of the magnet and keeper as I and $-$ I, and the force of attraction of one surface on the other per unit area of surface is given by $2\pi I^2$ (page 88). Hence, if A denote the surface area of the poles, and the intensity of magnetisation at every point on the surface be the same, the lifting power of the magnet in dynes is given by $2\pi I^2 A$. Since B, the magnetic induction in the iron, is equal to $4\pi I$ (H being zero), we have $I = B/4\pi$, and the lifting power $2\pi I^2 A$ can be expressed in the form

$$\left.\begin{array}{l}\text{Lifting power or attraction}\\ \text{between pole and armature}\end{array}\right\} = \frac{B^2 A}{8\pi}\text{ dynes.}$$

Note that in Fig. 98 (page 88), the field near N due to N is $2\pi I$ and if S be there with magnetism I per unit area the force on unit area of S is (field $\times$ pole) $2\pi I \times I = 2\pi I^2$ as stated above.

CHAPTER XVIII

ALTERNATING AND VARYING CURRENTS

THE subject of alternating currents is somewhat complex, and a logical treatment demands a fair amount of mathematical manipulation. In this book we will deal only with a few essential points in as simple a way as possible. Before proceeding further read again Art. 5 of Chapter I.

The variations in strength and direction of an alternating current can be represented, as we have seen, by a curve such as in Fig. 14: the current variations follow a simple harmonic law. Note the terms *cycle, period,* and *frequency* (page 22). The alternating current will, of course, be accompanied by an alternating E.M.F. or P.D. *of the same frequency* which can be represented in the same way. But there is one point in connexion with this which may be just mentioned now: it will be explained presently:—

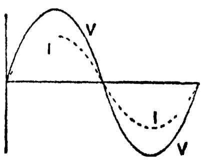


FIG. 476.

In some cases the alternating voltage and its current are *in step* with each other so that they both reach their maximum and both their zero values at the same time: this would be the case if the circuit had no inductance or capacitance—only resistance (Fig. 476). In most cases, however, *the alternating current does not keep in step with the applied alternating voltage,* so that the maximum current does not flow just at the instant that the voltage is a maximum. Thus if the circuit has inductance the maximum current tends to occur *after* the voltage has reached its maximum and the current is then said to lag. If there is capacitance in the circuit the maximum current tends to occur *before* the voltage has got up to maximum value, and the current is then said to lead.

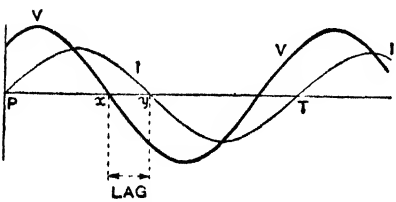


FIG. 477.

Fig. 477, for example, roughly indicates the P.D. and current curves when the circuit has inductance as well as resistance: the current (I) reaches its maximum *after* the voltage (v) has reached its maximum—it is lagging behind the voltage. The " lag " or " lead " is often expressed in terms of the period. Thus if the maximum current occurred $\frac{1}{8}$ of a period after the

maximum volts we would say that the lag was $\frac{1}{6}$ period. If the frequency was 50 we might say also that the lag was $\frac{1}{6}$ of $\frac{1}{50} = \frac{1}{300}$ second. Again, the distance PT along the horizontal line in Fig. 477 corresponding to a complete cycle represents in practice, as will be seen presently, a complete turn of 2π radians or 360°: so we can say the lag ($x.y.$) is $\frac{1}{6}$ of 360° = 60°. The "lag" and "lead" are spoken of as the phase difference between the applied volts and the current.

It is sometimes necessary to convert alternating currents into continuous currents, *e.g.* in electro-chemical work, charging accumulators, etc.: for this some form of rectifier must be employed: these are dealt with in Chapters XIX., XXI.

1. Coil Rotating in a Magnetic Field

Some important points about A.C. can be deduced if we first consider a rectangle of wire ABCD in between the poles of a magnet

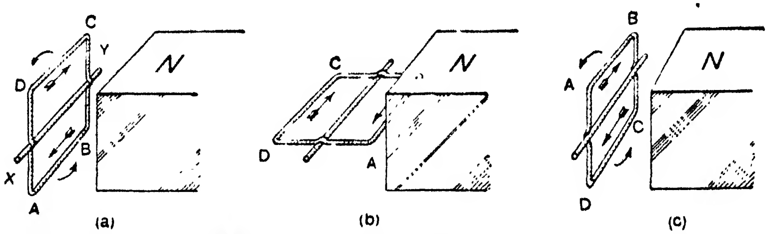


FIG. 478. In (a), the side AB has *just commenced* to rise and CD to fall. In (c), CD has *just commenced* to rise and AB to fall.

(only one pole is drawn in Fig. 478) to be capable of being rotated about the horizontal axis XY. Imagine AB (Fig. 478 (a)) begins to rise on the right and DC to fall on the left. At (a) a large number of magnetic lines from N are passing *through* the coil; at (b) after 90° rotation the coil face is lying along the lines and no lines are going *through* the face: at (c) after 180° rotation a large number are again passing *through* the coil. Thus as the coil rotates the number of magnetic lines through it is constantly changing and therefore induced E.M.F.'s will be developed in it. The coil is closed and current flows: for simplicity we will speak of the induced currents.

Take now the first 180° of rotation from (a) to (c), and consider the face of the coil which, at the start, is opposite N in (a). During this 180° of movement *this face* is turning away from N. The induced current will tend to bring it back. To do this, this face must be a south, *i.e.* the induced current must flow clockwise *at*

this face or in the direction BADC. By applying Fleming's right-hand rule we obtain the same result: thus in (*b*) the side BA is moving upwards and DC downwards, and the rule gives the induced current from B to A and from D to C.

Consider now the second 180° of rotation. DC in (*c*) begins to rise and AB begins to fall. The induced current in DC is from C to D which is opposite to what it was in DC before, and in AB it is from A to B, which is also opposite to what it was in AB before. In other words, *if we consider the same face of the coil as before*, the induced current during the second half of the rotation goes round counter-clockwise, which is opposite to the direction in which it flowed during the first half-rotation.

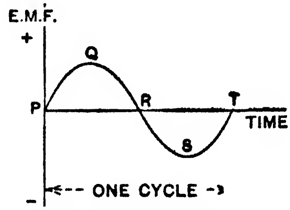


FIG. 479. P corresponds to coil vertical, Q to coil horizontal, R to coil vertical, and so on.

Thus during one-half of each complete revolution from position (*a*) the induced current flows in one direction in the coil, and during the other half it flows in the opposite direction, and the reversal of current direction takes place when the plane of the coil is at right angles to the magnetic lines (positions (*a*) and (*c*)), *i.e.* in the position where, really, the greatest number of lines are passing *through* the coil. As a matter of fact (see below), the induced E.M.F. and current are zero at (*a*), rise to a maximum at (*b*), and fall to zero at (*c*): then they reverse, rise to a maximum when the coil is horizontal, and fall to zero when the coil is again vertical, and so on (Fig. 479).

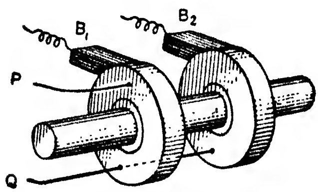


FIG. 480.

The alternating E.M.F. induced in the coil can be applied to an external circuit so that an alternating current flows in it. A gap would be made in the coil and the two free ends, say P and Q, joined to two metal rings called *slip rings* fixed to the axle but well insulated from it: a copper "brush" (carbon in practice) rests on each ring and serves to lead the current to and from the external circuit (Fig. 480).

We may determine the value of the induced E.M.F. and current at any instant in the case of a coil rotating in a magnetic field as follows:—In Fig. 481 let AB be the starting position of the coil with its face at right angles to the field. Let H = field strength,

a = face area: the flux N through the coil is the maximum, viz. $\mu a\text{H} = a\text{H}$ (since $\mu = 1$ for air), and when it has rotated through an angle α in time t sec. the flux $\text{N} = a\text{H} \cos \alpha$. The induced E.M.F. ($e$) *at this instant* is given by:—

$$e = -\frac{d\text{N}}{dt} = a\text{H} \sin \alpha \frac{d\alpha}{dt} = a\text{H}\omega \sin \alpha,$$

since $d\alpha/dt$ is the angular velocity ω. This is clearly a *maximum* when $\sin \alpha = 1$, *i.e.* when $\alpha = \pi/2$ or $3\pi/2$, and the coil therefore in the position EF, and the maximum value of the E.M.F. (E_{max}) is $\text{E}_{\text{max}} = a\text{H}\omega$. The E.M.F. has its *minimum* value (zero) when $\sin \alpha = 0$, *i.e.* when $\alpha = 0$, π, or 2π, and the coil therefore in the position AB.

If the coil has n turns the linkages in the position AB is $na\text{H}$: making this change in the mathematics gives for the above $e = na\text{H}\omega \sin \alpha$ and $\text{E}_{\text{max}} = na\text{H}\omega$.

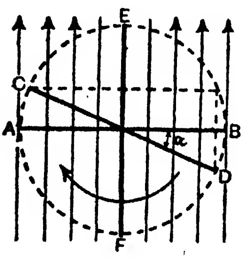


FIG. 481.

If the frequency of the alternating E.M.F. (and current) be f then the coil makes f revolutions per second since each revolution produces a cycle of E.M.F. (and current): thus $\omega = 2\pi f$, and, of course, α may be written ωt where t = time to rotate through α from the starting position AB. Hence for the **instantaneous values** of the E.M.F. (e) and current (i) we can write:—

$$e = \text{E}_{\text{max}} \sin \omega t = \text{E}_{\text{max}} \sin 2\pi f t \quad \ldots\ldots\ldots\ldots\ldots (1)$$

$$i = (\text{E}_{\text{max}} \sin \omega t)/\text{R} = \text{I}_{\text{max}} \sin \omega t = \text{I}_{\text{max}} \sin 2\pi f t \quad \ldots. (2)$$

$$\text{E}_{\text{max}} = na\text{H}\omega = 2\pi f na\text{H} : \text{I}_{\text{max}} = \text{E}_{\text{max}}/\text{R} \quad \ldots\ldots\ldots (3)$$

Average E.M.F. In the above we have dealt with the *instantaneous values* e and i of the E.M.F and current, and with the *maximum values* V_{max} and I_{max}. The average values may be found thus:—Imagine the coil to rotate through 90° from AB to EF. The linkages change from $na\text{H}$ to zero; hence the change in the linkages in a quarter turn is $na\text{H}$. The quantity of electricity set in motion is therefore $na\text{H}/\text{R}$; hence in one complete revolution the quantity circulated will be $4na\text{H}/\text{R}$, and if the coil makes f revolutions per second the quantity per second will be $4na\text{H}f/\text{R}$. Since, however, quantity is equal to average current multiplied by time in seconds, we have:—

$$\text{Average current} = 4na\text{H}f/\text{R}: \quad \text{Average E.M.F.} = 4na\text{H}f \quad \ldots. (4)$$

Now E_{max} has been proved equal to $2\pi f na\text{H}$: thus the average E.M.F. (and current) is $2/\pi$, *i.e.* ·637 of the maximum E.M.F. (and current).

2. Measuring Alternating E.M.F.'s and Currents

In Chapter XI. we saw that certain instruments could be used for alternating currents whilst others could not, and that amongst the former hot-wire instruments were suitable (p. 373). If D.C. be passed through them, the heat produced and therefore the reading of the instrument depends on the *square of the current*: if A.C., which is always changing be passed, the reading will depend upon a *mean square*. In measuring A.C., then, we use *the square root of the mean value of the square of the current*, and refer to it as the *root mean square* (R.M.S.) or *virtual value*. The following will make the idea clear.

(1) Suppose an A.C. is passed through a hot-wire ammeter and that the reading is 100. Now what exactly does this mean?—for we know that the current is going through all sorts of values from zero to a certain maximum value in both directions. Well, for A.C. work we take it that *an alternating current of one ampere is one which will produce the same heat in a resistance as a steady current of one ampere will produce in the same time*, and we call this a **virtual ampere** or **effective ampere** or "*root mean square*" (*R.M.S.*) *ampere*. If our hot-wire ammeter reads 100, it means 100 virtual or effective amperes: the actual current is sometimes less and sometimes more, but *from a heat point of view* the fluctuating current has the same effect as a steady current of 100 amperes would have. It will be shown presently that when the ammeter reads 100 the current is actually fluctuating between $100 \times \sqrt{2}$, *i.e.* 141·4 amperes first in one direction and then in the other direction. Remember:—

$$\text{Maximum amperes} = \sqrt{2} \times (\text{Virtual amperes or ammeter reading}),$$

$$\textit{i.e.}\ \text{Maximum amperes} = \text{Virtual amperes} \times 1{\cdot}414.$$

$$\text{Virtual amperes} = \frac{\text{Max. amperes}}{1{\cdot}414} = {\cdot}707 \times \text{the max. amperes.}$$

The following may help to fix the idea. Consider an A.C. which is fluctuating between a maximum value of 1 ampere in one direction and 1 ampere in the other. Its curve (*e.g.* Fig. 14) is a **sine curve**, and for this curve the following holds:—The *average value of the current* for a half-period is ·637 ampere, *i.e.* ·637 of the maximum value of 1 ampere. On the other hand, the heat at any instant depends on the square of the current at that instant, and the *average of the squares of the currents* at every instant is ·499849, the square root of which is ·707. As far as heating is concerned, this A.C. fluctuating between $\pm$ 1 ampere is equivalent to a steady current of ·707 ampere, *i.e.* Virtual amperes = ·707 × the maximum amperes.

Similar remarks apply to the measurement of an alternating E.M.F. or P.D. **A virtual volt or effective volt** *is one which when applied to the ends of a resistance results in the same heating effect as in the case of a steady P.D. of one volt applied for the same time* (*or which produces the same deflection on an electrostatic voltmeter*), and the virtual value of the voltage is ·707 of the maximum voltage, whilst maximum volts = virtual volts × 1·414.

(2) Now for a more detailed treatment:—Let a curve AOB (Fig. 482) be drawn for a half period but with ordinates representing the *square* of the current at any instant. Then the area of the strip *abcd* represents the value of I^2dt, where dt is the very short time represented by *ab* and I^2 is the square of the current strength represented by *ac* or *bd*. The area of the curve AOBA therefore represents $\Sigma I^2.dt$ for the half-period, and the value of $\Sigma I^2.dt$, divided by the time of the half-period, gives the *mean square* value for the current, and the square root of this gives the square root of the mean square value.

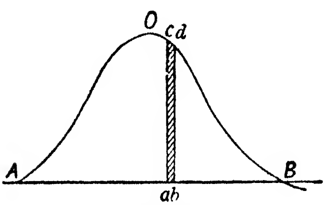


FIG. 482.

The reason for taking this value is clear. The heat developed by a current I in a time dt in a resistance R is $I^2R.dt$ or $R.I^2dt$, hence the heat developed in a resistance R by an alternating current during a half-period is $R\Sigma I^2dt$, and if **I** be the square root of the mean square current for the half-period t, then $I^2t = \Sigma I^2.dt$. The heat developed in the resistance R by the alternating current whose square root of mean square value is **I** is therefore the same as the heat developed by a steady continuous current of strength **I** in the same time. That is, *the alternating current is measured by the strength of the steady current which would produce the same heating effect.* The square root of the mean square value is the *virtual* value or *root mean square* (R.M.S.) value, and is the value given by alternating current instruments.

The mathematical student will be able to extend the preceding as follows:—Denoting the virtual current by **I**, the maximum by I_m, and the half-period by t_1, we have (since $i = I_m \sin \omega t$):—

$$I = \left(\frac{\Sigma i^2 dt}{t_1}\right)^{\frac{1}{2}} = \left(\int_0^{t_1} \frac{I^2_m \sin^2 \omega t.dt}{t_1}\right)^{\frac{1}{2}}$$

$$= \left(\frac{I^2_m}{t_1}\int_0^{t_1} \sin^2 \omega t.dt\right)^{\frac{1}{2}} = \left(\frac{I^2_m}{\omega t_1}\int_0^{\pi} \sin^2 \omega t.d\,(\omega t)\right)^{\frac{1}{2}};$$

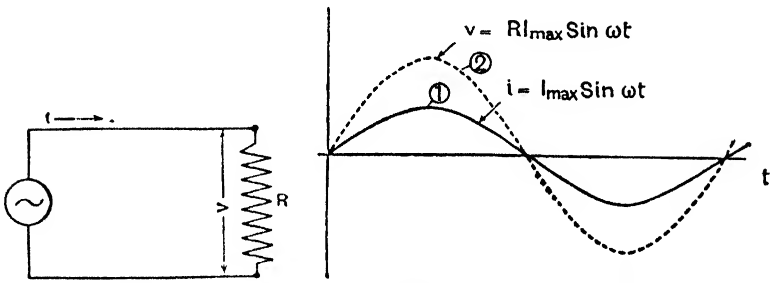
$$\therefore\ \mathrm{I} = \left(\frac{\mathrm{I}^2{}_m}{\omega t_1}\cdot\frac{\pi}{2}\right)^{\frac{1}{2}} = \frac{\mathrm{I}_m}{\sqrt{2}} \text{ since } \omega t_1 = \pi;$$

$\therefore$ I (virtual) $= \cdot 707\mathrm{I}_{max}$ and $\mathrm{I}_{max} = \sqrt{2}\mathrm{I} = 1{\cdot}414\mathrm{I}$ (virtual)

(Remember also I (average) $= \cdot 637\mathrm{I}_{max}$).

3. A.C. in a Circuit with Resistance only

In Art. 1 we dealt with the alternating E.M.F. (and current) *induced* in a coil rotating in a magnetic field, such induced E.M.F. at any instant being given by the *rate of change of flux* at that instant ($e = -d\mathrm{N}/dt$). We come now to the consideration of various circuits to which alternating P.D.'s are *applied*, say by means of an alternator, and in which therefore alternating currents are caused to flow. Now an electric circuit in general has three main properties, viz. **resistance, inductance,** and **capacitance.** In dealing with continuous currents we usually have to bother only

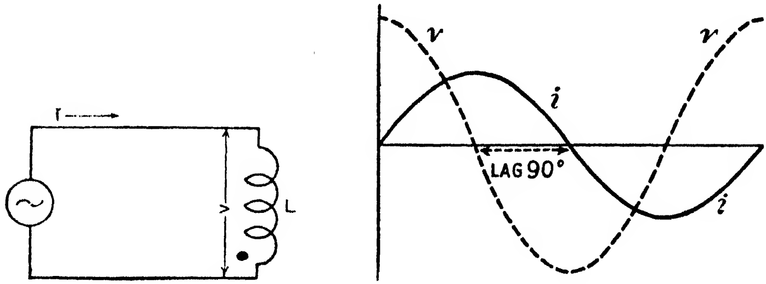


(a) FIG. 483. (b)

about the resistance, for the other two mainly have an effect at starting and stopping. With alternating currents, however, which are *always changing*, inductance and capacitance must be considered, for both are concerned with *rates of change*, the former with rate of change of current, the latter with rate of change of potential, *i.e.* of P.D. on the condenser, say.

Throughout the following investigations we shall use the letters i and v to denote *instantaneous values* of current and applied P.D., *i.e.* the values at any instant, I_{max} and V_{max} to denote maximum values, and I_v and V_v to denote virtual values, unless otherwise indicated.

Consider first a circuit having resistance only (Fig. 483(*a*)). Let us assume that A.C., of which one cycle is represented by the curve (1) of Fig. 483 (*b*), flows in it, and determine what P.D. will be required *at every instant* across the terminals of the resistance R in order to



(*a*) FIG. 484. (*b*)

maintain the current. At an instant when the current has a value $+i$ the fall of potential (v) across the resistance will be Ri. Thus v is zero when i is zero, and a maximum when i is a maximum, and as the current can be represented mathematically by $i = I_{max} \sin \omega t$ (Art. 1) the fall of potential across the resistance will be given by $v = RI_{max} \sin \omega t$ (dotted curve in Fig. 483(*b*)): the current and applied P.D. are *in phase*. Thus we have—

$$i = I_{max} \sin \omega t = I_{max} \sin 2\pi ft \quad \ldots\ldots\ldots\ldots(1)$$

$$v = RI_{max} \sin \omega t = RI_{max} \sin 2\pi ft \ldots\ldots\ldots\ldots(2)$$

$$v \text{ is a maximum when } \sin \omega t = 1;$$

$$\therefore\ V_{max} = RI_{max} \text{ and } I_{max} = V_{max}/R \ldots\ldots\ldots\ldots(3)$$

4. A.C. in a Circuit with Inductance Only

First consider an alternating current $i = I_{max} \sin \omega t$ to be flowing in an inductance L (Fig. 484(*a*)). Now we have seen that when a current in a wire which has inductance is changing, there is an " opposition " E.M.F. (e) *induced* in the wire given by $e = -L \times$ rate of change of current (the minus means it is an opposing E.M.F. when the current is increasing and a direct E.M.F. when the current is decreasing: it is usually spoken of, however, as the " back " E.M.F.). Thus for the back E.M.F. we have:—

$$e = -L\frac{di}{dt} = -L(\omega I_{max} \cos \omega t) = -\omega LI_{max} \cos \omega t.$$

Now coming to the *applied P.D.* which will be necessary to keep the current $i = I_{max} \sin \omega t$ flowing in the circuit, it is clear (since resistance is zero) that the applied P.D. (v) *at every instant* must simply be *equal and opposite in sign* to the back E.M.F. (e) *at*

that instant: thus the applied voltage must be $+ \omega LI_{max} \cos \omega t$. In this case then of *inductance only* in the circuit we have:—

$$i = I_{max} \sin \omega t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$v = \omega LI_{max} \cos \omega t \quad or \quad v = \omega LI_{max} \sin (\omega t + \pi/2) \quad \ldots (2)$$

Here, then, the voltage *leads* the current by 90° or the current *lags* behind the applied voltage by 90°: there is a " phase difference " of 90°, *the current lagging*.

It follows that v is a maximum when $\cos \omega t = 1$, and if $\cos \omega t = 1$, then $\sin \omega t = 0$, and therefore i is zero: thus i is zero when v is a maximum, which again indicates a phase difference of 90°. Putting $\cos \omega t = 1$ we get:—

$$V_{max} = \omega LI_{max} = 2\pi f LI_{max} : I_{max} = V_{max} / 2\pi f L \quad \ldots (3)$$

Clearly, if we write the applied P.D. in the form:—

$$v = V_{max} \sin \omega t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

$$\text{then} \quad i = I_{max} \sin (\omega t - \pi/2) \quad \ldots\ldots\ldots\ldots (5)$$

which again indicates *the current lagging by* 90°.

Fig. 484(*b*) shows the curves for a circuit with inductance only. Note that whilst the curves are in the correct relative positions indicating the phase relations, they are not drawn to scale: the same applies to other curves which follow.

The above and Fig. 484 may be clearer to a beginner if he considers separately (1) the rate of change of current (di/dt) and (2) the back E.M.F. (e) and their curves.

(1) Rate of Change of Current. If we choose an instant t_1 for the current curve (1) Fig. 485, and note the change in current di in a very short time dt, then the rate of change of current at the instant t_1 is di/dt, or it is the slope of the tangent to the curve at time t_1. Note that at t_1 the rate of change is positive, *i.e.* the current is increasing in the positive direction. At time t_2 (where $a = \omega t = \pi/2$) it is clear that the rate of change is zero, the tangent being

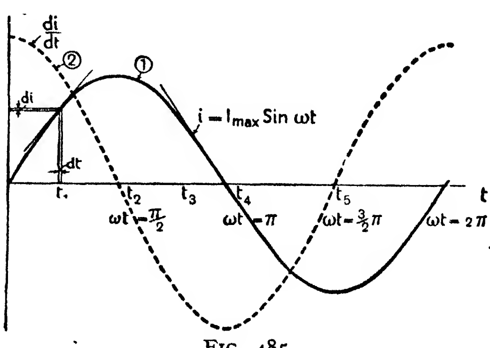


FIG. 485.

horizontal, while at t_3 the slope is negative, and the negative slope goes on increasing until t_4 ($\omega t = \pi$) is reached, when it begins to decrease again, falling to zero at t_5. If the slope is measured at a large number of points a curve of rate of change of current against time can then be plotted and will have the form of the dotted curve (2) in Fig. 485. Note that the *rate of change* is least (zero) when the current is a maximum, and greatest when the current is at zero value and reversal.

(2) Back E.M.F. Now back E.M.F. $= -L\,di/dt$, so that the back E.M.F. (e) at any instant in any case can be obtained by plotting the rate of change of current curve and multiplying its ordinates by $-$ L (note the sign $-$: the back E.M.F. at any moment has such a direction that it *opposes* the change). The back E.M.F. curve will thus be similar to the rate of change of current curve *but reversed*, and Fig. 486 shows roughly its form and position.

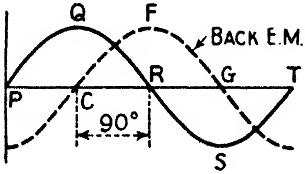


FIG. 486.

To maintain the current in the preceding problem the *applied* P.D. (v) at every instant must be *equal and opposite in sign* to the back E.M.F. (e) at the same instant. The *applied P.D.* curve is therefore positioned as shown in Fig. 484, and the *current* is lagging 90° behind this applied P.D.

Since $I_{max} = V_{max}/\omega L$ we see by comparison with the usual $I = E/R$ that ωL or $2\pi f L$ corresponds to R: it is called the *reactance* of the inductance or the **inductive reactance.**

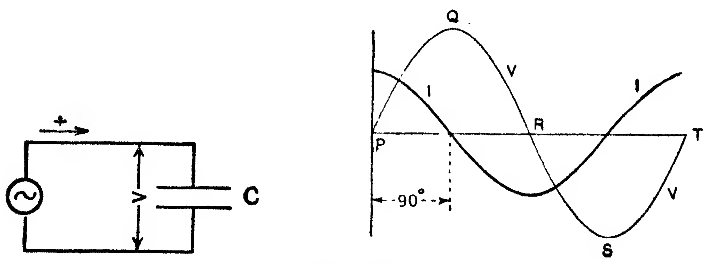


FIG. 487.

5. A.C. in a Circuit with Capacitance Only

For simplicity we will imagine the circuit to consist simply of a condenser of capacitance C, and that an alternating E.M.F. given by $v = V_{max} \sin \omega t$ is applied to it (Fig. 487). Now the P.D. between the plates at any instant must be equal to the applied E.M.F. at that instant. But for a condenser we know that P.D.

between plates = Q/C (page 205), so that if q be the charge at the instant considered we have:—

$$q/C = v = V_{max} \sin \omega t \text{ or } q = V_{max}C \sin \omega t;$$

$$\therefore\ i = \frac{dq}{dt} = V_{max}C\ (\omega \cos \omega t) = \frac{V_{max}}{1/\omega C} \sin (\omega t + \pi/2).$$

This is a maximum when $\cos \omega t = 1$ in which case:—

$$I_{max} = V_{max}\omega C = \frac{V_{max}}{1/\omega C}.$$ Summarising, then, we have, if—

$$v = V_{max} \sin \omega t \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

then $$i = I_{max} \sin (\omega t + \pi/2) \ldots\ldots\ldots\ldots\ldots (2)$$

and the current therefore *leads* the applied P.D. by 90°. This case is indicated in Fig. 487.

Further, since $I_{max} = V_{max}/(1/\omega C)$ we see by comparison with the usual relation $I = E/R$ that $1/\omega C$ or $1/2\pi fC$ corresponds to the resistance of the condenser: it is known as the condenser *reactance* or as the **capacitance reactance.**

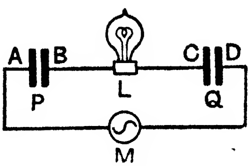


FIG. 488.

A Digression. For the beginner a point may be noted here in an elementary way about currents in circuits with condensers. If a condenser be joined to a battery it becomes charged up to the same P.D. as the battery poles but there is no *continuous* flow because the dielectric is an insulator. Now consider, for convenience, the A.C. circuit of Fig. 488. During the first half cycle of the A.C., electrons drift in the direction, say, from M to A (making A negative), an equal number move from B to C (making B positive and C negative), and an equal number from D to M (making D positive). During this half cycle, therefore, there has been a "flow" of electrons through the lamp in the direction from B to C. Clearly, during the next half cycle the flow of electrons will again occur, but in the opposite direction, viz. M to D, C to B, and A to M: during this half cycle, therefore, there has been a flow of electrons through the lamp from C to B. Thus the movement of electrons (*i.e.* the current) to and fro in the alternator part of the circuit causes a corresponding movement of electrons (*i.e.* current) to and fro in the part BC: there is an alternating current in the lamp and it lights up.

6. A.C. in a Circuit with Resistance and Inductance

We shall now consider the case of an A.C. circuit consisting of a source of E.M.F., a resistance R, and an inductance L in series (Fig. 489(a)). Assume that a current $i = I_{max} \sin \omega t$ flows in the

circuit. At any instant the total fall of potential round the circuit in the positive direction from 1 to 4 will be the sum of the falls of potential from 1 to 2 and 3 to 4. Thus the applied voltage (v) at any instant must be Ri to overcome the resistance, and $L\,di/dt$ *equal and opposite* to the back E.M.F. $-L\,di/dt$: we therefore have—

$$v = Ri + L\frac{di}{dt} \quad \dots\dots\dots\dots\dots\dots \quad (a)$$

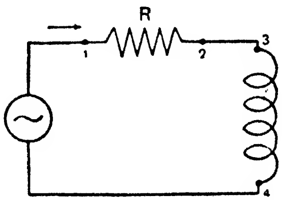
$$\text{But } i = I_{max} \sin \omega t \text{ and } di/dt = \omega I_{max} \cos \omega t;$$

$$\therefore\ v = RI_{max} \sin \omega t + \omega L I_{max} \cos \omega t.$$

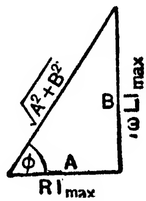
To solve this, let—

$$RI_{max} = A,\ \omega L I_{max} = B, \text{ and } B/A = \tan\theta \text{ (Fig. 489(}b\text{)).}$$

$$\text{Then } A = \sqrt{A^2 + B^2}\cos\theta \text{ and } B = \sqrt{A^2 + B^2}\sin\theta.$$



(*a*) FIG. 489. (*b*)



$$\text{But } v = A \sin\omega t + B\cos\omega t;$$

$$\therefore\ v = \sqrt{A^2 + B^2}\cos\theta \sin\omega t + \sqrt{A^2 + B^2}\sin\theta\cos\omega t,$$

$$\text{or } v = \sqrt{A^2 + B^2}(\cos\theta\sin\omega t + \sin\theta\cos\omega t).$$

And on substituting for A and B we get—

$$v = (\sqrt{R^2 + \omega^2 L^2})\, I_{max}(\cos\theta\sin\omega t + \sin\theta\cos\omega t);$$

$$\therefore\ v = I_{max}\sqrt{R^2 + \omega^2 L^2}\sin(\omega t + \theta) \quad \dots\dots\dots \quad (b)$$

Now v is a maximum when $\sin(\omega t + \theta) = 1$, and we have $V_{max} = I_{max}\sqrt{R^2 + \omega^2 L^2}$: thus (*b*) can be written—

$$v = V_{max}\sin(\omega t + \theta),$$

$$\text{and as } i = I_{max}\sin\omega t$$

it is clear that the phase of the voltage is in advance of that of the current, or the current lags behind the applied volts by an angle θ which is such that

$$\tan\theta = B/A = \omega L I_{max}/RI_{max} = \omega L/R = 2\pi f L/R. \dots. \quad (c)$$

We can also say that the lag is $\theta/2\pi$ of a complete period. Fig. 477 depicts the phase relations in this case.

It follows that if the applied P.D. (v) be written:—

$$v = V_{max} \sin \omega t = V_{max} \sin 2\pi ft \quad \ldots\ldots\ldots\ldots(1)$$

$$\text{then } i = I_{max} \sin (\omega t - \theta) = I_{max} \sin (2\pi ft - \theta) \quad \ldots\ldots(2)$$

which again indicates that the current lags behind the applied P.D. by an angle θ whose tangent is $2\pi f L/R$. Further, it is important to note specially that:—

$$I_{max} = \frac{V_{max}}{\sqrt{R^2 + \omega^2 L^2}} = \frac{V_{max}}{\sqrt{R^2 + (2\pi f L)^2}} \quad \ldots\ldots\ldots(3)$$

$$\text{tan angle of lag} = \tan \theta = \frac{\omega L}{R} = \frac{2\pi f L}{R} \quad \ldots\ldots\ldots\ldots(4)$$

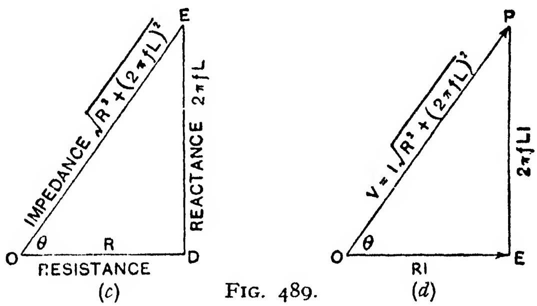


FIG. 489.

and since $I_{max} = \sqrt{2}\, I_{virtual}$ and $V_{max} = \sqrt{2}\, V_{virtual}$ it is clear that we can also apply these to virtual values or instrument readings.

The quantity ωL or $2\pi f L$ is called, as already stated, the **reactance** or **inductive reactance,** and is expressed in practice as so many ohms. The whole denominator in (3), viz. $\sqrt{R^2 + (2\pi f L)^2}$, is called the **impedance,** and is expressed in ohms: from the analogy with $I = E/R$ for direct currents we might say that impedance is the effective "resistance" encountered by an A.C. and that it involves both ohmic resistance and reactance. $2\pi f LI$ is referred to as the **reactive E.M.F. or P.D.** (just as RI is called the *resistance P.D.*) and, of course, is expressed as so many volts. Summarising, we have:—

$$\textbf{Maximum current in amperes} = \frac{\textbf{Maximum applied volts}}{\textbf{Impedance}}$$

$$\textbf{Impedance} = \sqrt{(\textbf{Resistance})^2 + (\textbf{Reactance})^2}$$

$$\text{Reactance} = 2\pi \times (\text{Frequency}) \times (\text{Self-inductance}) = 2\pi f \text{L};$$

$$\therefore \text{ Impedance} = \sqrt{\text{R}^2 + (2\pi f \text{L})^2}$$

$$\text{tan angle of lag} = \tan \theta = \frac{\text{Reactance}}{\text{Resistance}} = \frac{2\pi f \text{L}}{\text{R}}.$$

These quantities and their relations are best *remembered* by the aid of the two right-angled triangles shown in Figs. 489 (*c*), (*d*).

It will be helpful and of interest to examine Fig. 490 which gives the complete set of curves for this case, but the curves will be better understood after reading Art. 10.

Example.—*A coil of wire of negligible resistance and inductance* ·02 *henry, and a wire of zero inductance and resistance* 12 *ohms are in series. If the impressed E.M.F. as given by a voltmeter be* 130 *volts and the frequency* 40

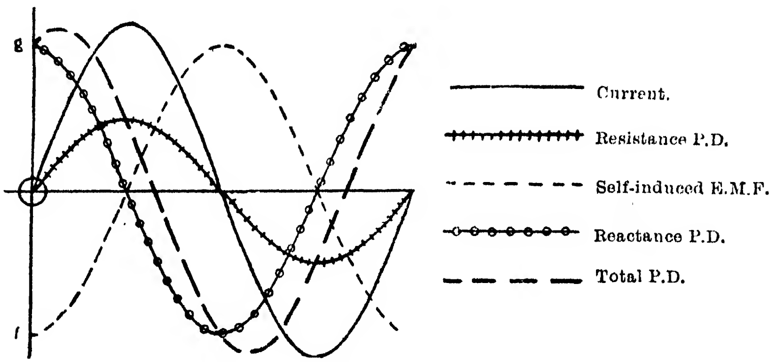


FIG. 490.

cycles per second, determine (a) *the current,* (b) *the lag,* (c) *the P.D. across the inductive resistance,* (d) *the P.D. across the non-inductive resistance.*

(*a*) For the current (using I and V for virtual values):—

$$\text{I} = \frac{\text{V}}{\sqrt{\text{R}^2 + (2\pi f \text{L})^2}} = \frac{130}{\sqrt{12^2 + (2\pi \times 40 \times \cdot 02)^2}} = 10 \text{ amperes.}$$

(*b*) The current lags behind the impressed E.M.F. by an angle θ such that:—

$$\tan \theta = \frac{2\pi f \text{L}}{\text{R}} = \frac{2\pi \times 40 \times \cdot 02}{12} = \cdot 417;$$

$$\therefore \theta = 22{\cdot}5^\circ \quad \text{and} \quad \text{Lag in time} = \frac{22{\cdot}5}{360} \times \frac{1}{40} = \frac{1}{640} \text{ second.}$$

(*c*) P.D. (E_1) on the non-inductive resistance is obtained in the usual way, viz.:—

$$\text{I} = \frac{\text{E}_1}{\text{R}}, \textit{ i.e. } 10 = \frac{\text{E}_1}{12}; \quad \therefore \text{E}_1 = 120 \text{ volts.}$$

(*d*) P.D. (E_2) for the inductive resistance is obtained from:—

$$I = \frac{E_2}{\sqrt{O + (2\pi f L)^2}}, \textit{ i.e. } 10 = \frac{E_2}{2\pi f L}; \quad \therefore E_2 = 50 \text{ volts.}$$

Note that the relation between the three pressures is represented by the triangle OPE of Fig. 489(*d*), OP representing 130, OE representing 120, and PE representing 50 ($130^2 = 120^2 + 50^2$). In the case of a *direct* current we would have, of course, $E_1 + E_2 = V$. The virtual values would have to be multiplied by 1·414 to get maximum values.

7. Vector Diagram Method of Solution for Art. 6.

The alternating P.D.'s across an inductance, a capacitance and a resistance in an A.C. circuit are all of the same frequency but are in general, as we have seen, of different magnitudes and differ in phase. These P.D.'s may therefore be regarded as vector quantities, *i.e.* quantities possessing both magnitude and direction, and may be added according to the parallelogram law of vectors. The following treatment should therefore be noted: we will work with maximum values calling them, for simplicity, I and V.

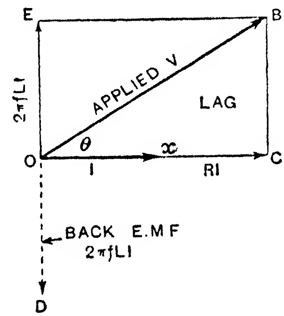


Fig. 491.

The voltage required will be RI to overcome the resistance, and there must also be a voltage *equal and opposite* to the back E.M.F., viz. $2\pi f$ LI, to overcome the back E.M.F. The current will be in step or in phase with the ohmic component RI, and, of course, the *back E.M.F.* is at 90° to this, *i.e.* 90° *behind* it. Suppose, then, we take OC (Fig. 491) to represent the voltage component RI in phase with the current O*x*. We next draw OD to represent the back E.M.F., viz. $2\pi f$ LI, the angle DOC being 90°, and then we draw OE equal and opposite to this: this represents the voltage component necessary to overcome the back E.M.F. The actual applied voltage to get the current through the circuit must evidently be the resultant of these two components OC and OE, and is given by the diagonal OB. From the figure we have:—

$$OB^2 = OC^2 + OE^2; \therefore V^2 = (RI)^2 + (2\pi f LI)^2 = I^2 (R^2 + 4\pi^2 f^2 L^2);$$

$$\therefore V = I\sqrt{R^2 + 4\pi^2 f^2 L^2} \quad \textit{and} \quad I = \frac{V}{\sqrt{R^2 + 4\pi^2 f^2 L^2}} \ldots (1)$$

The current I is in phase with RI and therefore is lagging behind the applied volts V by an angle θ. This is the lag of the current;

$$\therefore \text{ tan angle of lag} = \tan\theta = \frac{2\pi f\,\mathrm{LI}}{\mathrm{RI}} = \frac{2\pi f\,\mathrm{L}}{\mathrm{R}} \quad \ldots\ldots(2)$$

If L = o then tan θ = o and θ = o°, *i.e.* the current is in phase with the applied E.M.F., and I = V/R: this is the case in Art. 3. If R = o then tan $\theta = \infty$ and θ = 90°, *i.e.* the current lags 90°: this is so in Art. 4, and in this case I = V/$2\pi f$ L or V = $2\pi f$ LI.

8. A.C. in a Circuit with Resistance and Capacitance

The case of a circuit containing resistance and capacitance can be worked out in a similar way. From preceding pages (see Art. 5) it follows that our equation for the applied voltage v (corresponding to (a), Art. 6) is:—

$$v = \mathrm{R}i + \frac{q}{\mathrm{C}},$$

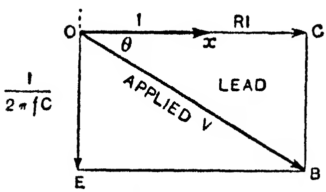


FIG. 492.

but for simplicity we will find the solution graphically by a vector diagram instead of analytically:—for simplicity we again use I and V for I_{max} and V_{max}.

Just as inductance causes *lag* of current, so capacitance causes *lead*, and it was shown in Art. 5 that $1/2\pi f$C was the *capacitance reactance* (corresponding to the resistance of the capacitance or condenser). The *E.M.F. set up by the capacitance reaction* (reactance E.M.F.) is therefore I × $1/2\pi f$C = I/$2\pi f$C. Thus Fig. 492 is the vector diagram: it is similar to Fig. 491, remembering that the capacitance effect works opposite to inductive effect. In this case the current (which is in phase with RI) *leads* the applied volts V by an angle θ, and from the figure:—

$$\text{tan angle of lead} = \tan\theta = \frac{\mathrm{I}}{2\pi f\mathrm{C}}\Big/\mathrm{RI} = \frac{1}{2\pi f\mathrm{CR}} \quad \ldots.(1)$$

$$\mathrm{V}^2 = (\mathrm{RI})^2 + \left(\frac{\mathrm{I}}{2\pi f\mathrm{C}}\right)^2; \quad \therefore \mathrm{V} = \mathrm{I}\sqrt{\mathrm{R}^2 + \left(\frac{1}{2\pi f\mathrm{C}}\right)^2}$$

$$\mathrm{I} = \frac{\mathrm{V}}{\sqrt{\mathrm{R}^2 + 1/4\pi^2 f^2\mathrm{C}^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

The reactance is, of course, $1/2\pi f$C, and the impedance is $\sqrt{\mathrm{R}^2 + (1/2\pi f\,\mathrm{C})^2}$.

If R = O, tan $\theta = \infty$, and $\theta = 90°$. Thus the current leads the voltage by 90°: this case was shown in Fig. 487. The expression (2) can of course be used for virtual values of I and V.

It is not easy for a beginner to see why capacitance causes lead. The capacitance (say a condenser) is charged up by the applied P.D., and when the latter dies down to zero (and is about to reverse) the capacitance starts a current in the opposite direction *before* the P.D. has got itself built up in the opposite direction, *i.e.* the current *leads.*

Example.—*What will be the magnitude and phase angle of the current flowing into a condenser of capacitance* 1 *microfarad when joined to A.C supply mains at* 50 *cycles.* $V_{max} = 600$.

$$I_{max} = \frac{V_{max}}{\sqrt{R^2 + 1/(2\pi f C)^2}} = \frac{V_{max}}{1/2\pi f C} \text{ since } R = 0.$$

$$V_{max} = 600 \text{ volts: } C = 1\mu F = 10^{-6} \text{ farad.}$$

Impedance (= reactance here) $= 1/2\pi f C = 1/(2\pi \times 50 \times 10^{-6}) = 3200$ ohm;

$$\therefore I_{max} = \frac{V_{max}}{\text{Impedance}} = \frac{600}{3200} = 0{\cdot}19 \text{ ampere,}$$

and this current *leads* by 90° (tan θ = reactance/resistance = ∞).

9. A.C. in Circuit with Resistance, Inductance, Capacitance

This is the most general case. Suppose an E.M.F., viz. $v = V_{max} \sin \omega t$, be applied to a circuit (Fig. 493) containing resistance R, inductance L, and capacitance C, then from the preceding we have that at any instant

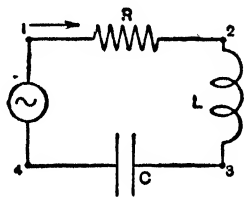


FIG. 493.

$$Ri + L\frac{di}{dt} + \frac{q}{C} = v = V_{max} \sin \omega t.$$

The solution for i in such a differential equation as this is beyond the scope of this book (see *Advanced Textbook of Electricity and Magnetism*): the solution leads to the results:—

$$\text{If } v = V_{max} \sin \omega t \quad \ldots\ldots (1)$$

$$i = \frac{V_{max}}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \sin(\omega t - \theta) = I_{max} \sin(\omega t - \theta) \quad \ldots (2)$$

$$\text{where } \tan \theta = \frac{\omega L - 1/\omega C}{R} = \frac{2\pi f L - 1/2\pi f C}{R} \quad \ldots\ldots (3)$$

$$I_{max} = \frac{V_{max}}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \quad \ldots\ldots (4)$$

From the expression for tan θ it follows that if $\omega L = 1/\omega C$, *i.e.* if $2\pi f L = 1/2\pi f C$ then tan $\theta = 0$ and $\theta = 0$: inductance effect and capacitance effect neutralise, there is neither lag nor lead and the current has the same

value as in a circuit of resistance R only. If $\omega L > 1/\omega C$ the current *lags*: if $\omega L < 1/\omega C$ the current *leads*.

A vector diagram solution (really a combination of Figs. 491, 492) will, however, serve the purpose of the student of this book: we again deal with maximum values calling them for simplicity I and V. The diagram is shown in Fig. 494, and the student should be able to reason out the figure. The P.D. for the resistance is RI, is represented by OC, and is in phase with the current. The P.D. to *overcome the back E.M.F.* of the inductance is $2\pi f LI$, and must, as already explained, be represented by OE drawn vertically upwards (90° in advance of the true current). Similarly the P.D. for the condenser is $I/2\pi f C$, and is represented by OF as in Art. 9. As these latter two are in opposite directions we take the difference OG. We have assumed that the inductive effect is greater than the capacitance effect, so that $OG = (2\pi f LI - I/2\pi f C)$. OB represents the applied volts V necessary to maintain the current in the circuit, and from the figure:—

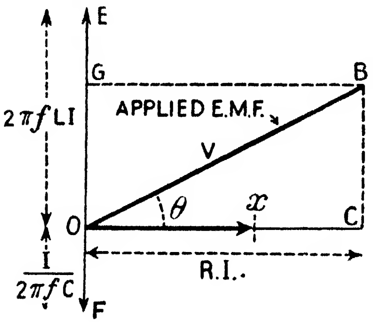


FIG. 494.

$$V^2 = OC^2 + OG^2 = I^2\left\{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2\right\};$$

$$\therefore\ I = \frac{V}{\sqrt{R^2 + (2\pi f L - 1/2\pi f C)^2}}.$$

The current is in step with RI so that in the figure the current *lags* behind the applied volts by the angle θ: this is because we assumed $2\pi f L$ to be greater than $1/2\pi f C$. If the latter is the greater of the two OG will be marked off downwards from O, and OB will slope downwards below OC and the current will lead the volts. If $2\pi f L = 1/2\pi f C$ the inductance and capacitance effects cancel, there will be neither lag nor lead as already stated—the current will be in phase with the applied volts.

When the last mentioned state of affairs occurs, *i.e.* when $2\pi f L = 1/2\pi f C$, the frequency $f = 1/2\pi\sqrt{LC}$ and the period $T = 2\pi\sqrt{LC}$. If the frequency f of the A.C. is fixed we can arrange that this occurs by altering L and C. The circuit is then said to be in resonance for a reason which will be seen later (Art. 13). The condition is of importance in wireless.

Note that, although, at resonance, the sum of the inductance and capacity voltages is zero (as they are equal in magnitude and opposite in phase) the individual voltages (*i.e.* the P.D. on the inductance and on the capacitance) may be large and may actually exceed the voltage of the A.C. supply. (It was the *difference* between the individual voltages, *i.e.* OG in Fig. 494, which was used in working out the value of OB the applied volts). See example (2) below.

The *impedance* is, of course, $\sqrt{R^2 + (2\pi f L - 1/2\pi f C)^2}$, the reactance $= (2\pi f L - 1/2\pi f C)$, and $\tan\theta =$ reactance/resistance $= (2\pi f L - 1/2\pi f C)/R$.

Examples.—(1) *A coil of resistance 100 ohms and inductance 0·16 henry is connected in series with a condenser of 16 microfarads capacitance across a 50 cycle supply,* $V_{max} = 600$. *Calculate the magnitude and phase of the current in the circuit. What capacity would be required to bring the current into phase with the supply volts, and what would the current be under those conditions?*

The vector diagram for the circuit will be as in Fig. 494.

$$R = 100, \quad \omega L = 2\pi \times 50 \times \cdot 16 = 50 \text{ ohms}$$

$$\frac{1}{\omega C} = \frac{1}{2\pi \times 50 \times 16 \times 10^{-6}} = 200 \text{ ohms};$$

$$\therefore \text{ Total impedance} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = 180 \text{ ohms};$$

$$\therefore I_{max} = \frac{V_{max}}{\text{Impedance}} = \frac{600}{180} = 3\cdot 33 \text{ amps.}$$

$$\text{Phase angle } \theta = \tan^{-1} \frac{\omega L - \frac{1}{\omega C}}{R} = \tan^{-1} - \frac{150}{100};$$

$$\therefore \theta = -56^\circ \text{ (from tables).}$$

Thus the current " leads " (θ is $-$) the voltage by 56° (note that $1/\omega C$ is *greater* numerically than ωL).

Now let C′ be the capacitance necessary to produce resonance.

$$\text{Then} \quad \omega L = \frac{1}{\omega C'}; \quad \therefore 50 = \frac{1}{2\pi 50 C'};$$

$$\therefore C' = \frac{1}{2\pi 50 \times 50} \times 10^6 = 63\cdot 5 \text{ micofarads } (\mu\text{Fd})$$

$$I'_{max} = \frac{V_{max}}{R} \text{ in this case} = \frac{600}{100} = 6 \text{ amperes,}$$

and the current is in phase with the supply volts.

(2) *A coil of resistance 20 ohms and inductive reactance 100 ohms, at the supply frequency, is connected in series with a condenser the capacitance of which has been adjusted to bring the current into phase with the supply voltage. The*

supply voltage is 140 *volts max. Calculate* (a) *the current drawn from the supply,* (b) *the voltage across the condenser, and* (c) *the voltage across the coil.*

Note that, as the inductive reactance and not the inductance of the coil is given, it is unnecessary to know the supply frequency. As the circuit is resonant the total impedance is simply the resistance of the coil, *i.e.* 20 ohms;

$$\therefore \text{ Current (Max)} = \frac{140}{20} = 7 \text{ amperes.}$$

$$\text{Voltage (Max) across condenser} = \frac{I_{max}}{\omega C}.$$

Now $1/\omega C$ is the reactance of the condenser, and as, for resonance, the capacity reactance of the circuit must be equal in magnitude to the inductive reactance, we have $1/\omega C = 100$ ohms;

$$\therefore \text{ Condenser voltage (Max)} = 7 \times 100 = 700 \text{ volts.}$$

$$\text{Coil voltage (Max)} = I\sqrt{R^2 + \omega^2L^2} = 7\sqrt{4 \times 10^2 + 10^4} = 714 \text{ volts.}$$

Note that condenser and coil voltages (at resonance) are each some *five times* as great as the total voltage of the supply.

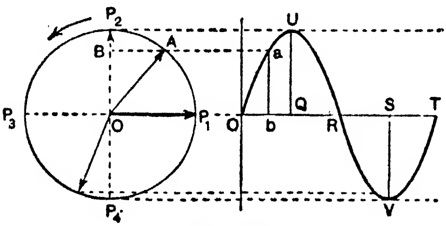


FIG. 495.

10. The Use of the Rotating Vector

The scope of vector diagrams can be extended by making use of the principle of the *rotating vector* as used in simple harmonic motion problems. Most students will understand this idea, but for the benefit of others we treat the matter as simply as possible.

Imagine a circle drawn with radius OA numerically equal to the *maximum* value of the current (I_{max}), and imagine OA to be a line (*radius vector*) which can revolve round O in a counter-clockwise direction (Fig. 495). Suppose OA is in the position OP_1 when the current is zero, and that it moves uniformly once round while the current executes one cycle. When the current is a maximum (QU) the vector will be in the position OP_2: when it is zero (R) the vector will be along OP_3: when it is a maximum in the other direction (SV) the vector will be along OP_4: when the current is again zero (T) having executed one cycle the vector will be back to its starting position OP_1, having done 360° rotation.

Again, when the vector is in any other position, say OA, the projection of OA on the vertical diameter P_2P_4 is equal to the strength of the current *at that instant, e.g.* by the time the vector has moved from OP_1 to OA the current has increased from zero at O to the value ab where ab = OB: the current value at a is $I_{max} \sin \omega t$, where t is the time from zero current, and from the figure OB = OA sin $AOP_1 = I_{max} \sin \omega t$. The same applies to all other positions of the vector.

Thus the current curve for a cycle can be drawn as follows:—Take a line OT to represent the period. Divide it into, say, 12 equal parts and erect perpendiculars at each point. With OA making angles of 30°, 60°, 90°, 120°, and so on, with OP_1 project A on to the corresponding perpendiculars. On joining these points the current curve is obtained.

Fig. 496 shows the vectors in the case of a circuit with resistance and inductance, the current lagging by an angle θ. The vector diagram indicated on the left is the same as Fig. 491. The two vectors Ox (max. current) and OB (max. volts) rotate together keeping at the angle θ apart. Their vertical projections as they rotate give the values of the current and voltage at the corresponding instants, and both curves are plotted in correct phase relation.

Similarly the resistance P.D. curve, back E.M.F. curve, reactance P.D. curve, etc., can be plotted in their proper phase relations by marking off the vertical projections of the corresponding vectors as the diagram rotates (see Fig. 490).

11. Power in an A.C. Circuit. Wattless Current

The *power* or rate of doing work in the case of a steady current is given by EI where E = E.M.F. or P.D., and I = current. In an alternating current circuit both the P.D. and current are varying, and they differ in phase. From preceding pages we have for an A.C. circuit:—

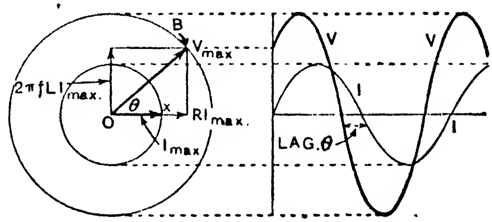


FIG. 496.

$$v = V_{max} \sin \omega t; \ i = I_{max} \sin (\omega t - \theta),$$

so that the power at any instant is given by:—

$$\text{Power} = vi = V_{max}I_{max} \sin \omega t \sin (\omega t - \theta);$$

$$\therefore \text{ Power} = \tfrac{1}{2}V_{max}I_{max} [\cos \theta - \cos (2\omega t - \theta)].$$

Now during a half-period the angle $2\omega t$ and therefore $(2\omega t - \theta)$ changes by 2π, and the mean value of its cosine for the half-period is, therefore, zero; hence the power may be taken as

$$\tfrac{1}{2}V_{max}I_{max} \cos \theta$$

for a time equal to any integral number of half-periods.

But if V_v and I_v be the *virtual* or *root-mean-square values*, *i.e.* the readings of a voltmeter and ammeter placed in the circuit, then $V_{max} = \sqrt{2}\, V_v$ and $I_{max} = \sqrt{2}\, I_v$. Thus:—

$$\text{Power} = \tfrac{1}{2}V_{max}I_{max} \cos \theta = \tfrac{1}{2} (\sqrt{2}V_v \times \sqrt{2}I_v) \cos \theta;$$

$$\therefore \text{ Power} = V_vI_v \cos \theta \ldots\ldots\ldots\ldots\ldots (1)$$

Now $V_v I_v \cos\theta$ measures the *true power* and is given by, say, a dynamometer type wattmeter in the circuit (page 350), whilst $V_v I_v$, the product of the voltmeter and ammeter readings, measures the *apparent power*: $\cos\theta$ measures what is termed the **power factor** of the circuit, so that

$$\text{Power factor} = \cos\theta = \text{True power/Apparent power} \quad \ldots(2)$$

In practice the true power is expressed in Kilowatts (K.W.) and the apparent power in *Kilovolt-amperes* (K.V.A.).

If a circuit were purely inductive the current would lag 90°, and if it were purely capacitive the current would lead 90°: in each case the true power would be zero since $\cos 90° = 0$. If the circuit, although not *purely* inductive say, is so very inductive that the current lags practically 90°, or if a circuit has such a capacitance that the current leads practically 90°, then the value of $\cos\theta$ is zero, but an ammeter will read something (I) and so will a voltmeter (V): the real power is $VI \cos\theta = 0$, and the apparent power is VI. Such a current is called a **wattless current**: it flows sometimes with the pressure and sometimes against it so that the net work done is zero assuming the resistance negligible.

A **choke coil**, as we have seen (page 463), has a big inductance and a small resistance, so that in an A.C. circuit the lag is almost 90° if the resistance is negligible, and a wattless current flows, the power $V_v I_v \cos\theta$ being almost zero. It may therefore be used for controlling an alternating current without that waste of energy on itself which would occur if an ordinary resistance were used (a condenser could also be used, but chokes are invariably employed). In the path of a continuous current a choke coil has very little effect for its resistance is very low (its inductive action only occurs at start and cut-off). In the path of an alternating current, however, its inductive action is important: its self-induced opposing E.M.F. reduces the effective pressure and can thus be used to reduce and control the A.C. without causing much energy waste.

Since the reactance of an inductance is proportional to the frequency a high frequency choke usually has an air core, whilst a low frequency choke has an iron core (note this, for example, in wireless receivers).

When the secondary of a transformer is open we practically have a wattless current in the primary.

12. Measurement of Inductance by Using Alternating Currents

When A.C. of known frequency is available several inductance measurements can be readily performed. Thus the self-inductance L of a coil may be found by first passing the A.C. through it and noting the R.M.S. or virtual amperes and volts by means of an A.C. ammeter and A.C. voltmeter.

Now $\quad V_{max}/I_{max} = \sqrt{R^2 + (2\pi f L)^2};$

$$\therefore \frac{V_{max}}{\sqrt{2}} \Big/ \frac{I_{max}}{\sqrt{2}} = \frac{V_{virtual}}{I_{virtual}} = \sqrt{R^2 + (2\pi f L)^2},$$

and thus $\sqrt{R^2 + (2\pi f L)^2}$ is determined. Disconnect, pass a steady continuous current through the coil, and measure the amperes and volts, thus determining R. Hence the reactance $2\pi f L$ is found and since f is known, L is determined.

As already indicated, several "bridge" or "null" methods of measurement are employed in which A.C. is used, the balance being indicated by employing a telephone. To avoid the use of a telephone in this type of experiment on inductance, capacitance and resistance measurement, Campbell used a **vibration galvanometer.** This is a modified moving coil instrument, the coil being very light and hung by a bifilar suspension. The tension and effective length of the suspension can be adjusted so that the frequency of vibration of the coil is the same as that of the A.C. used. Thus a *very feeble* current in the galvanometer will, owing to *resonance*, produce a big vibration of the coil (shown by a wide band of light on the scale). The instrument therefore readily and accurately shows when two points in an A.C. circuit are at the same potential and no current is flowing through it.

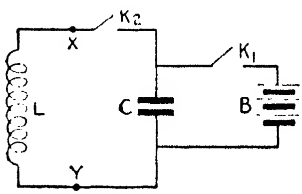


FIG. 497.

13. Natural Electrical Oscillations

It will be convenient here to consider *very briefly* natural electrical oscillations; *i.e.* alternating currents which, instead of having a frequency determined by that of the supply from an alternator (A.C. generator), determine their own frequency by the nature of the circuit. This may occur as the result of a single disturbance in the electrical state of a circuit containing inductance and capacitance.

In Fig. 497 L is an inductance and C a condenser. Imagine C to be charged by B and then K_1 opened. There is, of course, electrical energy stored in the condenser, *i.e.* in its associated electrostatic field which is permeated with electric lines. On closing K_2 a rush of current takes place discharging the condenser, and there is now a magnetic field and magnetic lines. The flow continues until the condenser is discharged, *i.e.* its two plates brought to the same potential, but the inductance of L *prolongs* the current, keeps up the flow in the same direction, with the result that the condenser becomes charged in the *opposite* direction, *i.e.* what was its positive plate is now its negative, and we have again

an electric field and electric lines. Then the process is repeated with the direction of flow reversed, and so on.

In this L.C. circuit, then, there is a high frequency electrical oscillation, and if the resistance were *nil* such an oscillation, once started, would go on indefinitely. In practice, however, there is *some* resistance, and the oscillations get weaker and die down.

Now this oscillating circuit has a natural frequency of its own just as a pendulum has. Take the instant when the maximum current I_{max} is flowing. The P.D. across L (say between X and Y) is $2\pi f LI_{max}$, and across C it is $I_{max}/2\pi f C$. But these are equal: hence:—

$$2\pi f LI_{max} = I_{max}/2\pi f C; \quad \therefore \text{ Frequency} = f = \frac{1}{2\pi\sqrt{LC}}.$$

Now going back to Art. 9—an alternating E.M.F. applied to a circuit with resistance, inductance, and capacitance—we saw that the impedance was $\sqrt{R^2 + (2\pi f L - 1/2\pi f C)^2}$, and is therefore least when $2\pi f L = 1/2\pi f C$, *i.e.* when f, *the frequency of the applied E.M.F.*, is equal to $1/2\pi\sqrt{LC}$: in this case there is neither lag nor lead, and the current will be large since the impedance consists only of the resistance.

But we have just seen that this same expression $1/2\pi\sqrt{LC}$ gives the *natural frequency* of a circuit of inductance L and capacitance C when the resistance is low. *Thus the alternating current in any circuit due to an applied E.M.F. is greatest when the frequency of the applied E.M.F. is the same as the natural frequency of the circuit.* The case is similar to that of *resonance* in sound. Such circuits in practice are known as *series resonance circuits.*

Series resonance circuits are used in " tuning in " in wireless receivers: there are variable condensers and inductances, and we alter the condensers (sometimes we also vary the inductance) until the natural frequency of the circuit $1/2\pi\sqrt{LC}$ is the same as that of the E.M.F. set up in the aerial by the radiation from any particular transmitting station.

14. Further Facts about Charging and Discharging Condensers

We return now to the charging of a condenser (by a battery) and to its discharge through various circuits.

In a *general way* the early scientists noted that when the plates of a charged condenser were connected by a large resistance, the discharge consisted of a unidirectional flow from the high potential plate to the other, the charges thus neutralising and the potentials of the plates both becoming zero (this was dealt with on page 437).

If, however, the resistance was below a certain value, the discharge consisted, not of a flow in one direction, but of a number of rapid oscillations or surgings to and fro. The first " rush " more than neutralises the opposite charge and charges the condenser in the opposite direction (due to the self-inductance continuing the flow in the same direction as the discharge current falls). This is followed by a reverse rush which again " overshoots the mark " and charges the condenser in the same way as it was originally, and so on. Each successive oscillation is weaker than the preceding; thus, after a number of such surgings the discharge is complete and the potentials of the plates equalised. At each oscillation the electrostatic energy of the condenser field is converted into electromagnetic energy accompanied by the dissipation of a small quantity of energy as heat in the conductor (and also by electric radiation as explained later). These oscillations take place *very rapidly*; the time of each complete vibration is the same, and this time is, of course, the *period*, the number of complete vibrations per second being the *frequency*.

Fedderson in 1857 verified the oscillatory character of the discharge spark of a Leyden jar across a small gap by viewing the gap in a rapidly rotating mirror. The gap, of course, is bright when current passes and dark when current ceases. The image in the mirror consisted of a band with bright and dark spaces: when a high resistance was included in the circuit the image became simply a bright band fading away in intensity.

We can now examine these points in a little more detail using various circuits to join the plates.

(1) CHARGE AND DISCHARGE THROUGH A NON-INDUCTIVE HIGH RESISTANCE.—The case of a charged condenser being *discharged* under these conditions was dealt with on page 437 (see also page 480). The discharge was continuous, the discharge current starting at maximum value and decaying exponentially. Using the notation on page 480 we had for the current at time t from " start "

$$I_t = I_m e^{-\frac{1}{CR}t},$$

and a similar expression applies to the *charging*, say by a battery of constant E.M.F. (E). The following is a brief proof of the charging.

There is a flow of charge through the resistance until the P.D. on the condenser (V) is equal (and, of course, opposite) to the E.M.F. (E) of the battery: the charging current starts at its maximum value (I_m) and falls to zero when $V = E$.

Now if Q_t and V_t be the charge and P.D. on the condenser at any instant, then $Q_t = V_tC$ and $dQ_t/dt = CdV_t/dt$. If I_t be the current at this instant $I_t = (E - V_t)/R = dQ_t/dt$: hence

$$C\frac{dV_t}{dt} = \frac{E - V_t}{R}; \quad \therefore \frac{dV_t}{E - V_t} = \frac{1}{CR}dt.$$

$$\text{But } d(E - V_t) = -dV_t; \quad \therefore \frac{d(E - V_t)}{(E - V_t)} = -\frac{1}{CR}dt.$$

Integrating this from the lower limits when t and $V_t = 0$,

$$\log_e \frac{E - V_t}{E} = -\frac{t}{CR}; \quad \therefore 1 - \frac{V_t}{E} = e^{-\frac{t}{CR}};$$

$$\therefore V_t = E\left(1 - e^{-\frac{1}{CR}t}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\text{And since } I_t = \frac{E - V_t}{R} = \frac{E}{R}.e^{-\frac{1}{CR}t}; \quad \therefore I_t = I_m e^{-\frac{1}{CR}t} \ldots\ldots\ldots (2)$$

Note that the wire joining the plates in both charge and discharge is free from inductance, and that a battery of *constant E.M.F.* is used in charging. (See again page 480.)

(2) USING A LOW RESISTANCE WIRE WITH INDUCTANCE.—We will assume the resistance so low that the dissipation of energy in it (I^2R loss) can be neglected. Consider the charged condenser to be discharging (charging battery removed). Let L = self-inductance, Q_m = maximum charge, I_m = maximum current. For simplicity in the printed text of the mathematics we will use Q and I for the charge and current *at any instant* (instead of Q_t and I_t). Now the energy in the field of a condenser is given by $Q^2/2C$, and in an electromagnetic field by $\frac{1}{2}LI^2$: hence assuming no loss by dissipation we have for the total energy (constant) at any instant when it is partly electrostatic and partly magnetic:—

$$\frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2 = \text{constant},$$

and we proceed with the solution of this as follows:—

Differentiating the above with respect to time we get:—

$$\frac{Q}{C}.\frac{dQ}{dt} + LI\frac{dI}{dt} = 0. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (a)$$

$$\text{Now } I = \frac{dQ}{dt}, \text{ and therefore } \frac{dI}{dt} = \frac{d\,(dQ/dt)}{dt};$$

$$\therefore \frac{Q}{C}.\frac{dQ}{dt} + L\frac{dQ}{dt}.\frac{d\,(dQ/dt)}{dt} = 0; \quad \therefore \frac{Q}{C} + L\frac{d\,(dQ/dt)}{dt} = 0, \ldots\ldots (b)$$

$$i.e. \quad \frac{d\left(\frac{dQ}{dt}\right)}{dt} = -\frac{1}{LC}Q.$$

This means that Q is a quantity such that if we differentiate it with respect to time to get dQ/dt and then differentiate dQ/dt with respect to time we get the original result (Q) multiplied by a constant ($-1/LC$). This at once suggests the form:—

$$Q = Q_m \sin \omega t,$$

because $dQ/dt = Q_m\omega \cos \omega t$ and $d\,(dQ/dt)/dt = -Q_m\omega^2 \sin \omega t$: this agrees with the above, the constant $1/LC$ being represented by ω^2.

Now from preceding pages the expression $Q = Q_m \sin \omega t$ indicates that Q is a periodic quantity varying harmonically between the limits Q_m and $-Q_m$, and the period of its variation is $2\pi/\omega$, for ωt changes from zero value to 2π in that time and goes through its full cycle of values within these limits. Now this period $2\pi/\omega$ is evidently the period of oscillation of the discharge, and since $\omega^2 = 1/LC$ the period of oscillation is given by $T = 2\pi\sqrt{LC}$.

In this case, then, of a condenser discharging through a circuit of negligible resistance but of inductance L, the discharge is oscillatory and the frequency is given by ($f = 1/T$):—

$$f = 1/2\pi\sqrt{LC}.$$

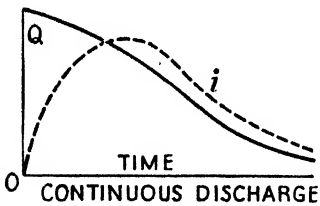


FIG. 498. R greater than $\sqrt{4L/C}$.

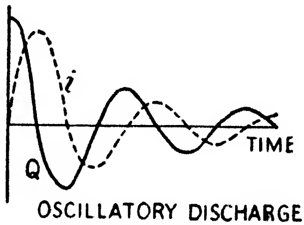


FIG. 499. R less than $\sqrt{4L/C}$.

(3) USING A WIRE WITH RESISTANCE AND INDUCTANCE.—The full solution of this case involves mathematics beyond the scope of this book. We have to include the I^2R rate of dissipation of energy to the other two rates of change of energy before equating to zero in equation (*a*). Writing (dQ/dt) for I and simplifying, equation (*b*) becomes

$$\frac{Q}{C} + L\frac{d\,(dQ/dt)}{dt} + R\frac{dQ}{dt} = 0 \quad \text{or} \quad \frac{Q}{C} + L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} = 0.$$

The mathematical student will be able to solve this (see *Advanced Textbook of Electricity and Magnetism*): we quote the results which the solution indicates:—

(*a*) If R^2 is greater than $4L/C$, *i.e.* if $R > \sqrt{4L/C}$, the discharge is *continuous*. The current starts from zero, reaches a maximum, and then steadily diminishes. The charge Q steadily diminishes. This case is shown in Fig. 498.

(*b*) If R^2 is less than $4L/C$, *i.e.* if $R < \sqrt{4L/C}$, the discharge is oscillatory. The period and frequency are

$$T = \frac{2\pi}{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}} \qquad f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}},$$

and if $R = 0$ we get $f = 1/2\pi\sqrt{LC}$ as in Case 2. Fig. 499 shows this discharge case. As a matter of interest, Fig. 500 shows the variation of the charge during the *charging* process in this case. Note that the maximum charge (first charging surge) is much greater than the final steady charge: hence it is that a condenser joined to supply mains (with small R and large L in circuit) may "break down" although its insulation can easily stand the final steady pressure: the remedy is to charge through a resistance which is afterwards cut out.

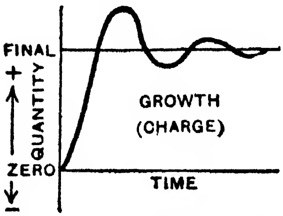


FIG. 500. Charging when R is less than $\sqrt{4L/C}$.

The student must note that care must be taken with the units in working out calculations involving the expressions in this section. Do not mix up e.s. and e.m. units in the same expression.

CHAPTER XIX

CONDUCTION OF ELECTRICITY IN GASES

WE have seen that, under normal conditions, air (and other gases) is a very poor conductor of electricity. Even a small gap in an electric circuit will often cut off the current: a well-insulated charged electroscope will retain its charge for some time: if two metal plates, one charged positively, the other negatively, be fixed parallel in air at a suitable distance apart, they will retain their charges for some time, showing that the air between does not permit electricity to pass across to neutralise the charges and bring the plates to the same potential. (Incidentally, there is, in fact, a *very gradual and very small leak*, but that will be understood presently.)

(1) Gases under Normal Conditions of Pressure

1. Ionisation of Gases

The reason for the above, as already explained (read again pages 129, 256), is that gas atoms retain all their electrons firmly, so that it is impossible for electrons to pass from atom to atom (strictly, *molecule* in the case of a gas—page 130)—*i.e.* there are no "detached parts" acting as carriers of electricity like electrons of a metal or ions of an electrolyte. Under certain conditions, however, electrons can be detached from some of the gas atoms leaving positive ions, and both are then free to move independently in the gas. If, then, our two oppositely charged plates be put in the gas the electrons will move towards the positive (high potential) plate and the positive ions towards the negative (low potential) plate. When they reach the respective plates they " give up their charges," thus diminishing both plate charges: the gas has thus become conducting. The process of producing ions and electrons in a gas thus making it conducting, is called, as already explained, the **ionisation** of the gas.

In moving through the gas an electron may become " loaded " by attaching to itself a neutral atom of the gas, thus forming a *negative ion* which, of course, will travel towards the positive plate (the positive ion may also become " loaded " in this way). As the *gas pressure* is reduced, however, the loaded electron throws off its

attendants and travels "free." Experiment shows, for example, that in the case of air, if V be the P.D. per cm. in volts in the space between the plates, and P be the gas pressure in mm. of mercury, then if V/P exceeds 0·2 any negative ion will throw off its load.

From the above it is clear that the *mass* of the positive ion (and also of the negative *ion*) will be comparable with that of the atom or molecule of the gas: the free electron is, of course, *considerably* lighter. In most cases of ionisation the process consists of the detachment of a single electron from the atom so that the charge on a positive ion is equal but opposite in sign to that of a negative ion and electron, and is thus independent of the nature of the gas, so that in this respect of "charge" the gas ion differs very much from the ion met with in electrolysis. (Incidentally, it is possible under special conditions to detach more than one electron from an atom, the charge on the resulting positive ion being then equal *numerically* to that of all the electrons detached: thus eight electrons have been obtained from an ionised mercury atom and under the unusual conditions which exist at the hottest stars *all* the electrons belonging even to complicated atoms are driven out.)

In their movement through the gas the detached free electrons and the negative ions will make collisions with other atoms, and if the P.D. between the plates be maintained at a *high value* the negative electrons (and ions) may be so accelerated and their velocity so increased that they are able to detach electrons from the atoms with which they collide, such electrons (and negative ions) also moving off and producing further ions and electrons by collision. With a strong field between the plates the positive ions also produce in their journey fresh ions and electrons by collisions. Thus the ionisation is *cumulative*, and with the increased available "carriers" the conductivity increases.

In order to ionise a gas atom a definite amount of energy is required, and this differs with different gases: it is greater, for example, with the inert gases helium and neon in which the electrons are firmly held in the atoms by their attracting nuclei, than it is with hydrogen and oxygen, where the electrons in the atoms are much less firmly held. In most of our experimental work ionisation is brought about by the collision between the gas atom and an ion or electron which, during its free movement in the gas, has been so accelerated by the electric forces of the field and so increased in velocity that it has acquired sufficient energy of motion to bring about the ionisation. The energy of motion acquired by an accelerated ion or electron depends on the field, *i.e.* the P.D. applied

(see below). It is therefore usual to state the energy necessary to ionise a particular gas atom, not in ergs, but in terms of the P.D. (volts) through which an ion or electron must accelerate freely in order to attain sufficient energy of motion to bring about the ionisation of the gas atom. This is termed the **ionisation potential** of the gas: a few values are: helium 24·8, neon 21·5, hydrogen 13·5, oxygen 12·8, mercury vapour 10 (volts).

Consider two parallel plates, one positive, the other negative, the P.D. between them being V. Imagine a particle of mass m and negative charge e initially at rest just outside the low potential plate, and that it is freely accelerated towards the other plate. The energy it acquires is, by the definition of V, equal to Ve. If v be its velocity of arrival at the plate, the energy is also $\frac{1}{2}mv^2$. Thus $Ve = \frac{1}{2}mv^2$ *or*

$$v = \sqrt{2\frac{e}{m}V}.$$

Thus since energy = Ve, all ions and electrons with the same charge e will acquire the same energy when they freely accelerate through the same potential difference V, but we see from the above that the *velocity* (v) they attain in acquiring this energy is greater the smaller the mass (m) of the ion, and therefore *the velocity attained is greatest in the case of the electron.*

In the course of its motion in the gas a positive ion may encounter an electron (or electrons) moving with sufficiently low velocity to be drawn by the forces of mutual attraction into the ion's electron system, so that the positive ion again becomes a neutral atom (molecule). This process, as already stated, is termed **recombination.** Naturally, as soon as ionisation has been produced, recombination commences, and both processes proceed simultaneously. With sufficiently intense fields, however, the rate at which fresh ions and free electrons are produced exceeds that at which they recombine, so that the total number of ions and free electrons, and with it the conductivity of the gas, increases. With a sufficiently high P.D. and this increased ionisation and conductivity of the gas we reach the condition of the electric discharge (brush, spark, etc.) dealt with in preceding chapters.

As indicated (pages 238, 371), when positive ions and electrons join up again in an ionised gas the energy which was necessary to separate them when the gas was ionised reappears as heat, whilst some is radiated away and under certain conditions the frequency of some of this radiation may fall within the range limits we call " light." Thus it is that we have the " light " of the gas discharge lamp, the *colour* depending on the kind of ions recombining—

with neon red-orange; with mercury vapour blue; with mercury vapour and neon bluish-white, and so on. Thus it is, also, that we have the colour of the electric spark and the lightning flash referred to in dealing with discharges under high P.D.'s in Chapter VIII.

We have referred to the very gradual and very small leak which occurs when a charged body is exposed to the air. This is due to the fact that a very small proportion of the air atoms, even in lower regions, is always ionised—about 1 in 10^{16} atoms—and it is brought about by *cosmic rays* which reach the earth from remote space outside the solar system, and by *traces of radioactive material*: it is known as **spontaneous ionisation.** This natural ionisation is not sufficient to interfere with, for example, our ordinary electrostatic experiments, but it is important for it assists the starting of further ionisation by collision when a suitable P.D. is applied.

The process of bringing about the ionisation of a gas by some external ionising agent (say X-rays—see page 557) can be partly studied by the following experiment:—

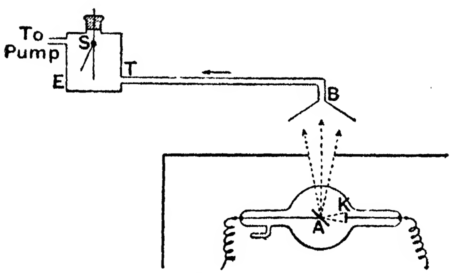


FIG. 501.

In Fig. 501, E is a charged electroscope, TB a tube which terminates in a funnel, below which is an X-ray bulb. The bulb is in a lead box provided with an opening just below B, the lead box being necessary to screen E from the direct action of the X-rays. The tube on the left of E communicates with a water-pump, so that air can be drawn through BT and E. When the bulb is on and the air drawn along BT is therefore exposed to the rays, the electroscope collapses, whether the charge on it is positive or negative, thus showing that the X-rays have made the air a conductor. If a plug of cotton-wool be placed in the tube BT the natural, very gradual, very small leak in E is *not* altered when the bulb is put on, *i.e.* the air does not show improved conductivity. If TB is a metallic tube with a wire stretched along its axis, and the tube and wire be maintained at a high potential difference, the same thing happens, *i.e.* there is no increase in conductivity when the bulb is put on.

These experiments indicate that the X-rays have produced in the gas charged "particles" which, when they enter the electroscope, neutralise the charge on the gold leaf. Since the gas, as a whole, is neutral, the charged particles must be of two kinds, positive and negative, the amount of charge carried by the positive particles being equal numerically to the amount of charge carried

by the negative particles. These charged "particles" are, of course, the positive and negative ions. When the cotton-wool is there they do not reach the electroscope: when a P.D. is established between a central wire and the tube, the wire being at the higher potential, the negative ions move to the wire and the positive ions to the tube, and again the electroscope is not affected.

When an ionising agent which has been acting on a gas is shut off, the conductivity of the gas does not immediately disappear, although it ceases in time: in the interval the ions remaining *slowly* recombine. This *slow* recombination of ions after the ionising agent ceases to act also occurs in the upper regions of the atmosphere. These regions are ionised by radiation from the sun, but the air is so rarified, and the ions and electrons therefore so far apart, that recombination after dark is *very slow*: some ionisation continues right throughout the night, and this is important in wireless transmission.

2. Methods of Producing Ionisation in Gases

The following is a brief summary of the chief methods by which a gas may be ionised and rendered conducting:—

(1) By *X-rays*, by *gamma* (γ) *rays* from radioactive material, by *cosmic rays* which reach us from remote space, and by *ultra-violet light*. The first three are also of the same nature as light but are much shorter in wave-length, and when they pass through a gas they are able to detach electrons from some of the atoms. The shorter the wave-length the more efficient it is as an ioniser. Ordinary light is of too big a wave-length to ionise a gas atom *directly* (it may ionise the atoms of certain substances placed in the gas, and the ejected electrons under a high P.D. may bring about further ionisation in the gas), but *ultra-violet* light can do so. An outstanding example is the upper regions of the atmosphere, which are ionised by ultra-violet rays from the sun. In this ionisation the ultra-violet is, however, rapidly absorbed, so that the lower regions are (fortunately) protected from it *to any marked extent*. (Although, in certain conditions, ultra-violet in moderation is used in medical treatment, it can be very detrimental to living matter.)

(2) By *alpha* (α) *particles* and *beta* (β) *particles* from radioactive material. An α particle is, as already stated (page 16), the *nucleus* of an atom of helium (2 protons + 2 neutrons), and a β particle is an electron, both ejected with high velocity from the nucleus of a radioactive atom. If a small quantity of radioactive

material (say radium) be brought near a charged electroscope the leaves very soon collapse.

(3) By *flames* and *heat in general.* Gases near flames are found to conduct. When a lighted match is placed between two oppositely charged plates they soon lose their charges. When a gas is *very strongly heated,* the intense thermal agitation results in violent collisions which in turn result in electrons being driven from the gas atoms, *i.e.* in ionisation and conductivity of the gas. The temperature of the gas must be very high, however, for this to occur, *i.e.* for electrons to be driven *directly* from the gas atoms, but it is quite usual in some stars. On the other hand, heating certain substances (*e.g.* platinum) in a gas causes the ionisation of the platinum atoms and the ejection of electrons, and this may bring about further ionisation in the gas.

(4) By the *electric discharge*: after the first spark the discharge takes place more readily, due to ionisation.

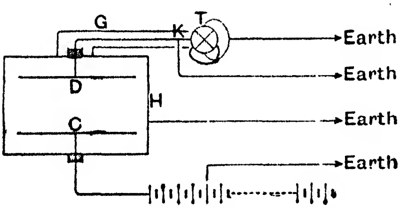


FIG. 502.

3. Saturation Current

In the usual experimental work the current through a gas, unless strongly ionised by radium salts, is too weak to be measured by a galvanometer. A quadrant electrometer or gold-leaf electroscope of the types shown in Figs. 218, 134 must be used instead. If C is the capacitance of the electrometer or electroscope and V the rise, say, of potential per second, CV is the current flowing into it. Care must be taken with the units: do not "mix up" e.s. and e.m. units. The following experiment will illustrate essential points:—

In Fig. 502, C and D are two insulated parallel metal discs placed inside an earthed metal vessel H. C is attached to one pole of a battery of many cells. By means of an earthed wire making contact with the plates of the cells the voltage on C can be varied within wide limits. D is well insulated and attached to one pair of quadrants of an electrometer T, the other pair of quadrants being earthed (an electroscope may be used instead). K is a key by which D may readily be earthed. G is an earthed tube-guard to protect the wire from D to T from external induction effects. The air in H being ionised by some ionising agent, C is charged successively to different voltages and the current flowing from C to D observed. If the plate C is connected to the negative pole of the battery it repels the negative ions (and

electrons) to D, which thus receives a negative charge. If C is positive D receives a positive charge. The results being plotted, a curve like Fig. 503 is obtained.

At first the current increases almost according to Ohm's law: this is shown by the rising part of the curve in Fig. 503. When the P.D. reaches a value represented by O*a* a certain current is flowing. When the P.D. increases beyond O*a*, however, the current remains constant, and it continues constant for a fairly big increase in P.D., viz. from O*a* to O*b*: this constant current is shown by the level part of the curve, and it is called the **saturation current.** When the P.D. exceeds the value O*b* the current again increases as shown by the rising curve.

When the P.D. is low only a few ions are able to reach the plates before recombining, and the current is small. As the P.D. increases more and more ions reach the plates before recombination, and the current increases. Between the P.D.'s O*a* and O*b* all the available ions are being driven to the plates, and the current is steady. Beyond O*b* the strong P.D. causes the ions (and electrons) to have such a velocity that *first* the electrons and negative ions *and then* the positive ions produce fresh ions and electrons by colliding with molecules and detaching electrons; production of "carriers" much exceeds recombination and current increases.

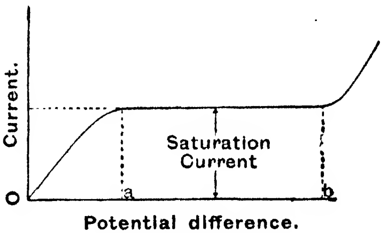


FIG. 503.

Of course, with still higher P.D.'s, increased ionisation and number of available carriers, the phenomena of the electric discharge already dealt with would set in.

If the ionisation is merely spontaneous ionisation, a P.D. of the order of about 10 volts per cm. produces the saturation current, but with agents such as X-rays, radioactive bodies, etc., the ionisation is so great that a much greater voltage is necessary for saturation. The same happens if the distance between the plates is increased, for more ions are produced in the greater space. Thus for the same P.D. between the plates *the saturation current is greater when the plates are drawn further apart.* The student should note this difference between conduction through gases and conduction through metals, viz. Ohm's law.

If the ionising agent is removed the current drops down to the normal air current in a short time, due to recombination of the ions (and electrons) remaining in the space.

If Q ions be produced per unit volume per second by the ionising agent, the total number produced per second in the space between the plates will be QAl, where A is the area of the plates and l the distance apart. *If all these are driven to the plates before any recombinations take place* we evidently have the saturation current I; hence, if e be the charge on an ion,

$$I = QAle.$$

The saturation current is proportional to l, as indicated above.

4. Determination of the Mobilities of Gaseous Ions

The determination of the mobilities of gas ions, *i.e.* their velocities under unit potential gradient (one volt per cm.) has been attempted by various methods, but full details are beyond the scope of this book.

One method is as follows. Imagine two parallel plates P and Q maintained at a fixed potential difference, and let ions be produced near P: ions of one sign will give up their charge to P, and the others will move towards Q. Just as they are about to reach Q imagine the field to be reversed; the ions will be driven back towards P and a fresh set of opposite sign will move towards Q. Just as they are about to reach Q imagine the field again reversed, and so on. By a careful arrangement of an experiment of this kind and a noting of the time between reversals, for which Q is just on the point of taking a charge, we obtain the time required for the ions to travel the distance between the plates, and, therefore, the velocity of the ions. Experiment shows that the plate Q takes a negative charge sooner than a positive one; the velocity of the negative ion is, therefore, the greater.

The following values may be quoted: the ionisation was by X-rays, and the velocities are in cm. per sec. under a potential gradient of one volt per cm.

	u_+	u_-		u_+	v_-
Air	1·36	1·87	Helium	1·42	2·03
Oxygen	1·36	1·80	Hydrogen	6·70	7·95

5. Ionisation Photographs

Much important research work has been carried out at the Cavendish Laboratory, Cambridge, on the tracks made by ionising agents (X-rays, α-particles, etc.) in passing through a gas which not only throws light on many of the facts mentioned about ionisation, but also on atomic structure. The principle involved is simply this:—Suppose α-particles, say, are passing through super-saturated air: the particles ionise the air, and *the ions produced*

in their path condense water vapour round them, forming droplets. These are illuminated for the short time necessary for a photograph to be taken and thus the tracks are portrayed.

The Frontispiece (*b*) shows the tracks of α-particles (nuclei of helium atoms from radioactive material: nucleus = 2 protons + 2 neutrons: atomic weight 4: free charge + 2) passing through air. The α-particles shot straight through about 7 cm. of air, ionising on their way, and going ***right through*** atoms without any (positive) nucleus of a gas atom repelling and deflecting them from their straight tracks. Towards the ends there may be a deflection showing that as they slow down they may approach so close to a nucleus that they are repelled from their straight path. This is shown in one track

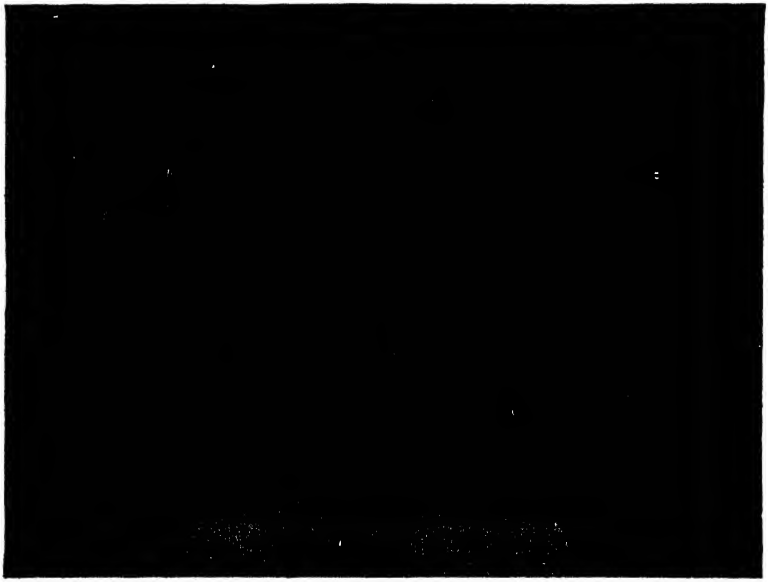


FIG. 504. Ionisation of Air by X-rays.

in the figure where the α-particle has collided with a nucleus: the track divides, one being the α-particle deflected, and the other the recoil of the nucleus. Fig. (*c*) also shows an α-particle deflected by a nucleus as it slows down.

If a fast moving α-particle is passing through a gas which has a comparatively small positive charge at the nucleus of its atom they may come so close that the α-particle drives a proton (positive) from the nucleus. This is shown in one track in the Frontispiece (*a*) : the firmer of the two tracks after the collision is the α-particle still ionising as it goes; the fainter is the ejected proton which, owing to its less positive charge, has less power to ionise the gas atoms which lie in its path. (The gas in this case was nitrogen.)

Fig. 504 depicts the ionisation of air by X-rays. The rays eject electrons from the gas atoms: these ionise as they go, and condensation occurs on the ions. As they slow down they are less able to resist deflection by near-by charges: hence the curls towards the ends of the tracks. Note that the little tracks indicated by the condensation are in all directions, and bear no relation to the path of the X-rays through the gas.

(2) GASES AT LOW GAS PRESSURE

6. Discharge through Gases at Low Pressure

As the pressure is reduced below atmospheric pressure, a gas becomes a much poorer insulator, and a less P.D. will produce a discharge. From previous pages this is what might be expected.

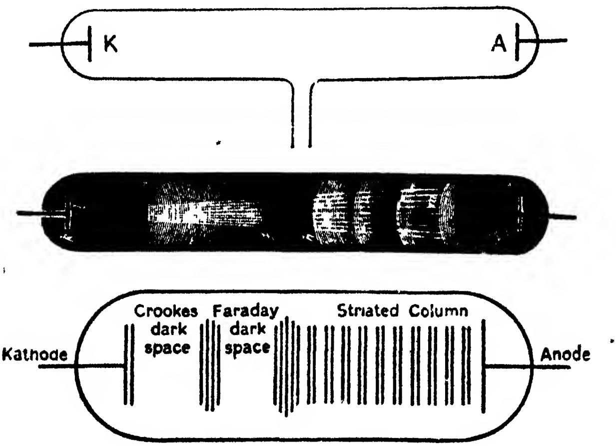


FIG. 505.

As the pressure is lowered the atoms of the gas are less packed together and their "free paths" between collisions (thermal agitation) become much greater. Any ions and electrons present will therefore be accelerated through a greater distance, and thus acquire the energy necessary to ionise the atoms with which they collide. The same applies to any electrons driven from the low potential (negative) electrode in the gas. Thus the low gas pressure causes conditions which assist ionisation and therefore a lower P.D. will bring about a discharge.

The discharge at low pressures may be studied thus:—Fig. 505 is a glass tube (about 25 cm. long) containing the gas, A and K are

two metal electrodes (A = anode, K = cathode) joined to the secondary of an induction coil, and the side tube communicates with a pump so that the tube may be gradually exhausted. The result, as exhaustion proceeds, is best observed in a darkened room. The main points may be briefly summarised as follows:—

(1) As the gas pressure falls there is at first very little change in the appearance of the spark discharge which is taking place between A and K, although the P.D. necessary to produce it becomes less. With further reduction of pressure (to about 2·5 cm. of mercury) the discharge begins to broaden out from its narrow spark path, and becomes paler.

(2) At lower pressures, say 1 cm. of mercury, the discharge is a broad luminous column stretching from the anode almost to the cathode; it is known as the **positive column.** The colour of the positive column depends upon the gas in the tube: with air—mauve: hydrogen—blue or red: nitrogen —red: carbon dioxide—white. Of course, during the discharge positive ions move to the cathode and electrons to the anode, producing further ionisation on their journey: the impact of the positive ions with the cathode assists the ejection of electrons from the metal, and these also are repelled towards the anode, ionising as they go. " Recombinations " are also taking place: hence the " light "—the *colour* depending on the kind of ions recombining.

(3) At still lower pressures the column breaks up into alternate bright shells and dark patches. There is a bright glow near the cathode, and between this and the first bright shell is a dark space known as the **Faraday dark space.**

(4) Next, the cathode glow moves further from it, and another dark space appears between it and the cathode known as **Crooke's dark space.** Electrons near, and those ejected from, the cathode are under intense repulsive electric force, and move away from it so rapidly that they travel a certain distance away before their velocity is reduced sufficiently to permit of their being absorbed into a positive ion's electron system, *i.e.* to permit of recombination.

(5) As the pressure is further reduced the electrons repelled from the cathode travel a further distance down the tube before they are sufficiently slowed down for recombination. Thus the Crooke's dark space extends until it practically fills the tube. What has the appearance of a bluish beam seems to be coming from the cathode, and it causes fluorescence where it strikes the glass—yellowish-green for soda glass, blue for lead glass.

(6) At still lower pressures the discharge diminishes, and an extremely large P.D. is necessary to get a discharge at all, the nearly complete vacuum being an exceedingly good insulator.

When the Crooke's dark space extends to fill the tube (5) a *stream of electrons is travelling with amazing speed away from the cathode*: these, when first discovered, were called **cathode " rays,"** but they are really *electrons* thrown out by the cathode or negative electrode. The P.D. is high, and the gas is so rarefied that the

acceleration of the tiny electrons is little interfered with in their passage along the tube; thus they attain *very great velocities.* When these fast-moving electrons of the cathode stream strike the glass they displace some of the *outer* electrons of the glass atoms, and when the atoms return to their normal condition they give out energy some of which is radiated as light; hence it is that we have the fluorescence of the glass. Further, the speed and consequent energy of motion of the electrons of the cathode stream may be so great that under suitable conditions they can *penetrate right into the atoms* of a solid with which they collide, and displace some of the *inner and more firmly bound electrons* of the atoms. In returning to their normal conditions energy is again radiated from the atom: but in this case it is an invisible radiation of the same nature as "visible light" but much more penetrating and of shorter wave-length. This radiation is known as X-rays or **Röntgen** rays (discovered by Röntgen in 1895). X-rays are dealt with in Chapter XX.

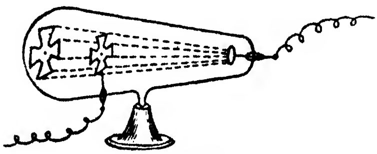


FIG. 506.

It may be noted in passing that the modern *cathode ray tube*, as it is called, is a device for the production of a narrow but intense stream of cathode rays, and it uses a *heated cathode*: this results in a copious supply of electrons from the cathode with lower P.D.'s between electrodes: it is dealt with later.

7. Observed Facts about the Cathode Rays

The cathode rays were discovered by Plücker in 1859, and the main properties may be briefly summarised as follows:—

(1) The rays are shot out by, and travel at a high speed normally from, the cathode, their direction being in no way connected with the position of the anode.

(2) They travel in straight lines and cast shadows of objects placed in their path; a hinged Maltese cross was placed opposite the cathode (Fig. 506) and a shadow appeared, as shown, on the end of the tube.

(3) A body placed in their path experiences a force tending to urge it away from the cathode. Thus rotation is produced in the case of the wheel fitted with vanes (Fig. 507), on which the rays fall.

(4) They produce heat when they fall upon matter.

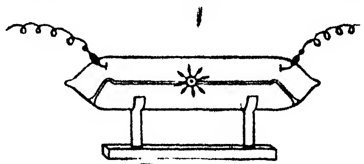


FIG. 507.

(5) They produce phosphorescence in glass, barium platinocyanide, and rare earths (cerium, lanthanum, etc.), and in many other substances.

(6) They ionise a gas.

(7) When they strike a solid substance—particularly one with heavy atoms (big positive charge at the nucleus)—the atoms become the source of a penetrating radiation akin to light but of shorter wave-length (X-rays).

(8) They are deflected by magnetic and electric fields, and the direction of deflection is that in which a stream of *negatively charged particles* would be deflected. That the "rays" are negative was proved by Sir J. J. Thomson as follows:—

The apparatus (Fig. 508) consisted of a vacuum tube with two bulbs. C is a cathode, and some of the rays leaving it pass through the slit S in a brass plug A, which is used as the anode, and strike the opposite wall of the larger bulb, giving a phosphorescent patch. Attached to this bulb is a side tube containing an earthed cylinder M, and within that, but insulated from it, another small cylinder connected to an electroscope or electrometer. The cylinders are out of the direct line of fire. A magnet is now brought up to the bulb and the rays are deviated until they pass through the slit of M into the insulated cylinder. The electrometer at once shows that the inner cylinder is being *negatively* charged, thus proving that the rays carry a negative charge. Of course, *the direction of deflection by the magnetic field* also verifies this.

(3) "MASS" AND "CHARGE" OF AN ELECTRON

8. Determination of Velocity and Value of (Charge/Mass) for the Cathode Stream Particles (Electrons)

Thomson devised a cathode ray experiment of the following type (Fig. 509):—C is the cathode, A the anode, and H a metal plug, the two latter being each pierced by a slit so that a narrow pencil of cathode rays passes through from C into the compartment on the right and produces a phosphorescent line on the glass at S. The centre portion is

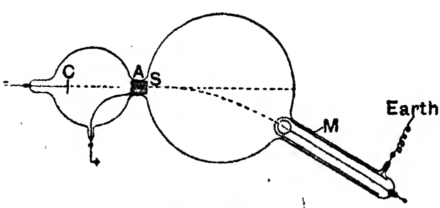


FIG. 508.

placed in a strong magnetic field (H) which is applied in the direction perpendicular to the plane of the paper and is uniform over the dotted area. The pencil of rays is deflected (at right angles to the field and the motion), and the phosphorescence now appears at S′. In passing through the field of the dotted area the pencil describes a circular arc, whilst beyond the arc it is straight. By noting the deflection SS′ and the horizontal distances, the radius (R) of the circular arc path can be calculated (see below).

The next step is to join the horizontal plates shown in the figure to a battery and earth, so as to establish an electrostatic field (E) at right angles to the magnetic field, its direction being such that the force on the charged particles of the pencil due to it is *opposite* to the force above. This field is then adjusted until S′ is brought back to S, and its intensity E is noted. It is readily shown (see

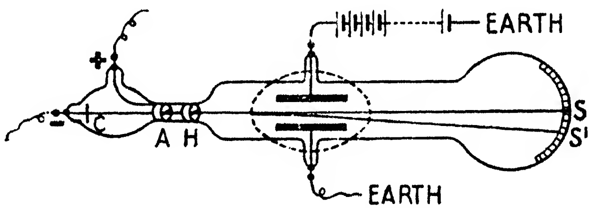


FIG 509.

below) that if v = velocity, e = charge, and m = mass:—

$$v = \frac{E}{H} \quad \text{and} \quad \frac{e}{m} = \frac{v}{RH},$$

so that the velocity and the ratio charge/mass for the negative particles (electrons) of the cathode stream are determined. *The value of e/m was found to be independent of the material of the cathode and of the gas in the tube.*

The expressions used above may be easily established. A charge e electrostatic units moving at v cm. per sec. is equivalent to a current ev electrostatic units = $ev/(3 \times 10^{10})$ e.m. units.

Now consider the case of a *stream of electrified particles shot across a magnetic field.*—Let n be the number of particles in unit length, m the mass of each, e its charge, v the velocity, and H the magnetic field supposed uniform and perpendicular to the direction of motion. The equivalent current is nev. The force exerted on unit length by the field is nevH, and is at right angles to both the motion and the field. The force on each particle is therefore evH. The particle will be deflected, and will move along an arc of a circle in a plane *perpendicular to the lines of magnetic force.* Let R be the radius

of this circle, then, since evH is the centripetal force on a particle of mass m, it follows by mechanical principles that:—

$$mv^2/R = evH; \quad \therefore \; e/m = v/RH \text{ (see above).}$$

Now R is found, as stated, from the deflection and horizontal distance. Thus if AB (Fig. 510) represents the line of moving particles before the field is on, and AC the line after the field is on, then $AB^2 = BC\,(2R - BC)$, from which, after measuring AB and BC, R can be found.

Again, if a particle of charge e be in an *electrostatic field* of intensity E the force on it is Ee *along the lines of the field*. If, then, we have both electrostatic and magnetic fields acting, and if we so arrange their directions and intensities that they produce deflections which are equal and opposite, we evidently have equal and opposite forces, and we can write $evH = Ee$, so that $v = E/H$ (see above), and is therefore determined. Hence all the factors in the expression v/RH are known, and the value of e/m is found.

The value obtained for the velocity v of the negative particles of the cathode stream naturally varied with the conditions of the experiment ranging, in a general way, from $2{\cdot}2 \times 10^9$ to $3{\cdot}6 \times 10^9$ cm. per sec., say $\frac{1}{10}$ of the velocity of light.

FIG. 510.

Thomson's earlier results for e/m were low (about $0{\cdot}77 \times 10^7$ e.m. units per grm.): later work gives e/m as $1{\cdot}759 \times 10^7$ e.m. units per grm. $= 1{\cdot}759 \times 10^8$ coulombs per grm. $= 5{\cdot}3 \times 10^{17}$ e.s. units per grm.

In Chapter XIII it was stated that the hydrogen ion in electrolysis carried a unit charge equal and opposite to that of an *electron*, and that the value of e/m for the *ion* was, say, $9{\cdot}65 \times 10^3$ e.m. units per grm. If, then, the negative particles of the cathode stream are electrons (as stated) so that their e is the same as for the hydrogen ion in electrolysis we have:—

$$\frac{\text{Mass of electron}}{\text{Mass of H ion}} = \frac{e/m \text{ for H ion}}{e/m \text{ for electron}} = \frac{9{\cdot}65 \times 10^3}{1{\cdot}759 \times 10^7} = \frac{1}{1850}, \text{ approx.}$$

so that the mass of the electron (in this case our particles of the cathode stream) is about 1/1850 of that of a hydrogen atom: this is the figure we have already taken in previous pages as the mass of an electron.

9. Determination of Charge and Mass of an Electron

We saw (page 535) that a *negative gas ion* was a detached *electron* usually "loaded" by attaching to itself a neutral atom (or molecule) of the gas, but that when the gas pressure was sufficiently lowered the electron threw off its attendant and travelled free: and we saw that a *positive gas ion* was an atom which had lost an electron.

Now if air saturated with water-vapour is subjected to a sudden (*i.e.* adiabatic) expansion condensation occurs, and the space is filled with a cloud. As already explained (page 257), the formation

of a cloud is impossible unless nuclei are provided on which the water may condense. In ordinary air dust particles provide most of the nuclei, but in dust-free ionised air the ions (and electrons) act as nuclei. With a certain expansion the condensation is only formed on the negative ions; with a larger expansion it occurs on both negative and positive ions. When such minute spheres fall through a gas they quickly attain a steady velocity, due to the upward force of viscosity balancing the downward force of gravitation, and Sir G. G. Stokes has calculated that the steady velocity of a sphere of density ρ and radius a falling in a medium of viscosity μ is given by

$$v = \tfrac{2}{9}.\frac{\rho g a^2}{\mu}.$$

Thomson and H. A. Wilson used this property to find the charge on a gas ion, *i.e.* the electron charge (e), and Wilson's method is briefly as follows:—The lower half of the vessel AB (Fig. 511) contains water, so that all the space above is saturated. By the tube E the vessel AB communicates with an expansion chamber and a manometer. X-rays from the bulb on the left can be sent along the space between C and D to ionise the gas there. The expansion apparatus is arranged so that condensation will only take place on the *negative ions*. The plates C and D are first joined so that they are at the same potential, and the X-rays are turned on for a short time to ionise the gas: then the rays are turned off.

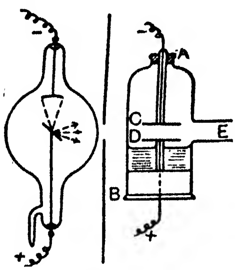


FIG. 511.

The expansion is now produced, thus forming a cloud, and the velocity v_1, with which the top of the cloud between C and D settles down, is noted. C is now connected to the negative and D to the positive pole of a battery, so that the electric field (E) hastens the fall of the cloud, and the velocity v_2 with which the top of the cloud now falls, is noted. Assuming that each drop has one electron for its nucleus, the value of e, the electronic charge, is readily determined.

Before the electric field is put on, the downward force on a particle is Mg, where M = mass and g = acceleration due to gravity. When the field is put on, the downward force on a particle is $Ee + Mg$. Hence:—

$$\frac{Ee + Mg}{Mg} = \frac{v_2}{v_1}; \quad \therefore e = \frac{Mg}{E}.\frac{v_2 - v_1}{v_1}.$$

Now $M = \frac{4}{3}\pi a^3\rho$, where a = radius of drop, and ρ = density, and therefore, using Stokes' formula, $a = \sqrt{9\mu v_1/2\rho g}$. Substituting in the expression for e above we get:—

$$e = 9\pi\sqrt{\frac{2\mu^3}{\rho g}}\,\frac{(v_2 - v_1)\sqrt{v_1}}{E},$$

and since E, v_1, and v_2 are known, whilst ρ = 1 grm. per c.cm., g = 981 cm./sec.^{2}, and μ for air = $1{\cdot}8 \times 10^{-4}$, the value of e is determined. Wilson's result for the charge (e) of an electron was $3{\cdot}1 \times 10^{-10}$ e.s. unit. Latest results give:—

Charge = $e = 4{\cdot}77 \times 10^{-10}$ e.s. unit = $1{\cdot}59 \times 10^{-20}$ e.m. unit,

or $e = 1{\cdot}59 \times 10^{-19}$ coulomb (page 375),

which is the value we obtained for the charge of the hydrogen and other monovalent ions in electrolysis.

Combining this result with the value of e/m found for the negative particles (electrons) of the cathode stream we get for the mass (m) of an electron—

$$\text{Mass} = m = \frac{1{\cdot}59 \times 10^{-20}}{1{\cdot}759 \times 10^{7}} = 9 \times 10^{-28} \text{ grm.}$$

On page 375 it was stated that from calculations based on the kinetic theory of gases the number (N) of atoms in one gram-equivalent of hydrogen was $6{\cdot}06 \times 10^{23}$, which gives for the mass of the hydrogen atom $1{\cdot}66 \times 10^{-24}$ grm. (N is known as **Avogadro's number**). Taking then the m value found for the electron, we arrive at a result already stated in preceding pages:—

$$\frac{\text{Mass of electron}}{\text{Mass of H atom}} = \frac{9 \times 10^{-28}}{1{\cdot}66 \times 10^{-24}} = \frac{1}{1850}.$$

Again, from Faraday's laws of electrolysis (Chapter XIII.) we know that when one gram-equivalent of a univalent radical is liberated 9650 e.m. units have passed, so that if e be the charge (e.m. units) on each ion, $Ne = 9650$, *i.e.* $(6{\cdot}06 \times 10^{23})\,e = 9650$, which gives $e = 1{\cdot}59 \times 10^{-20}$ e.m. units, the value obtained for the electron (see page 375).

Millikan (U.S.A.) improved upon Wilson's method of finding e in what are referred to as his **balanced oil drop experiments.** He used oil drops instead of water drops, the drops passing in between the plates (say C and D of Fig. 511) through a hole in the upper one; and by a suitable adjustment of the P.D. between the plates, and arranging that the electric field *opposed* the motion of the drops, a drop could be kept in view for some considerable time and its motion observed through a telescope provided with a graduated scale in the eye-piece. Millikan's apparatus is shown in Fig. 512, some of the minor details being omitted.

For full details of Millikan's experiments and calculations, and for the work of other experimenters, see *Advanced Textbook of Electricity and Magnetism.*

10. Positive Rays. The Mass Spectrograph

If the cathode in the tube of Fig. 505 be perforated (Fig. 513), streamers may be seen behind it if the gas pressure is within certain limits. These produce phosphorescence, penetrate *thin* aluminium foil, and consist of *positively charged particles* travelling in the direction anode to cathode. They are referred to as positive rays or "canal" rays.

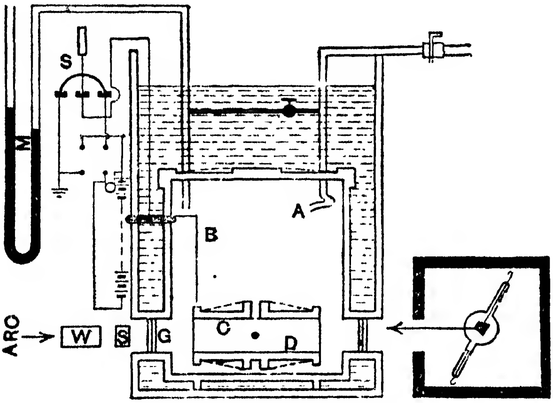


FIG 512. B — brass vessel surrounded by a constant temperature oil bath. A — atomizer through which oil spray is blown into B: droplets of oil are formed and occasionally one passes through the hole in C into the space between C and D where it is illuminated by light from an arc and its movement observed. Air between C and D is ionised by X-rays (tube on the right). M = manometer.

Measurements were carried out by Thomson on the positive rays somewhat similar to those for the electrons of the cathode stream, etc., and their e/m, e, m, and v values were determined. *Their e/m value was not constant, but depended on the gas in the tube.* Their *mass* also varied with the gas, *being about the same as that of the atoms and molecules of the gas used.* Their *charge* in many cases was equal and opposite to the electron charge e: in others it was $+2e$, sometimes $+3e$, and so on (up to $+8e$ in the case of mercury vapour, and in most cases traces of this were present, for mercury was used in the pumps). The positive rays are in fact *positive ions*—atoms (and molecules) *of the gas* which have lost one or more electrons.

Aston made measurements on the rays, using what is known as the **mass spectrograph** (Fig. 514). A narrow band of the positive rays is passed between two slits, S_1S_2, then between the two oppositely charged plates P_1P_2, and finally through a magnetic field at O at right angles to the electric field. In passing through the two fields the rays are deflected according to their e/m value, and those with the same value are brought to the same focus (F) on the photographic plate GB recording a line: from this the masses of the positive ions can be determined (see *Advanced Textbook of Electricity and*

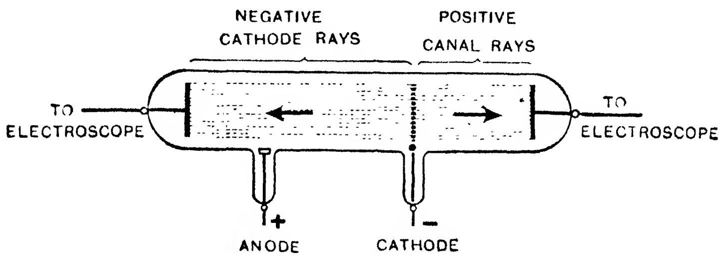


FIG 513.

Magnetism). If **isotopes** (page 13) are present in an element examined, several lines are recorded, each corresponding to an isotope: in this way it has been found that about forty elements exist as isotopes.

(4) OTHER CASES OF ELECTRON EMISSION

11. The Photo-Electric Cell

The free electrons in a good conductor, say a copper plate or wire, are moving at random with high velocities, and it might be

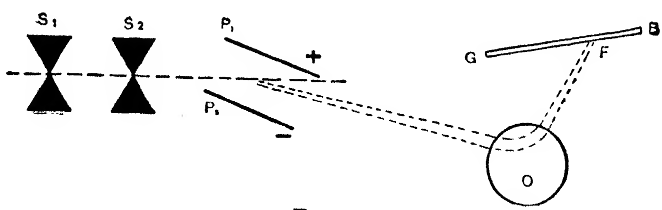


FIG. 514.

expected that some would be constantly escaping, at least temporarily, from the surface. Actually, however, there is an electric field at the surface tending to repel any electrons approaching it, and an electron must, in order to escape, possess sufficient kinetic energy to surmount the potential barrier. Under ordinary conditions the number of electrons which attain sufficient velocity and

energy to do this is quite negligibly small, but there are methods by which the electron energy can be increased so that electrons pass copiously from the surface.

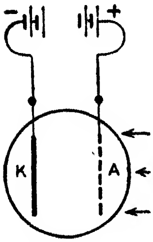


FIG. 515.

In some quite early experiments it was found that certain metals exposed to ultra-violet light emitted electrons from their surface: exposure to X-rays has the same effect. This phenomenon is known as **photo-electric emission** or the **photo-electric effect,** and is due to the fact that the surface electrons are set into forced vibration by the electro-magnetic radiation and acquire sufficient velocity and energy to overcome the surface barrier and escape. Measurements of e/m and e (and m) verify that the ejected particles are the universal electrons.

Ultra-violet light (short wave and high frequency) is more effective in the above than ordinary light because its period and frequency are nearer to those of the electrons in the atoms. Further experiments showed, however, that the metals *potassium*, *rubidium*, and *caesium* gave good results with ordinary light, and this fact is utilised in the **photo-electric cell.**

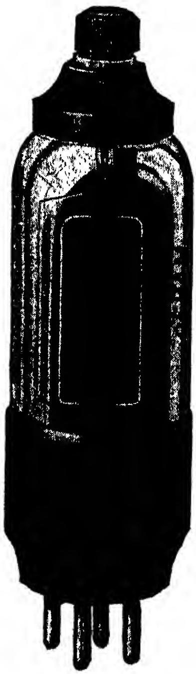


FIG. 516.

In Fig. 515 K is a metal plate (silver or copper) coated with potassium or caesium, and A is a metal grid, the two being in an evacuated glass vessel and joined to a battery so that A is the anode and K the cathode: no current passes, for the space between A and K is insulating. If light, however, passes through the grid holes and falls on K, the latter ejects electrons. These are repelled by K and attracted by A, so that current flows—an *electronic* current in the direction —ve pole of battery to K, through the cell from K to A, and from A to the + pole of the battery. The more intense the light, the greater the rate of emission of electrons and the greater the current. Sometimes an inert gas, generally argon or helium, at a low pressure is enclosed in the containing vessel in which case ionisation by collision occurs, electrons are added to the stream, and greater currents are produced. Fig. 516 shows a practical type of cell, but there are other forms.

These cells are used in television transmission (page 585), and for the automatic switching of street lighting circuits, the circuit arrangement being such that when the cell current falls below a certain value (due to failing daylight) the street lighting circuit is automatically switched on.

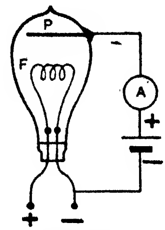


FIG. 517.

12. The Thermionic Valve. A.C. Rectifiers

Another method of increasing the velocity and energy of the electrons of the metal and enabling them to overcome the surface potential barrier and escape, is by raising the temperature (increased thermal agitation), and a well known example is the **thermionic valve,** which in various forms is used in wireless transmission and reception, and as *rectifiers* of A.C., *i.e.* for converting A.C. into unidirectional or direct current. Only a brief reference to the *simplest* type will suffice here: fuller details are given on pages 579-583.

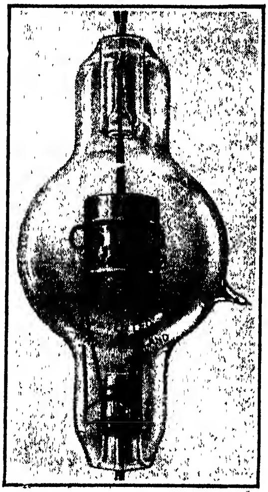


FIG. 518.

The first practical valve (called a *diode* valve) was made by Fleming (Fig. 517). It consists of a tungsten filament F (cathode) and a metal plate P (anode or plate) in an evacuated vessel. The filament is heated by a battery joined to the ends + and —, whilst another battery (say) shown on the right has *its positive pole joined to the plate* and its negative pole to the filament. The very hot filament throws out electrons which are attracted by the positive plate, so that we have an *electronic* current in the direction F to P in the valve and P through the battery to F in the external circuit, and the indicating instrument A will be deflected: the current in the outside circuit is referred to as the *plate or anode current.*

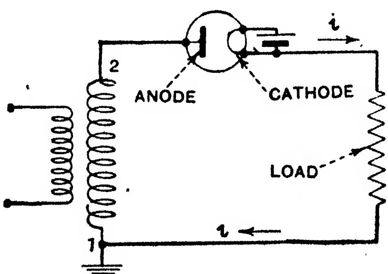


FIG. 519.

If the connexions of the battery on the right be reversed, the negative pole being joined to P, the current through the

valve will stop, for P being negative repels the electrons from F, so that they cannot get across. If an alternating P.D. be used instead of the battery on the right, current will flow through when P is positive with respect to the filament, but not when P is negative. The diode can therefore be used as a **rectifier of A.C.** in the sense that it allows current in one direction to flow, but not current in the other direction. A typical diode rectifying valve for obtaining high voltage D.C. from A.C. mains is shown in Fig. 518: valves for wireless receivers are dealt with on page 579.

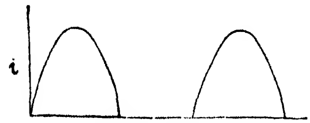


FIG. 520.

Fig. 519 shows a circuit with a diode valve in it to act as a rectifier of A.C. The alternating P.D. is applied to the primary (on the left) of a transformer, and it induces an alternating P.D. in the secondary which is joined to the circuit (the load) through a valve as shown. When the direction of the A.C. is such that the end 2 is positive, current passes through the valve and the circuit (load), but not when the A.C. reverses and 2 is negative: a pulsating unidirectional current as in Fig. 520 is therefore obtained.

A better arrangement is shown in Fig. 521, where two valves are used and a centre-tapped transformer. A little consideration will show that when one plate is positive the other is negative, and this is reversed when the current reverses, so that one valve is passing current when the other is refusing, with the result indicated in Fig. 522.

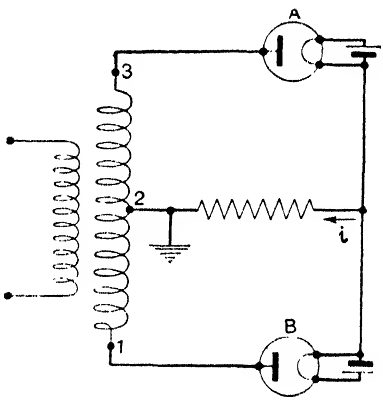


FIG. 521.

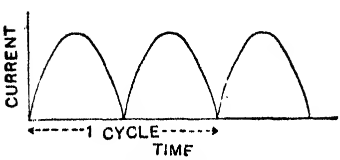


FIG. 522.

Several crystals — carborundum, galenta, bornite, etc.—when in contact with a metal will pass a current across the contact in one direction only, and certain pairs of crystals in contact act in the same way.

Again, if one side of a copper plate be oxidised it is found that *electrons* will only pass in the direction from the copper to the copper oxide. In practice several plates or discs are used. The device is known as a copper-oxide rectifier.

CHAPTER XX

X-RAYS, RADIOACTIVITY AND ATOMIC STRUCTURE

SOON after the exhaustion of the tube of Fig. 505 reaches the point at which the dark space at the cathode extends to the anode (5) Röntgen found that a photographic plate lying near was affected; he attributed this to some unknown form of radiation emanating from the tube, and to it he applied the term **X-rays.** It is now known, as already stated, that X-rays or **Röntgen rays** are an *electromagnetic radiation akin to light* (electromagnetic waves which travel through space—or the aether—with the velocity of light) which originates at a solid substance when it is bombarded by the very fast moving electrons of the cathode stream. A very early type of **X-ray tube** is shown in Fig. 523. The aluminium cathode on the left is concave and a platinum plate (which may be the anode) is placed at the centre of curvature (so that the cathode rays are focused at a point on it) with its plane at an angle of 45° to the axis; the X-rays arise at the point struck as indicated.

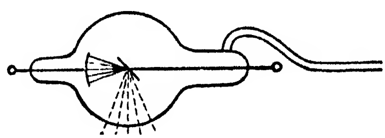


FIG. 523.

1. Facts about the Röntgen or X-rays

(1) They are *not* deflected by a magnetic or electric field. This differentiates them from cathode rays; they are not charged particles.

(2) They can pass through many solid substances. The transparency of substances to X-rays depends upon their density—the denser the material the more opaque it is to the rays. Thus lead 1 mm. thick is practically opaque to moderately penetrating rays, but the same thickness of aluminium is transparent. Soda glass is transparent, lead glass is opaque. Ebonite (which is composed of atoms of low atomic weight) is transparent to them. Flesh is much more transparent to the rays than bones.

(3) They excite fluorescence in many substances, *e.g.* barium platino-cyanide, zinc sulphide, etc. If the rays fall on a screen coated with one of these and the hand be interposed, a shadow of the bones appears on the screen, the fluorescence being less here owing to the rays being more absorbed by the bones.

(4) They ionise a gas and have a photo-electric effect, just as, for example, ultra-violet light has.

(5) They effect a photographic plate in a similar way to light. If the hand be interposed in the path of the rays (as in (3)) the bones will appear darker and stand out pronounced in the finished print: hence their use in surgery to detect fractures, foreign bodies, diseased organs (*e.g.* examining the lungs in tuberculosis), etc.: see radiograph, page 209. Similarly they can be used to investigate the structure of metals, the cause of weaknesses in structures, and to detect cracks and blow holes in metal plates. They are also of service in dental work, the cure of rodent ulcer, lupus, etc.

(6) Very little trace of regular reflection, refraction or diffraction was observed, but later, and with difficulty, some signs of polarisation were detected (see below).

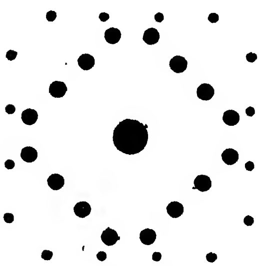


FIG. 524.

(7) When they fall on a material some are *scattered*, and these have a penetrating power somewhat similar to that of the incident X-rays. In addition to these scattered X-rays new rays called characteristic X-rays start from the material: these X-rays are different for different materials, being more penetrating for the materials of high atomic weight, each material giving a particular set of X-rays, known as the Series K, Series L, Series M, etc., rays.

(8) Investigation shows that they are of the same nature as light but of shorter wave-length (higher frequency) and more penetrating: their velocity is the same as that of light (also of cosmic rays, gamma (γ) rays, and " radio " waves), and their wave-length of the order 5×10^{-9} cm. (" visible " light is of the order 5×10^{-5} cm.—but see page 570).

The wave-length of light can be measured by a " diffraction grating " (as is known to all students of *Light*), the distance between the grating lines being of the same order as the wave-length: the difficulty with X-rays was to get a grating with lines near enough together for the much shorter waves. Lane suggested that the symmetrical arrangement of the atoms of a crystal might be used as a grating: hence a narrow beam of X-rays was passed through various crystals, and the rays emerging were caused to fall on a photographic plate. In passing through, the rays undergo successive scatterings at atoms in successive layers, so that on rejoining and emerging the waves may be such that they reinforce or destroy each other. Fig. 524 gives one result, the central spot being due to the undiffracted rays. From the separation of the

spots, the known constants for the crystal used, and the theory of the grating (see a book on *Light*) the wave-length can be determined.

2. The X-Ray Tube

The practical form of X-ray tube has undergone considerable modification in recent years.

A tube largely used some years ago (and still met with) is shown in Fig. 525. The cathode on the right is a concave cup of aluminium. The **anti-cathode** or **target**, as it is usually called, is tungsten (high atomic weight): it acts as an anode although another anode is also provided. Since the cathode rays have a considerable heating effect the anti-cathode is usually fitted (Fig. 527) with a *water cooling* device (sometimes fins are fitted and *air cooling* employed). To work the tube an induction coil giving of the order 100,000 volts may be

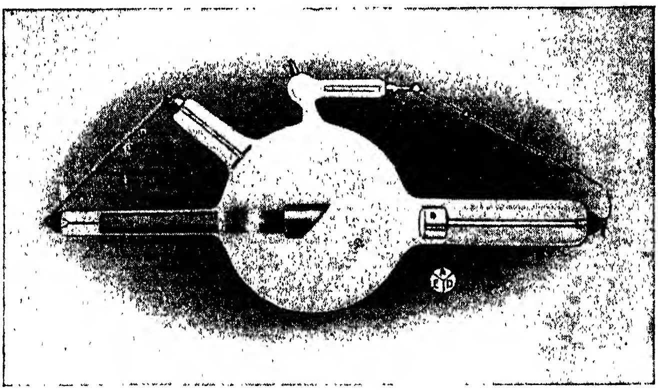


FIG. 525. A "Gas" X-Ray Tube.

used, but as the secondary E.M.F. of the coil is not strictly unidirectional (see Fig. 450), a valve (page 555) is included in the tube circuit to prevent the reverse current. During the working of the tube the residual gas pressure decreases and may become so low that discharge passes with difficulty unless the applied E.M.F. be increased. To overcome this a rod of palladium is frequently fitted in the tube and heated by a Bunsen, in which case some hydrogen is absorbed from the coal gas, diffuses into the tube, and raises the gas pressure inside. Another method is to have a side tube containing asbestos in which gases have been occluded: when the residual gas in the main tube becomes too low in pressure, and its resistance therefore too high, the discharge passes through the side tube, rather than through the main tube from anode to cathode, liberates some gas and restores the working conditions.

The modern X-ray tube makes use of thermionic emission to increase the supply of electrons by using an electrically heated cathode (see the thermionic valve, page 555), and one of the latest

types is shown in Figs. 526, 527, the former being partly cut away to show the essential internal arrangement. The vacuum chamber is a glass cylinder. On the right (Fig. 526) is the tungsten-faced target, and on the left is the electrically heated cathode in the form of a helix of tungsten wire mounted in a trough. Surrounding this central active part is a metal cylinder in which is an aperture through which the X-rays pass: this cylinder (it is at cathode potential) acts as a shield to the glass, preventing electrons (and

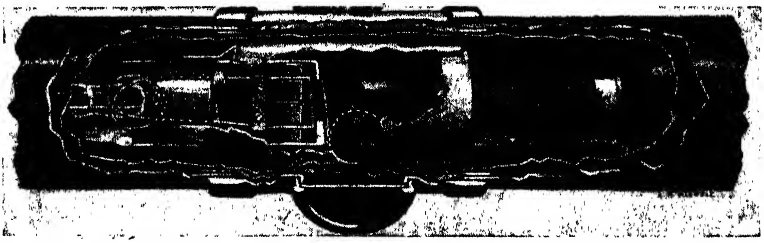


FIG. 526. Construction of a Modern X-Ray Tube (Cuthbert Andrews).

ions) from striking and damaging the glass tube. Next to the glass is a cylinder of lead (with aperture), and finally comes the outer case, the central portion of which is of brass and the end portions of insulating and radiation-proof material. Fig. 527 shows a complete tube fitted with water-cooling for the anti-cathode: other patterns use air-cooling.

As the modern tube depends for its fast moving electrons on

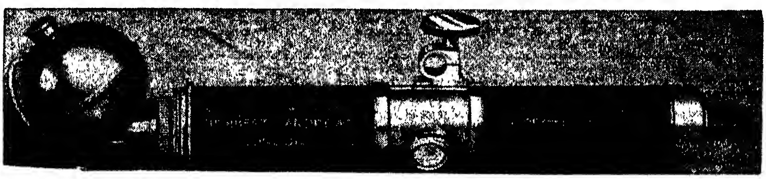


FIG. 527 The "Water-cooled" Anti-cathode Pattern.

emission from a heated cathode, it acts as its own rectifier or valve, and A.C. may therefore be used with them. X-rays can inflect injury to the skin if exposure to them is too long or frequent.

3. The Origin of X-Rays

On pages 7 and 8 we saw that the electrons outside the nucleus of an atom normally occupy a series of "energy shells *or* layers *or* levels," the K level being innermost, nearest the nucleus, that it

requires more energy to detach an electron from an inner level than from an outer (naturally—being nearer the positive nucleus), and that it is the outer-level electrons which mainly take part in our experiments. Suppose, however, that in some way or other an electron *has been* ejected from one of the inner levels and made to occupy the next outer level: energy would be required to do this. Now suppose that under the attraction of the nucleus an electron "rushes" into the first level to fill the vacancy and bring the atom back to its normal state: then an equal amount of energy is given out, this energy being mainly *radiated* from the atom and passing onwards as an electromagnetic wave with the velocity of light. When an electron "rushes" to a level near the nucleus the energy emitted is greater than if the rush takes place between two levels further from the nucleus, and the radiation given out is of higher frequency and shorter wave-length.

It is difficult to dislodge an electron from the K level of an atom, *but it is done* by the *very fast moving* electrons in the cathode stream when they bombard the anti-cathode, for they have considerable energy and penetrate right into the atoms of the anti-cathode. Thus the ensuing "electron-rushes" (either by the atom's own electrons from other levels or by electrons in the stream) are *"rushes" in connexion with innermost levels,* and the energy given out is very great: that is the radiation given out is of high frequency and short wave-length and very penetrating—in fact it is our X-rays. Further, the energy liberated when an electron enters, say, the K level of an atom which has a big positive charge on its nucleus will be greater than if it enters the K level of an atom with a less positive nucleus: it will be bigger with tungsten (nucleus free charge + 78) than with copper (+ 29). Hence the anti-cathode of an X-ray tube is made of a material of high atomic weight—tungsten or molybdenum.

To remove a K electron from tungsten the electrons of the cathode stream must travel under a P.D. of about 100,000 volts so that the applied voltage must be of this order. With a less voltage the K electrons will not be displaced, only the L or M, etc., and the radiation given out will not be so penetrating. The radiation from an atom which we call "light" is due to outer-level electron disturbances: light is therefore of lower frequency and longer wave-length than X-rays and not so penetrating. Very penetrating X-rays are spoken of as "hard," less penetrating as "soft."

The Quantum Theory. If E be the energy liberated by an "electron rush" as indicated above, and *f* is the frequency of the radiation given out

then E is proportional to f or $E = hf$, where h is a constant known as **Planck's constant**: its value is $6{\cdot}55 \times 10^{-27}$ erg-sec. In modern Physics we say that the radiation due to a " rush " in this way of one electron travels out (with the velocity of light) carrying a **quantum** of energy represented by hf. This fact that radiation passes out in bundles or quanta of energy each $= hf$ has led to the suggestion that radiation should be regarded as of the " particle " character rather than as waves. The **quantum theory** cannot, however, explain certain facts (*e.g.* interference and diffraction) as the wave theory can. The conclusion seems to be that in its *transmission* radiation must be regarded as a wave-motion, whilst in its *emission* and *absorption* the quantum theory applies. But this is a subject—and it is an important one in modern Physics —which is outside the scope of this book. Incidentally, just as *radiation* (wave motion) appears under certain conditions to be corpuscular, *i.e.* to be *bundles* or *quanta* or *photons* of energy, so our fundamental particle, the *electron*, appears under certain conditions as if it were a wave motion: but this, again—a subject known as **wave mechanics**—is beyond this book.

4. Radioactivity

Before proceeding with this section the student must read again, and thoroughly grasp, the facts given in an elementary way about atomic structure on pages 5-17: this is necessary in order to understand, and realise the significance of, the details which follow. Note the " energy shells " or " energy levels " of the electrons outside the nucleus of the atom; the numerical equality between the " net " or " free " positive charge at the nucleus and the total negative charge of all the electrons outside; the connexion between the *atomic number* and the nucleus *free* positive charge and that between the *mass number* (corresponding to *atomic weight*) and the *total* positive charge of the nucleus; the formations of *isotopes*; and the groupings of the protons and electrons *in* the nucleus giving as the probable " make up " of a nucleus alpha (α) particles (helium nuclei), protons (hydrogen nuclei), electrons, and neutrons. All these points are of importance at this stage.

The work of the preceding pages which has involved " changes " taking place in an atom has been concerned with changes in connexion with electrons *outside* the nucleus. As already indicated, the *nucleus itself* is very stable and in many cases has, so far, not been broken up, although it has been done in the laboratory in some cases and research is still going on in that direction. However, there are some substances with *very heavy atoms* in which the nuclei are not so stable: in fact their nuclei are gradually undergoing a breaking-up process without any effort on our part. These atoms are said to be **radioactive**, and to be undergoing " radioactive disintegration." It would appear that the atomic number of an element must be at least 82 for this *spontaneous* disintegration, *i.e.* by Nature and not by man, to take place.

In 1896 Becquerel discovered that ores of ***uranium*** gave out "something" which affected a photographic plate and ionised a gas, and in 1898 he discovered the same for ***thorium***. Then M. and Mme Curie working with pitchblende (a uranium ore) discovered a new element to which they gave the name ***radium*** which showed similar properties, and a further one which they called ***polonium***. Yet another was noted by Debierne who named it ***actinium***. These five substances led to some brilliant research work about that time.

Investigations on the "emanations" of these ***radioactive substances*** rapidly proceeded, and it was found that they were of three kinds—

(1) A part fairly easily absorbed by metal foil, *e.g.* by about ·01 cm. of aluminium foil. To this the name **alpha** (α) **rays** was given (we now refer to them as *α particles*): they were deflected by electrical and magnetic fields in directions which showed that they were *positively charged* (Figs. 528, 529).

(2) A part much more difficult to absorb: it took, for example, about ·5 cm. of aluminium for complete absorption. To this the name **beta** (β) **rays** was given (now referred to as *β particles*): they were found to be ***negatively charged*** (Figs. 528, 529).

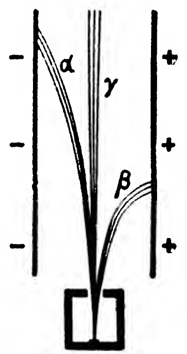


FIG. 528. Electric.

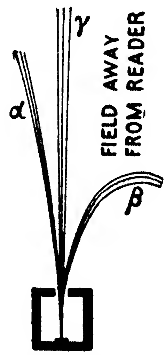


FIG. 529. Magnetic.

(3) A part hardly absorbed at all by the foil—which, in fact, can sometimes penetrate 30 cm. of iron. To this the name **gamma** (γ) **rays** was given: they were found to be *uncharged* (Figs. 528, 529).

Further investigation (details of which are not necessary for the purpose of this book) on the e/m, e, m, and v values proved that the α particle is the ***nucleus of a helium atom which is ejected from the nucleus*** of the atom of the substance with a velocity of the order of 10^9 cm. per sec. The β particle ***is an electron also ejected from the nucleus*** of the atom of the substance with a velocity which may be as high as ·998 of the velocity of light. The γ rays ***are electromagnetic waves*** (like light) which are ***set up when an electron*** (β ***particle***) ***is suddenly ejected from the nucleus*** of the atom of the

substance: they are penetrating radiation of higher frequency and shorter wave-length (of the order 10^{-10} cm.) than X-rays, and travel with the velocity of light. Note particularly that in these radioactive phenomena we are encountering ejections *from the nucleus* of the atom—the disintegration of atomic nuclei.

(1) *Ejection of an α particle from a nucleus*:—Consider the nucleus of an atom of a radioactive substance, say radium, ejecting an α particle. The α particle is a helium nucleus. Now the helium nucleus has a *free* or a *net* positive charge numerically equal to 2 electron charges, and the *mass number* corresponding to the *atomic weight* is 4 (the atomic weight is practically due to the nucleus in all atoms). So far as mass number or atomic weight is concerned, then, it will be 4 less than it was before the α particle was ejected. Further, the nucleus free positive charge has decreased by 2 owing to the ejection of the α particle: this means that there are now 2 electrons outside the nucleus more than it can balance, and these 2 outside electrons will leave the atom. In short, when the radium nucleus throws out the α particle the atomic weight is less (4 less), the atomic number is less (2 less), and there are less electrons outside the nucleus (2 less): and remember that the chemical properties of any substance depend on these outer electrons (*i.e.* on atomic number). In fact *after the ejection of the α particle the atom is no longer a radium atom: it is a new atom, a new substance,* and it is called **radon** or *radium-emanation*.

Radon also is radioactive: it ejects an α particle from its nucleus and becomes another atom—another substance called *radium A*. Then follow in succession further transformations from one kind of atom to another, and the final product is *lead* which undergoes (practically) no further radioactive change. During the successive changes from the radium to lead *five* α particles are ejected. This means that the atomic weight of this lead should be 20 less than radium: the figures actually are radium 226, this lead 206.

As a matter of fact, radium itself comes from uranium by a series of successive disintegrations. During the process three α particles are expelled on the whole, and this means that the atomic weight of radium should be 12 less than uranium. The atomic weight of uranium is about 238 and of radium 226.

(2) *Ejection of a β particle from a nucleus*:—When a β particle, *i.e.* an electron, is ejected from a nucleus, the atomic weight will (practically) not change but the *free positive charge* of the nucleus (atomic number) will *increase* by one, *i.e.* by an amount numerically

equal to the *negative* electron expelled. It will therefore be able to balance one more outer electron than it could balance before, and hence the atom again acquires different properties—it is the atom of another substance.

Some of the transformations which take place in radioactive phenomena are brought about by the ejection of α particles, some by the ejection of β particles and some by the ejection of both, and of course γ rays accompany some of the changes. We can now consider a set of transformations in greater detail.

5. Radioactive Changes

(1) URANIUM SERIES.—Uranium, our heaviest element (atomic weight 238: atomic number 92), is slowly—very slowly—disintegrating. The nucleus of its atom ejects an α particle and becomes a new atom—an atom of *Uranium* X_1, with an atomic weight 4 less (*i.e.* 234) and an atomic number 2 less (*i.e.* 90) than Uranium. If we started with a given amount of uranium, then in about 5000 million years half the atoms would have disintegrated, whilst after the next 5000 million years half the remaining atoms would have disintegrated, and so on: we say that the *half value period* of uranium is 5×10^9 years.

Uranium X_1 disintegrates (half value period = 24·6 days) into *Uranium* X_2 by ejecting a β particle (electron) from the nucleus of its atom: this does not alter the atomic weight but it *increases* the atomic number by 1 (*i.e.* to 91). Uranium X_2 disintegrates (half period = 1·2 minutes) into *Uranium II.* by ejecting a β particle which again does not alter the atomic weight but *increases* the atomic number by 1 (*i.e.* to 92). Thus the atomic weight of Uranium II. is 4 less than Uranium, but its atomic number (*free* +ve at nucleus and balanced electrons outside) is the same, viz. 92: Uranium II. is therefore an isotope of Uranium.

Uranium II. disintegrates by the ejection of an α particle into *Ionium*, and this, by an α particle ejection, into *Radium*. Between Uranium and Radium 3α particles have been ejected so that the atomic weight of Radium should be 226, which it is. Further, the 3α particles mean a decrease of 6 in atomic number, but 2β particles were ejected, raising the atomic number by 2, so that the atomic number of Radium should be 4 less than Uranium, *i.e.* 88, as it is. The student should now be able to follow the whole of the disintegrations of this series from the following table and Fig. 530.

In the table the first number below each is the mass number (corresponding to atomic weight) and the second the half value period (y = years, d = days,

h = hours, m = minutes). Fig. 530 will provide a further help to the student: horizontal arrows mean the ejection of an α particle (with a *drop* in atomic weight of 4 and in atomic number of 2): vertical arrows mean the ejection of a β particle (with no change in atomic weight but a *rise* in atomic number of 1): substances vertically under each other have the same atomic weight, whilst those joined by sloping dotted lines have the same atomic number, *i.e.* are *isotopes.* The final product is lead (Radium G) of atomic weight 206, atomic number 82.

Ur. $\xrightarrow{\alpha}$ Ur. X_1 $\xrightarrow{\beta}$ Ur. X_2 $\xrightarrow{\beta}$ Ur. II. $\xrightarrow{\alpha}$

Ur.: 238, $5 \times 10^9 y.$; Ur. X_1: 234, 24·6*d.*; Ur. X_2: 234, 1·2*m.*; Ur. II.: 234, $4 \times 10^6 y.$

Io. $\xrightarrow{\alpha}$ Ra. $\xrightarrow{\alpha}$ Ra. Em. (Radon) $\xrightarrow{\alpha}$ Ra. A. $\xrightarrow{\alpha}$

Io.: 230, $10^5 y.$; Ra.: 226, 1730*y.*; Ra. Em.: 222, 3·85*d.*; Ra. A.: 218, 3*m.*

Ra. B $\xrightarrow{\beta}$ Ra. C $\xrightarrow{\beta}$ Ra. C′ $\xrightarrow{\alpha}$ Ra. D $\xrightarrow{\beta}$

Ra. C $\xrightarrow{\alpha}$ Ra. C″ $\xrightarrow{\beta}$ Ra. D

Ra. B: 214, 26·7*m.*; Ra. C: 214, 19·5*m.*; Ra. C′: 214; Ra. C″: 210, Rapid.; Ra. D: 210, 16*y*

Ra. E. $\xrightarrow{\beta}$ Ra. F (Polonium) $\xrightarrow{\alpha}$ Pb (Ra G)

Ra. E.: 210, 4·85*d.*; Ra. F: 210, 136*d.*; Pb: 206

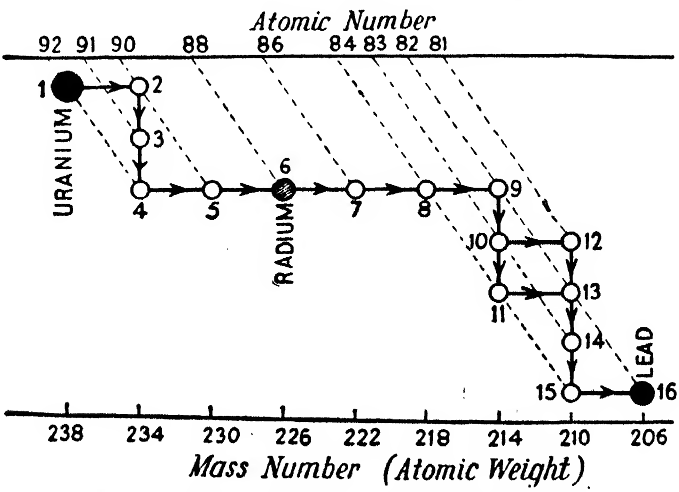


FIG. 530. (1) Uranium; (2) Uranium X_1; (3) Uranium X_2; (4) Uranium II. (5) Ionium; (6) Radium; (7) Radium Emanation (Radon); (8) Radium A; (9) Radium B; (10) Radium C; (11) Radium C′; (12) Radium C″; (13) Radium D; (14) Radium E; (15) Radium F; (16) Radium G (lead).

(2) THORIUM SERIES.—The disintegrations here will be readily understood from the preceding and from the following table: elements with a dotted line connexion are isotopes (same atomic number). The final product is lead (Thorium D) of atomic weight 208, atomic number 82.

ELEMENT	ATOMIC WEIGHT	ATOMIC NUMBER	TYPE OF EJECTIONS
Thorium	232	90	α
Mesothorium 1 ..	228	88	β
Mesothorium 2 ..	228	89	β
Radiothorium ..	228	90	α
Thorium X ..	224	88	α
Thoron	220	86	α
Thorium A ..	216	84	α
Thorium B ..	212	82	β
Thorium C ..	212	83	α.β
Thorium C′	208	81	β
Thorium C″	212	84	α
Lead (Thorium D)	208	82	—

(3) ACTINIUM SERIES.—The starting point in this Series is *Protoactinium* (atomic weight 231, atomic number 91). This disintegrates to actinium and successive disintegrations follow, the final product being lead (Actinium D) of atomic weight 207 and atomic number 82.

ELEMENT	ATOMIC WEIGHT	ATOMIC NUMBER	EJECTIONS
Protoactinium ..	231	91	α
Actinium	227	89	β
Radioactinium ..	227	90	α
Actinium X ..	223	88	α
Actinon	219	86	α
Actinium A ..	215	84	α
Actinium B ..	211	82	β
Actinium C ..	211	83	α.β
Actinium C′	211	84	α
Actinium C″	207	81	β
Lead (Actinium D)	207	82	—

Note that the end product in each of the above cases is an isotope of lead (atomic number 82), the three atomic weights being 206, 208, and 207: the atomic weight of ordinary lead is 207·22.

6. Disintegrating Nuclei in the Laboratory

In 1919 Rutherford showed that it was possible to break up an atomic nucleus *in the laboratory* (as distinct from *natural* radioactivity) by bombarding it by fast moving α particles from a radioactive substance. If a fast moving (positive) α particle collides with a nucleus which has only a small positive charge (say *nitrogen*—atomic number 7, atomic weight 14) it may eject a (positive) proton from the nucleus: this only happens if the nucleus positive charge is small so that the α particle can get quite close to it before being repelled, and even then the ejection of the proton is not a frequent occurrence (see page 543, Art. 5). This ejection of a proton from, say, the nucleus of the nitrogen atom would decrease the atomic weight by 1 and the atomic number (free +ve at nucleus) by 1, so that we again have a transformation to another atom just as in radioactivity.

However, when an α particle ejects a proton it generally joins up into the nucleus itself, *and this makes the transformation different.* Taking the nitrogen case above, the nucleus gains 2 +ve charges due to the α particle absorbed and it loses + 1 due to the ejected proton, so the net gain is + 1. The mass number increases by 4 due to the α particle, and decreases by 1 (ejected proton), so the increase is 3. Our new nucleus has therefore an atomic number of $(7 + 1) = 8$, and a mass number $(14 + 3) = 17$. But oxygen has an atomic number 8 and atomic weight 16. We have therefore *by an experiment* brought about a change from a nitrogen atom to an oxygen atom (isotope): this isotope of oxygen (atomic weight 17) is present in ordinary air to a certain extent.

Again, experiments showed that when the nuclei of certain *very light* atoms (*e.g.* boron and beryllium) were bombarded by α particles **neutrons** (*i.e.* particles having *the same mass as a proton but no charge*) were ejected: a neutron is a proton and electron combined. Now when a nucleus of, say, boron (atomic weight 11, atomic number 5) is bombarded by an α particle the latter joins up with the nucleus. The mass number increases by 4, due to the α particle, and decreases by 1 (the proton of the ejected neutron), so the net increase is 3. The nucleus gains 2 +ve charges due to the α particle, whilst the loss of the neutron (+ 1 and − 1) will not affect its resultant free charge, so the total *free* charge change is an increase of + 2. Our new nucleus has therefore a mass number 14 and atomic number 7, *i.e.* the boron nucleus has changed to a nitrogen nucleus, and the boron atom to an atom of nitrogen.

Using high speed protons (+ve) from a hydrogen discharge tube, and working with P.D.'s up to 500,000 volts (and over) nuclei of various elements have been bombarded and disintegrated. Thus when a rapidly moving proton bombards and enters the nucleus of an atom of lithium the nucleus is split up into two α particles which rush away with high velocity (Art. 7).

Gamma rays can produce disintegration of atomic nuclei: thus the nucleus of heavy hydrogen (the *diplon*—atomic weight 2, atomic number 1) has been broken up into a proton and a neutron by γ rays (page 15). Neutrons also have been used as the bombarding bullets in experiments (page 16).

7. Another Glance at the Nucleus of the Atom

In the sections on atomic structure (pages 11-17) it was indicated that the +ve and −ve charges—the protons and electrons—constituting the fundamental "make-up" of the nucleus of an atom were grouped or combined in various ways, and this is supported by the facts of radioactivity and the allied laboratory research dealt with in this chapter. To illustrate the conception we will examine in greater detail one or two of the elements mentioned above and in Chapter I.

(*a*) The *atom of helium* has an atomic weight 4, an atomic number (free +ve at nucleus) 2, and 2 electrons outside the nucleus (and its nucleus is, of course, the *α particle*). The nucleus can therefore be accounted for by 4 protons + 2 electrons, but it can also be explained by 2 protons + 2 neutrons as diagrammatically represented in Fig. 9 (*b*), and this is now usually taken as the make up.

(*b*) The *atom of nitrogen* has an atomic weight 14, an atomic number 7, and there are 7 electrons outside. Its nucleus is therefore 14 protons + 7 electrons, but it is also accounted for by:—

$$3\alpha \text{ particles} + 2 \text{ protons} + 1 \text{ electron},$$

the 3α particles accounting for 12 protons and 6 electrons: it can also be accounted for by—

$$3\alpha \text{ particles} + 1 \text{ proton} + 1 \text{ neutron},$$

the neutron consisting of 1 proton + 1 electron, and this is now generally taken as the make-up of the nucleus.

Again, we have seen that when the nitrogen nucleus is bombarded by α particles an α particle joins up with it, ejects a proton, and the result is the nucleus of an atom of oxygen (isotope) of atomic weight 17, and, of course, atomic number 8: this may be represented as follows where P = proton, E = electron, and N = neutron:—

$$\underbrace{(1\alpha,\ i.e.\ 2P + 2N)}_{\text{Bombarding } \alpha \text{ particle.}} + \underbrace{(3\alpha + 1P + 1N)}_{\text{Nitrogen nucleus.}} \rightarrow \underbrace{(4\alpha + 1N)}_{\text{Oxygen nucleus.}} + \underbrace{1P.}_{\text{Ejected proton.}}$$

The 4α particles + 1 neutron of the oxygen nucleus account for (16P + 8E) + (1P + 1E), *i.e.* for 17 protons + 9 electrons, giving an atom of atomic weight 17 and free positive at the nucleus 8 (atomic number) with 8 electrons outside (oxygen isotope). It is usual to represent the nucleus change above as follows:—

$$_2He^4 + {}_7N^{14} \longrightarrow {}_8O^{17} + {}_1H^1,$$

the small number in front being the atomic number, and the small number behind, the mass number (corresponding to atomic weight): the α particle is, of course, a helium nucleus, and the proton a hydrogen nucleus. We can also represent the change pictorially (Fig. 531), as we did with the nuclei of Fig. 9.

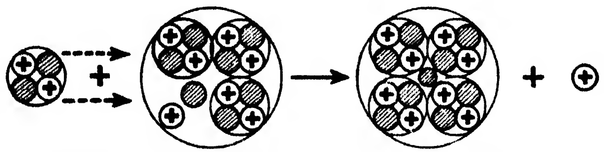


FIG. 531.

(*c*) One more example will suffice. Lithium has an atomic weight 7 and an atomic number 3, so its atom is explained by 7 protons + 4 electrons in the nucleus and 3 electrons outside: its nucleus would also be explained by—

1α particle + 1 proton + 2 neutrons.

Further, we have seen that when the lithium nucleus is bombarded by a fast moving proton two α particles are the result. In this bombardment the proton joins up with the nucleus and the change may be represented thus:—

$$\underset{\text{Bombarding proton.}}{(1P)} + \underset{\text{Lithium nucleus.}}{(1\alpha + 1P + 2N)} \longrightarrow \underset{\alpha\text{ particle.}}{(1\alpha)} + \underset{\alpha\text{ particle.}}{(2P + 2N)},$$

$$or \quad {}_1H^1 + {}_3Li^7 \longrightarrow {}_2He^4 + {}_2He^4.$$

8. The Range of Electromagnetic Waves

It will be interesting to summarise here the electromagnetic radiations mentioned in this and preceding chapters. All are akin to light, have the same velocity (3×10^{10} cm. per sec.), and differ only in frequency and wave-length. They are given in *increasing* order of wave-length and *decreasing* order of frequency (Velocity = Wave-length $\times$ Frequency). The figures quoted for wave-lengths and frequencies are only approximate and of a general character for comparisons, for each group spreads over a range of values.

(1) COSMIC RAYS.—These are the shortest in wave-length and the highest in frequency, and reach us from remote space. Millikan considers they are due to protons and electrons joining up to form nuclei when atoms of helium, etc., are "born." Their frequency is enormous, and the wave-length of the order 2×10^{-12} cm.

Cosmic rays seem to have a specially interesting property, viz. that they *sometimes* break up a nucleus and eject what is called a positron—a particle with a *positive unit charge* (like a proton) but having *the same small mass as an electron*. Positrons are rarely met with and are very elusive, having an exceedingly short life: we do not yet know much about them, or the full explanation of their origin, or how they fit into the story of the atom.

(2) GAMMA RAYS FROM RADIOACTIVE BODIES.—These are produced when an electron is ejected *from the nucleus* of the atom of these bodies. Their frequency is of the order 2×10^{18} to 2×10^{21}, and their wave-length again a very small fraction of a millimetre, although longer than the cosmic (about 4×10^{-10} cm.).

(3) X-RAYS.—These are produced when the *inner energy level of a heavy atom* (*i.e.* one with a large positive nucleus) having lost an electron captures another to fill the vacancy. Their frequency is of the order 10^{16} to 10^{19}, and their wave-length still a small fraction of a millimetre, (of the order 10^{-9} cm.).

(4) ULTRA-VIOLET RAYS, LIGHT RAYS, INFRA-RED RAYS.—The rays of the visible spectrum range from the shortest waves, violet (with a wave-length of ·0004 mm.), through indigo, blue, green, yellow, and orange to the longest waves, red (with a wave-length of ·0008 mm.). The ultra-violet are waves shorter than the violet, and the infra-red waves longer than the red. These rays are produced when *one of the more outer energy levels* of an atom captures an electron to fill a vacancy caused by an electron having been removed from it. Including the ultra-violet and infra-red the frequency ranges from 10^{12} to 10^{16}.

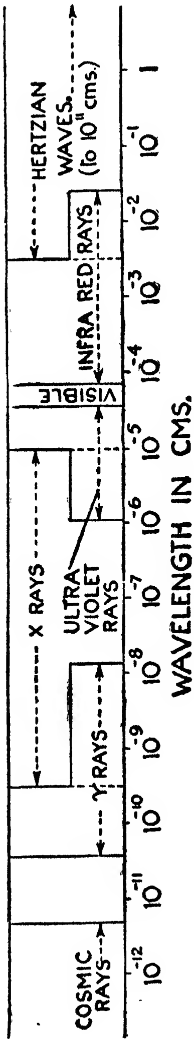


FIG. 532.

(5) WIRELESS (HERTZIAN) WAVES. — These are radiated *from conducting circuits* in which very high frequency electric currents are circulating (high frequency electrical oscillations). Including the very short wireless waves used in laboratories and in television, we may say the frequency ranges from about thirty thousand million (very short waves) to 100,000 (long waves). Still longer waves are those from an alternator. This group is dealt with in the next chapter.

CHAPTER XXI

RADIATION FROM HIGH FREQUENCY CURRENT CIRCUITS—WIRELESS AND TELEVISION

THESE subjects have made such strides in recent years that they necessitate works specifically devoted to them. As no modern book on Electricity, however, is complete without some reference to them, we will briefly glance at a few essential principles. Read again Arts. 13 and 14 of Chapter XVIII.

1. Damped Waves from H.F. Oscillatory Current Circuits

On pages 529-534 it was shown that if a charged condenser of capacitance C be discharged through a circuit of resistance R and inductance L the discharge is *oscillatory* (and of high frequency—H.F.) if R is *less than* $\sqrt{4L/C}$, the frequency being given by:—

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}},$$

which becomes $f = 1/2\pi\sqrt{LC}$ if R be zero or negligible. At each oscillation the electrostatic energy of the charged condenser is converted into electromagnetic energy (as current flows), reconverted into electrostatic (condenser recharged in opposite direction), and so on. During the process some energy is dissipated as heat in the resistance. Further, these periodic changes at H.F. in the electric and magnetic fields give rise to an *electromagnetic radiation* which is propagated through space as a wave motion with the velocity of light. Thus the oscillations during the discharging process gradually weaken and die down: such are referred to as **damped oscillations,** and the waves sent out as **damped waves.** Fig. 533 might represent such a set of oscillations and waves.

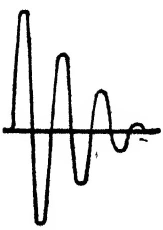


FIG. 533.

If we modify the L and C circuit of Fig. 497, imagine a spark gap in it, and picture the electric and magnetic lines during the oscillatory spark discharge of C (Fig. 534), the student will be better able to visualise the facts stated above. Note that at discharge the condenser is charged first one way then the other, that we have tubes (or lines) of electric force at (*a*), (*c*), (*e*), and magnetic tubes at (*b*) and (*d*), that the two sets of " strains " are at right angles

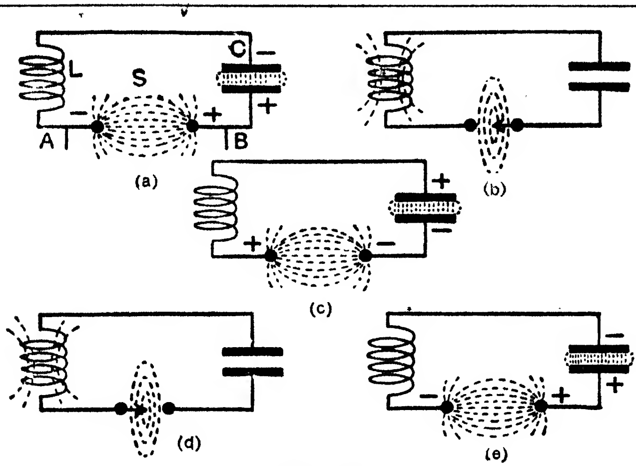


FIG. 534.

to each other, and that each is varying periodically—rising, falling, reversing. And Maxwell showed mathematically that under such conditions and with a suitable circuit we are bound to have an "electromagnetic disturbance" spreading out—a "radiation travelling as a wave motion through space (or in the *aether* of space)," with the velocity of light.

Imagine, now, that the two wires A and B (Fig. 534) are joined to a battery and a key. On closing the key the condenser is charged until the P.D. can overcome the gap resistance: then a spark discharge occurs which is oscillatory and waves pass out. When the oscillations have died down, the battery again charges up the condenser, and the action is repeated. (Choke coils would have to be included in the battery circuit to prevent oscillations passing back through the battery.)

Such a succession of oscillations (and waves) might be represented as in Fig. 535. We are assuming for simplicity later that the frequency of the oscillations at *each* discharge is 1,000,000 cycles per second (the "radio or wireless frequency," as we call it) and that the sparking occurs at the rate of 1000 per second.

An oscillatory circuit such as Figs. 497 or 534, where the condenser plates are fairly close together and the electric field does not therefore extend to any appreciable distance, does not send out much energy as "radiation."

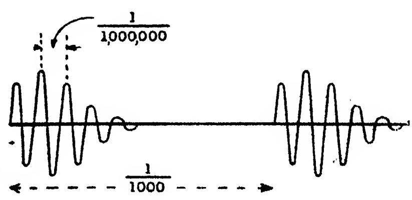


FIG. 535.

If, however, the condenser plates be opened out, forming an "open oscillatory circuit," we readily get energy passed out in the form of waves, and this is done in practice. See Fig. 536.

2. Wireless Telegraphy with Damped Waves

In 1888 Hertz published an account of certain experiments of his on such waves, and these are often looked upon as the starting point in *practical* "wireless."

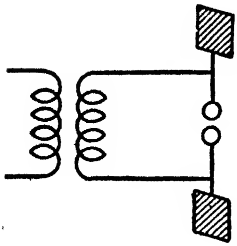


FIG. 536.

Hertz's oscillator or transmitter (Fig. 536) consisted of two metal plates (40 cm. square) connected by wires (30 cm. in length) to the spark gap (2-3 cm.). The gap was joined to the secondary of an induction coil. The capacitance between the two plates and the inductance of the connexions between them formed the necessary capacitance and inductance for the oscillatory circuit. When the induction coil is worked, a P.D. is set up across the gap, then a spark passes, and the potential is equalised by very rapid oscillations of electricity (electrons) in this circuit, and waves pass out.

To detect the waves at a distance Hertz used a circle of thick wire (Fig. 537) fitted with a gap. When the waves from the oscillator passed this receiver or detector they set up oscillations in it, and sparks were obtained at the gap. Further, he found that the sparks were strongest if he used a receiver having such dimensions, that its natural frequency as an oscillator was equal to the frequency of his oscillator. This is again an example of tuning or resonance.

Following this, Sir Oliver Lodge did some brilliant pioneer work, and with his improved apparatus demonstrated the transmission of waves over much greater distances at the British Association in 1894.

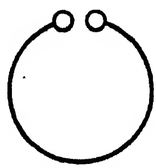


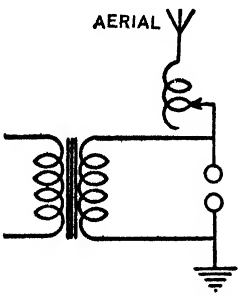
FIG. 537.

About 1896 Marconi discovered that if one of the spark gap terminals in the transmitter were connected to a plate buried in the ground, the earth could be used as one plate of the condenser; and also that the higher the other plate the greater was the distance over which he could transmit the waves. He also found that an upper plate was unnecessary, and that a vertical wire (the aerial) supported by a kite or masts gave similar results: a horizontal aerial acted in the same way. The aerial and earth form a condenser, the wire being one coat, the earth the other, and the air in between the dielectric. Fig. 538 shows the principle of Marconi's transmitting arrangement, the variable inductance in the aerial circuit enabling the circuit to

be adjusted to have various oscillation frequencies and thus to send out various wave-lengths.

A better transmitting arrangement is indicated in Fig. 539. Here the coil P of the oscillating circuit is " coupled " to a coil S in the aerial circuit, *i.e.* P is the primary and S the secondary of a transformer. The H.F. oscillations in the condenser circuit induce H.F. oscillating E.M.F.'s in the aerial circuit, and if the latter be " tuned to resonance " with the condenser circuit the aerial acts as a powerful radiator of a train of waves (Fig. 535).

When the waves from the transmitting aerial fall upon a receiving aerial they set up H.F. oscillatory E.M.F.'s and currents in it corresponding to those at the transmitting end, the receiving aerial circuit being again " tuned " to have the same frequency as that of the arriving waves. This is done by variable condensers (C) and inductances (L) at the receiver. Since wave-length = velocity/fre-



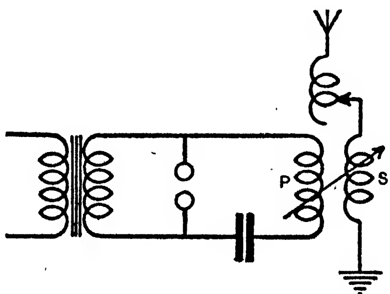


FIG. 538. Induction coil on Left in both Figures. FIG. 539.

quency, and frequency = say $1/2\pi\sqrt{LC}$ we have wave-length $\propto \sqrt{LC}$, so that increasing L or increasing C will tune the receiver to respond with maximum intensity to a longer wave.

In addition to the tuned receiving aerial there must be some device to register the arrival of the waves (and enable intelligible signals to be received) such as, for example, a telephone or a sounder. Now the oscillating currents we are dealing with are of much too high a frequency (of the order a million a second) to work, say, a telephone directly, for the vibrating disc would not respond to the rapid *changes in direction* of the current: thus the telephones would not respond to current changes like (*a*) in Fig. 540. If, however, the flow in one direction be suppressed, we will have a *rectified* or *unidirectional* but varying current (Fig. 540 (*b*)). The telephones will still be unable to respond to the rapid variations of the rectified current, but these may be looked upon as equivalent

to a (comparatively) slowly varying one direction current as at (*c*), and to this the telephones respond. Using the numerical value in Fig. 535, we have a rush of current through the telephones every $\frac{1}{1000}$ second, which tends to attract the iron disc as it rises and to allow the disc to go back as it falls. The telephones can respond to this (1000 per sec.), so that they give a note of this frequency (spark frequency).

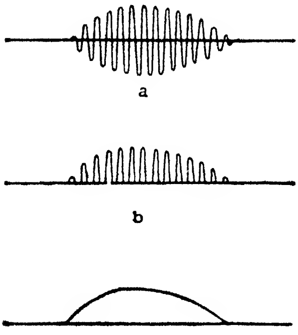


FIG. 540.

Clearly, a crystal or a valve may be used as the rectifier (Chapter XIX.), and Fig. 541 shows the principle of a very simple receiving circuit using a crystal. The condenser across the telephones is to by-pass any high frequency current away from the telephones. Valves are now mainly used as rectifiers.

The germ of **wireless telegraphy with damped oscillations and waves** will now be understood. The charging key in the transmitting circuit is kept closed for long or for short periods, thus sending out long or short trains of waves and producing long or short signals in the telephones or other receiving instrument, and combinations of longs and shorts (" dashes " and " dots ") form letters according to the Morse code. The time the operator keeps the key down in sending a dot signal is, say, $\frac{1}{8}$ sec., and in Fig. 535 we assumed there were 1000 spark discharges per second, so that even during the sending of a dot signal a definite train of waves passes out.

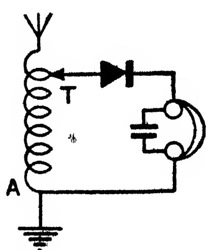


FIG. 541.

3. Wireless Telegraphy with Continuous Waves

Although cases of radio-*telegraphy* are still met with in which *damped* oscillations and waves are employed, all modern installations use ***undamped*** or **continuous oscillations and waves** as depicted in Fig. 542 (*a*). The methods of producing ***very high frequency*** oscillatory currents which are ***continuous*** and do not die down are referred to later, but one point should be noted at this stage so far as ***telegraphy*** is concerned. If such a wave as shown in Fig. 542 (*a*) falls on the receiving aerial a corresponding current is set up in it which is

rectified by the crystal, and practically a "uniform" unidirectional current passes through the telephones (not a "rising and falling" current as at (*c*) in Fig. 540). The telephones would thus merely give a click when the current began and ended but no continuous note while the current was passing—the "body" of the signal would be absent—and "longs" and "shorts" would be extremely difficult to read.

One method of surmounting this is to break up the wave into short trains of waves as in Fig. 542 (*b*), but what is known as the **heterodyne method** is now mainly in use. In this, the incoming oscillations (A of Fig. 543) are combined with others (B) of a slightly different frequency, and the effect of the two is to produce an oscillating current which increases and decreases at a rate depending on *the difference in the frequencies* (C): the telephones respond (*i.e.* to D) as in Art. 2.

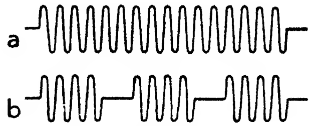


FIG. 542.

4. **Wireless Telephony and Broadcasting with Continuous Waves**

We will take as our starting point the fact that H.F. alternating currents which do not decrease in amplitude and die down are caused to circulate in the transmitting aerial: the wave passing out

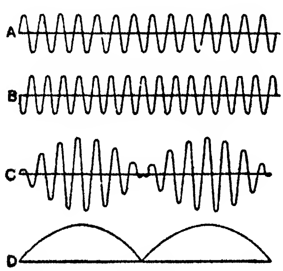


FIG. 543.

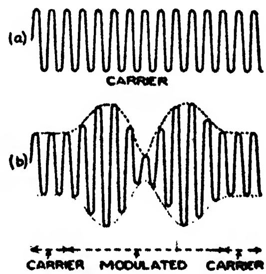


FIG. 544.

is called the **carrier-wave** of the station, and may be represented by (*a*) in Fig. 544.

The method of producing these H.F. *continuous* oscillations is referred to in Art. 6. What in effect is then done at the station is this:—The H.F. voltage of the **master oscillator,** as it is called, is passed to a series of valves each of which magnifies or amplifies the H.F. voltage passed to it from the preceding valve, and finally the magnified H.F. voltage is applied to the tuned aerial by coupling (Fig. 539) the last oscillatory circuit to it.

Joined in various ways to the transmitting apparatus and its aerial is a microphone. When a person speaks into the microphone (*e.g.* in broadcasting) the sound waves cause changes in the microphone current, and therefore corresponding variations in the aerial current, so that we have a H.F. oscillatory current in the transmitting aerial, but its strength is varying in a complicated way according to the words spoken. Thus the wave is similarly affected, and when it reaches the receiving aerial it sets up H.F. currents varying in strength according to the words spoken at the transmitting end. We say that the voice of the speaker or broadcaster **modulates the carrier wave**: this modulation of the carrier may be represented as at (*b*) in Fig. 544.

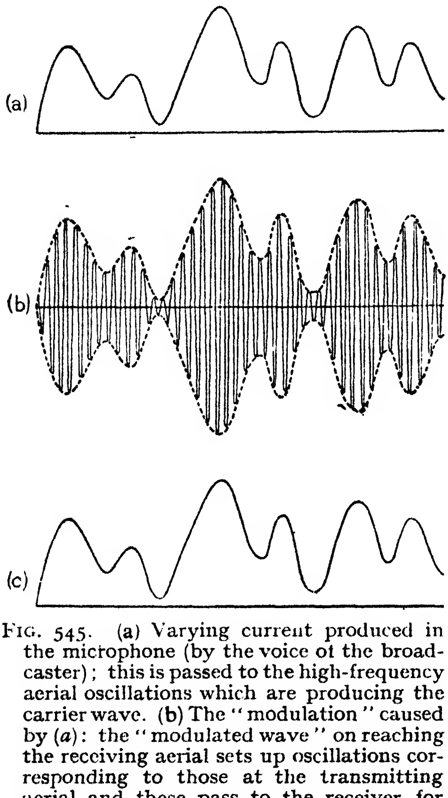


FIG. 545. (a) Varying current produced in the microphone (by the voice of the broadcaster); this is passed to the high-frequency aerial oscillations which are producing the carrier wave. (b) The "modulation" caused by (*a*): the "modulated wave" on reaching the receiving aerial sets up oscillations corresponding to those at the transmitting aerial and these pass to the receiver for amplification and rectification. (c) The varying current passed finally to the loud speaker: the sound obtained due to (*c*) is an exact copy (or should be) of the sound which caused (*a*).

The electrical oscillations in the receiving aerial (which has, of course, been tuned to the arriving wave) pass to the *rectifier* (the crystal of Fig. 541, but usually a valve known as the *detector valve*) which suppresses the surges in one direction leaving the others, still varying according to the speech, etc., to pass and to be applied to the telephones or loud speaker: the latter respond to the resulting slower strength variations and vibrate accordingly, and the words, etc., are reproduced. The curves in Fig. 545 will make the idea clear: the telephones respond to (*c*). Study this figure carefully.

Just as at the sending end the microphone variations and the oscillations are amplified or magnified by valves before passing to the transmitting aerial, so at the receiver the signals are amplified by valves both before and after the detector valve. We can now consider how the valve is used for this rectification and amplification.

Sidebands. But one more point should first be noted, particularly so far as broadcasting is concerned. The carrier wave for, say, the London Regional had (pre-war) a frequency of 877,000 cycles or 877 kilocycles. Now suppose the wave is modulated by, say, a vocalist producing in front of the microphone a note of 1200 cycles frequency (soprano). It can be shown that in the aerial there is now *what amounts to* the original frequency of 877,000 cycles and two others, one 1200 above the carrier frequency (viz. 878,200 cycles), the other 1200 below (viz. 875,800)—a total frequency spread of 2400 cycles. When we consider the complex modulation produced by an orchestra, it will be clear that whilst each station must have a fixed wave-length and frequency for its carrier, a certain *sideband* allowance must be given below and above the station frequency to accommodate the modulation effects, and the spacing between stations on that account is 9000 cycles or 9 kilocycles frequency. Thus the next station on one side of the London Regional is Graz (Austria) with a carrier frequency of 886 kilocycles—9 more than London, whilst the next on the other side is Poznan (Poland) with a carrier frequency of 868 kilocycles—9 less than London (this is the *pre-war* international agreement).

The mathematical student will be able to follow the " proof " of the above. Let the carrier frequency be p, and its *amplitude* (extent of " swing ") when there is no modulation, be A. Let the frequency of the modulation be ω and its amplitude K. The equation of the tangent curve will then be $y = (\mathrm{A} + \mathrm{K} \sin \omega t)$.

$$\text{Amplitude of H.F. current} = (\mathrm{A} + \mathrm{K} \sin \omega t).$$

$$\text{Equation of H.F. current: } i = (\mathrm{A} + \mathrm{K} \sin \omega t) \sin pt,$$

$$\text{i.e. } i = \mathrm{A}\,(1 + m \sin \omega t) \sin pt, \text{ where } m = \mathrm{K}/\mathrm{A};$$

$$\therefore\ i = \mathrm{A} \sin pt + \mathrm{A}m \sin pt \sin \omega t;$$

$$\therefore\ i = \mathrm{A} \sin pt - \tfrac{1}{2}\mathrm{A}m \cos (p + \omega)\, t + \tfrac{1}{2}\mathrm{A}m \cos (p - \omega)\, t.$$

Thus the high frequency wave varying in amplitude at low (*e.g.* " sound ") frequency can be represented as consisting of three waves: p is the " carrier " frequency, and the wave A sin pt is the *carrier wave*: $(p + \omega)$ is termed the *upper " side " frequency*, and $(p - \omega)$ the *lower " side " frequency*: all this was illustrated numerically above. The side frequencies when a carrier is modulated are spoken of as the **sidebands.**

5. The Action of a Triode Valve

The *diode valve* was dealt with in Chapter XIX.: the ordinary valve used in wireless, however, is the **triode** (or some modification of it). It consists of a *plate* or *anode* (P) and a *filament* or *cathode*

(F), but a wire *grid* (G) is placed between them, as shown diagrammatically in Fig. 546 (the filament is, of course, electrically heated by, say, a battery—the *low tension battery* of a receiver—not shown in the figure). Let the heated filament and plate be joined to the battery on the right (*high tension battery*), P being positive, so that electrons pass from F to P in the valve, and a plate current flows.

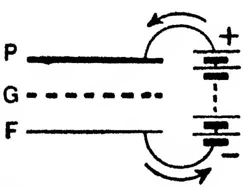


FIG. 546. *Conventional* current indicated.

Imagine G acquires from some outside source a negative potential. It will repel the electrons coming from F, and the plate current will decrease. If the potential of G becomes positive it will attract the electrons from F, and (as P is at a higher potential in practice), many continue their motion through the grid holes to the plate; thus the plate current will increase. To summarise—*lowering the grid potential reduces the plate current: raising the grid potential increases the plate current* (study Fig. 547). The characteristic curve of the valve is indicated in Fig. 548. When the grid has the negative potential OB it repels practically all the electrons from F, and there is no plate current. As the potential of the grid rises to zero (O), the plate current increases to the value OA. As the grid potential becomes more positive the plate current rises according to the curve ACD.

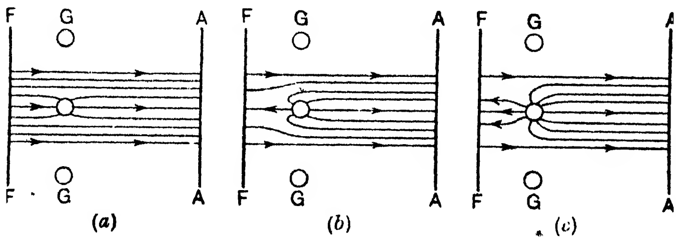


FIG. 547. (*a*) Grid slightly + ve; (*b*) Grid slightly − ve; (*c*) Grid more − ve. F = Cathode (filament), G = Grid, A = Anode. For simplicity the grid "holes" are exaggerated, and only a portion of the field depicted. Arrows show direction of force on an electron.

In a receiver the varying oscillations in the aerial due to the arriving waves are passed on to the grid of a valve so that the grid potential is varying accordingly: this immediately produces corresponding variations in the plate circuit, and it is plate variations which, after the processes of rectification and amplification have been done, are finally passed to the telephones or loud speaker.

(1) USING A TRIODE FOR AMPLIFYING.—Consider the *steepest* part of the valve curve shown in Fig. 549. Suppose the grid (when no signals are being received) to be at a negative potential OA (this is done, say, by joining the grid to the negative of a small dry battery known as the *grid bias battery*). A steady plate current will be passing, represented by AF. Now suppose wireless waves arrive and oscillations such as shown at P are conducted to the grid. These will vary the voltage of the grid, the positive half-waves causing the grid potential to be higher and equal to OB, say, and the negative half-waves causing it to be an equal amount lower, *i.e.* equal to OC. When the grid potential is OB the plate current will have increased to BE, and when the grid potential is OC the plate current will have fallen to CD. Thus as the grid *potential* swings to and fro (due to the arriving waves) between OB and OC, the plate *current* swings between BE and CD. As we are working on the *steep part* of the curve, a small jump from C to B causes a big jump from D to E. It will be seen by comparing the two curves P and Q, where P represents the *potential swings applied to the grid* from the aerial, and Q the *current swings in the plate circuit*, that the swing of Q will be greater the steeper the curve: this *indicates* amplification. *For amplification the valve should be so arranged that the steep part of its curve is used.*

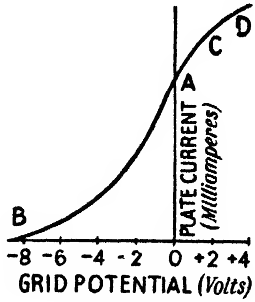


FIG. 548.

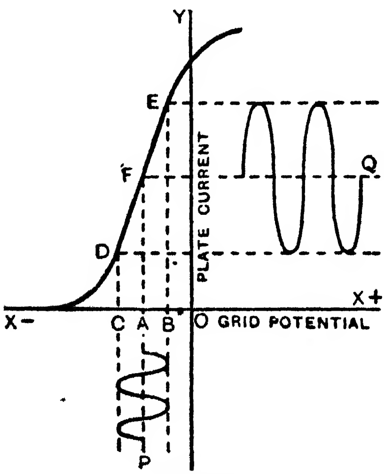


FIG. 549.

To get a proper idea as to the "magnification" the *current* swings Q must be converted into *potential* swings and these compared with the grid *potential* swings P which cause them. If a small change x volts applied to the grid causes a certain change in the plate current, and a change of y volts on the plate is necessary to bring about the *same* change in the plate current, then y is invariably bigger than x and the ratio of y to x is usually taken (although not strictly accurate)

as the **amplification factor** (μ) of the valve. Further, if a small change y in the plate potential causes a small change z in the plate current, the ratio y to z measures the **differential resistance** or *impedance* (R_a) of the valve. Now it can be proved that if R be the resistance in the plate circuit of our valve, R_a the differential resistance of the valve, and μ the amplification factor:—

$$\text{Change in plate potential} = \mu \frac{R}{R_a + R} \times \text{Change in grid potential,}$$

and if R_a is small compared with R so that we can neglect it:—

$$\text{Change in plate potential} = \mu \times \text{Change in grid potential.}$$

Thus if a valve is arranged for amplification and worked under these conditions, and if the manufacturer quotes its amplification factor as 50, then a change of 1 volt *potential* applied to the grid produces a change of 50 volts in the plate *potential*. This is "amplification."

(2) USING A TRIODE FOR RECTIFICATION.—Consider now the parts of the valve curve where the *bending* is greatest, *i.e.* the "knees." To simplify the idea we will first exaggerate these

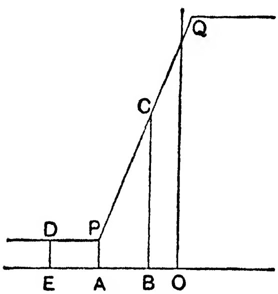


FIG. 550.

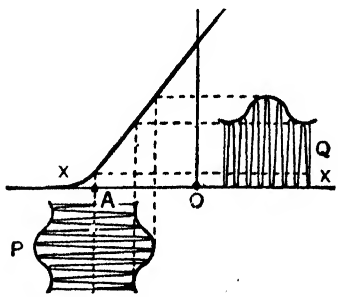
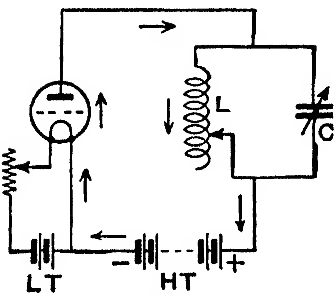


FIG. 551.

"knees" P and Q as in Fig. 550. With this "curve" let the grid potential be negative and equal to OA when no waves are passing, in which case the plate current is AP. When wireless waves arrive the negative half-wave lowers the potential of the grid to OE, but the plate current does not alter—it is ED, the same as AP. When the positive half-wave arrives the grid potential rises to B and the plate current becomes BC. We thus get a plate current variation from AP to BC when the positive pulses arrive, but no change when the negative pulses arrive. The valve is therefore responding, as it were, to swings in one direction only: it is *rectifying*. Fig. 551 shows the case less exaggerated: the swings at Q below XX are practically wiped out. To utilise this rectification action *the value must be arranged so that the bending part of the curve is used*. There are other methods of using a valve for rectification, but the above will answer the purpose of this book.

There are now many modifications of the triode or three electrode valve (plate, grid, filament), *e.g.* the screened-grid valve has two grids, the pentode valve has three grids, and so on, but for these and their specific purpose some book on Wireless must be consulted. (See *Wireless : Its Principles and Practice.*)

In a simple receiver the comparatively weak aerial oscillations are applied between the grid and filament of the first valve arranged, say, for amplification (**high frequency amplifying valve**); the magnified and corresponding variations in the plate circuit of this valve are applied to the grid of the next valve arranged, say, for rectification (**"detector"** valve); the plate variations of this are applied to the grid of the next valve arranged for amplification (**low frequency amplifying valve**), and so on, and finally the plate circuit for the last valve (**output valve**) contains, or is coupled to, the loud speaker. There are various methods of "coupling-up" the valves, and there are many refinements and improvements in receiver design and operation (*e.g.* the super-heterodyne receiver). But, here again, for details some book on Wireless must be consulted.



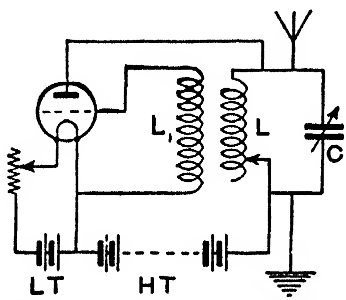


FIG. 552. C = Variable Condenser. FIG. 553.

6. Using a Valve to Produce Continuous Oscillations and Waves

Consider first our capacitance (C) and inductance (L) circuit of Fig. 497 to be placed in the plate circuit of a triode valve (Fig. 552) at the transmitting station. As the filament is being heated it will be giving off electrons and a *steady* plate current (electronic) will be flowing as indicated, whereas what we require is to set up an *oscillatory* current in the LC circuit which will not die down. Now take another coil L_1, join it to the grid and filament, and push it up against L (Fig. 553). The current in L acts inductively on L_1 and starts a momentary current in it: this current varies the potential of the grid, and this change in the grid potential varies the plate current. The sudden change in the previous steady plate current starts up oscillations in the LC circuit which is what is required. Then the oscillations in LC, acting inductively on L_1, produce oscillations in L_1 in unison with them. These latter now vary the

potential of the grid in unison with the LC oscillations, and this again produces changes in the plate current in unison with the LC oscillations. Evidently the combined result of all these actions and reactions is to build up strong oscillations in the LC circuit, which do not die down. For scientific accuracy there are other points about this which should be considered, but they do not interfere with the broad idea we wish to convey. In practice, the simple switching-on of the H.T. supply will start the action.

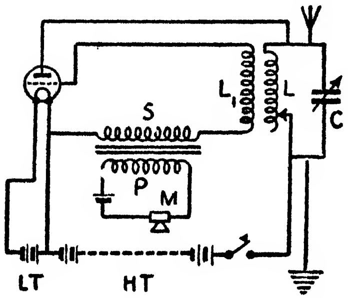


FIG. 554.

A transmitting aerial could be placed in the LC oscillating circuit as in Fig. 553 or (better) a tuned aerial could be coupled to L as in Fig. 539. Finally, a microphone must be introduced so that variations in the microphone current produced by the voice may vary the strength of the oscillations in LC and modulate the carrier wave, and one method is indicated in Fig. 554. Here the secondary S of the microphone is put into the grid circuit. Current is induced in S varying according to the words spoken: these vary the potential of the grid accordingly, which again varies the strength of the oscillations in LC, and the wave is modulated.

(*a*) As already indicated various stages of amplification are employed before the oscillations are passed to the transmitting aerial. Further, it must be remembered that the valves used in transmitting stations are on a much larger scale than those used in domestic receivers. Thus at the B.B.C. Brookmans Park Station one type of valve is 2½ ft. high, weighs 7 lb., has a water-cooled anode, and the H.T. anode voltage is 12,000: and there are much larger ones.

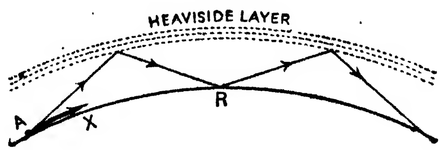


FIG. 555.

(*b*) Strictly, a valve does not *generate* oscillations itself, but it makes it possible for an oscillatory circuit to be *maintained* in *undamped* oscillation with energy supplied by a direct current source.

(*c*) Owing to the curvature of the earth one would not expect wireless waves from a station to travel even a quarter of the way round it, but signals have been received which have passed several times round the earth. A *perfect* conductor would be a perfect reflector of waves, and the ionised upper atmosphere is conducting. Now there is in the upper regions an ionised layer known as the **Kennelly-Heaviside layer** which turns the waves down again, and it is by successive reflections here and at the earth that wireless waves

can pass right round (Fig. 555). Above this is another layer—the **Appleton layer**—which has a similar effect on the ultra-short waves used in television.

7. The Idea of "Scanning" in Television

Television makes use of that property of the eye known as *persistence of vision*, viz. that any impression produced on the retina does not disappear immediately the object is taken away, but lasts for about $\frac{1}{16}$ to $\frac{1}{10}$ second.

Suppose a person standing in a dark room, and that a *narrow* beam of light is projected on his face. Suppose this light spot is moved quickly from right to left over the upper part of the head: then suppose it is moved quickly across again, from right to left, but that this second journey is a little below the first one, its upper edge just touching the bottom edge of the first journey: then suppose a third journey is made across, and so on. Finally, imagine these journeys to take place so rapidly that the whole face is covered in $\frac{1}{16}$ of a second, and that the operation is kept going on at the rate of 16 complete "scans" over the face per second: then the whole face would be visible by persistence of vision, although the spot-light is only on one small area at any instant. This principle of scanning is employed in Television.

The above idea can be carried a little further, thus introducing the *germ* of television. Suppose that in front of the person there is a photo-electric cell so situated that light reflected from the face falls on the cathode. The amount of light reflected by the face at any instant depends on where the very small spot-light happens to be: thus more light will be reflected by the teeth than by the cheek, and more by the latter than by the dark hair. This varying light falling on the cathode causes the cell to give a varying current depending on the light and shade of the face in question. This varying current could then be treated in the same way as the varying current from the microphone in sound broadcasting, *i.e.* it could be amplified and caused to modulate the carrier wave of a transmitting station. At the receiving end the H.F. oscillations in the aerial, *varying in strength in a complicated way according to the light and shade of the face at the sending end*, would be passed to a wireless receiver, but the output of the receiver, instead of passing to a loud speaker for conversion into a varying sound, would be passed to another device for conversion into a varying light which owing to persistence of vision would "build up" into a picture of the face at the sending end. This other "device" is usually a *cathode ray tube* (Art. 9).

The above method was used (with some slight modifications) in the early television transmissions of the B.B.C. known as "low definition (L.D.) television." This, however, is now superseded by "high definition (H.D.) television" as explained in Arts. 8, 9.

8. At the Sending End in Modern (H.D.) Television

What is known as the **iconoscope camera method** at the sending end has been developed in this country by the Marconi E.M.I. in their **emitron camera** which is used by the B.B.C.

The iconoscope consists of an evacuated glass vessel (Fig. 556), at the narrow end of which is a cathode which is heated. In front of the cathode is an anode or two, each with a hole at the centre and maintained at a positive potential. The electrons given off by the cathode are accelerated towards the anode, so that we have a very narrow beam of electrons shooting through the anode holes into the wider part of the tube. It is this narrow beam of electrons which is used to do the " scanning " operations.

In the wide portion of the tube is a plate M on which an optical image of the person or scene to be televised is focused by photographic lenses. M consists of a thin sheet of mica backed by a sheet of metal known as the **signal plate.** The front of the mica is first sprayed with silver oxide powder which is then heated and reduced to silver in the form of *tiny* globules: these are next oxidised so that they are insulated from each other, and then given a coating of caesium. Thus the front of the mica sheet is a tremendous number of tiny silver globules all insulated from each other, and each made sensitive to light, like the cathode of a photo-electric cell, by the caesium.

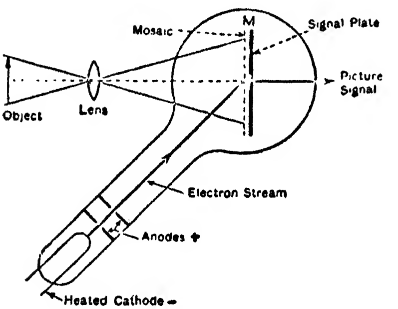


FIG. 556.

Each silver globule forms, with the signal plate behind and the mica between, a small condenser. When therefore an optical image of the scene is projected on M each element of the surface ejects electrons according to the light and shade of that part of the image which falls on it, and each tiny condenser is therefore " charged " accordingly. We have a "picture" on M made up of "charges."

The narrow electron beam from the cathode is now caused to scan the surface of M in the usual way. The beam is moved steadily along horizontally, say from right to left, and then *very quickly* triggered back again to begin its second horizontal journey: meanwhile it has been slightly moved downwards, so that the second journey is below the first and edge to edge with it. When, say, 240 lines have been scanned in this way it is rapidly triggered back to the starting point to begin its second scan (frequently there are 25 scans in a second). These two movements are brought about by two pairs of coils suitably placed outside the tube and carrying the necessary varying currents to act on the electron beam as required.

Consider then the scanning electron beam coming for an instant on one of the tiny condenser elements. Electrons (negative) pass from the beam into the condenser, discharging it, and consequently an equal current impulse

appears at the signal plate behind. This happens with each element in turn. Thus during the scanning we get a varying current in the signal plate circuit, the current at any instant depending on the charge on the tiny condenser which the beam is discharging at that instant, which again depends on the light and shade of the part of the image there. In short, we get a corresponding current picture signal in the signal plate circuit. This picture signal is passed to amplifiers, and finally modulates the carrier wave on which the vision is sent out. The whole is contained in a single camera as seen in the illustration on page 431.

9. At the Receiving End in Modern (H.D.) Television

For the reception of television we require a wireless receiver and a device in place of the telephones or loud speaker for converting the varying output into *light*, thus building up the scene, and the **cathode-ray tube** is almost invariably used for the purpose to-day.

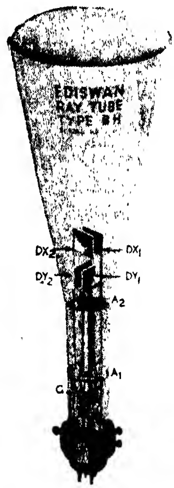


FIG. 557.

The cathode-ray tube is shown in Fig. 557, and the picture is built up on the large end of the tube. At the extreme narrow end is a filament cathode which is heated by a current, and gives off electrons. In front of the cathode (Fig. 558) is a disc anode A_1 (pierced with a hole), a second anode A_2 in the form of a cylinder, and a third anode A_3 in the form of a disc with a central hole. These anodes are at positive potentials, A_3 being at a higher potential than A_2, and A_2 higher than A_1. The positive anodes attract the electrons, the result being that we have a narrow stream of electrons shooting through the anode holes and passing on to strike the end of the tube. Moreover a cylinder S (called the shield) surrounds K, and is given a negative bias: it therefore repels the electrons and squeezes them

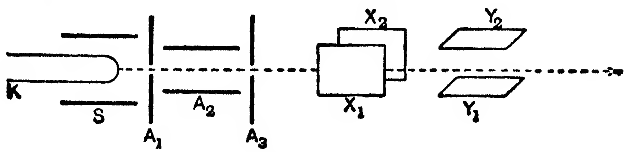


FIG. 558.

into a narrow beam to pass along the axis. The effect of the various anodes is to focus the beam at the end where it strikes.

The end of the tube is coated with a fluorescent material, so that we get a bright patch of light on the "screen," as it is called, at the point where the cathode-ray stream strikes it, and the more intense the stream the brighter is the patch. If the end of the stream be quickly moved horizontally along

the screen we see (by persistence of vision) a bright line, and so on. Two pairs of *deflecting plates*, as they are called, X_1X_2 and Y_1Y_2, are fixed as indicated for a reason which will be seen presently.

Now, in practice, the cathode-stream is caused to "scan" the end of the tube. It is moved along the first line, and then is very rapidly triggered back to begin its second line journey, this journey being edge to edge with the first one, and so on. When 240 lines, say, have been traced it is rapidly triggered back to the starting point to begin its second complete scan, and this continues at the rate of, say, 25 complete scans per second. These movements are brought about by the two pairs of deflecting plates to which varying potentials are given in order to move the electrons about as required: the varying P.D.'s on X_1X_2 move the beam horizontally and the varying P.D.'s on Y_1Y_2 move it gradually vertically. The result so far, then, is that by persistence of vision we see on the screen, not a moving spot of light, but a complete area of light.

The output of the receiver, *i.e.* the picture signal, is passed to the shield S. This varies the potential of S, varies its effect on the electron stream, and varies, therefore, the *intensity of the stream.* Thus we get a varying illumination on the screen depending on the varying signal from the receiver—in other words, an image of the object televised is built up on the screen.

This account of television is necessarily very brief: for details some book on the subject must be consulted (e.g. *Television Up-to-Date*, by R. W. Hutchinson). There is, however, one more point which must be mentioned.

Sidebands in Television. On page 579 reference was made to *sidebands*, and we saw that to accommodate these sound broadcasting stations were separated from each other by 9 kilocycles. The eye, however, is much more sensitive than the ear, and it can be shown that for television 1000 kilocycles at least on each side of the carrier is necessary. The medium broadcasting band for sound is from 200-500 metres, a total *frequency space* of, say, 900 kilocycles with about 100 stations. *One* television station requires more than the whole of this space.

There is, however, plenty of elbow room if we use the ultra-short waves between 3 and 10 metres wave-length. The frequency space here is from $(300,000 \div 3) = 100,000$ kilocycles to 30,000 kilocycles, *i.e.* the space is 70,000 kilocycles—room for many stations with 1000 kilocycle sidebands. In the television from the Alexandra Palace the vision carrier was 6·6 metres wave-length, and the sound 7·2 metres. A receiver for television reproduces both sound and vision, only one aerial being used.

CHAPTER XXII

FURTHER PRACTICAL APPLICATIONS

THE subjects dealt with in this chapter each constitute, or are associated with, some highly developed branch of applied electricity, so that in this book only a glance at one or two essential principles is possible: for details, specialised works on the various branches must be consulted.

1. Microphone Transmitters, Telephone Receivers, Loud Speakers

(1) THE MICROPHONE.—This appliance is used at the sending end in "telephone" communication, and the general *principle* of the modern **carbon microphone** transmitter is shown in Fig. 559. A number of granules of carbon are enclosed in a chamber between two suitable plates. The latter are joined through a key and battery to a primary coil P, and the secondary coil S is joined to the outside circuit which contains the receiver—say the "telephones." Suppose the key is closed and a steady (small) current is flowing from the battery. When a person speaks in front of the diaphragm M the latter vibrates according to the sound waves. This produces compressions and decompressions of the granules, which vary the resistance and the current flowing. This varying current, depending on the words spoken, passes through P and produces by induction a corresponding varying current in S which passes along the outside circuit, through the telephone receiver at the receiving end, and the movement of the latter sets the air in corresponding vibration reproducing the sound.

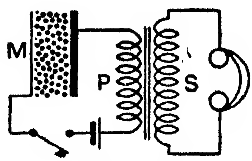


FIG. 559.

The **Reisz microphone** employs a thin layer of powdered carbon spread on a slab of marble and covered by a rubber sheet which takes the place of the diaphragm above. It works on the same principle, but does not tend to strengthen some notes at the expense of others as the granule instrument does.

The **magnetophone** is another type of microphone. It consists of a small coil of wire on the back of a freely moving plate. The coil is in the magnetic field between the poles of a magnet. The sound waves cause the plate and coil to move, and varying currents are induced in the coil. This is largely used in present-day broadcasting.

(2) THE TELEPHONE RECEIVER.—What is practically the original *Graham Bell telephone* (1876) is still used as the receiver in telephone communication. It consists (Fig. 560) of a permanent U-shaped magnet M fitted with two soft iron pole pieces P which are magnetised by induction and attract a thin soft iron disc D situated just in front of them. On the pole pieces are wound two coils CC which carry the incoming current, and of course this is a *varying* one depending on the words spoken into the microphone at the sending end. The varying current round the coils CC varies the strength of the magnet and thus varies its pull on the soft iron disc. D therefore vibrates according to the varying current, *i.e.* according to the words spoken, and this vibration being communicated to the air, sound waves are set up reproducing the words. Another type is shown in Fig. 561. M is of cobalt steel, and D is a stalloy disc: the containing case is of ebonite or some special composition. The wires leading from the coils C to the terminals are not shown in the figures. The instrument was originally also used as a transmitter, but it has been superseded by the microphone for this purpose.

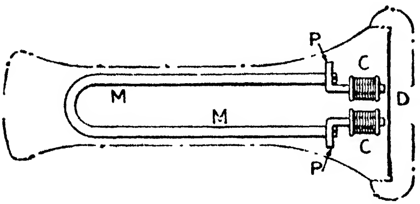


FIG. 560.

(3) A SIMPLE TELEPHONE CIRCUIT.—For two persons A and B to carry on a *conversation* A must have both a microphone and a receiver, and so must B, and there must be some device at both ends so that A can "call up" B or B call up A when required. A simple practical circuit is shown in Fig. 562. R and R_1 are the receivers (ear-pieces), whilst MT and M_1T_1 are the transmitters (mouth-pieces). The figure depicts the hand-form in which R and MT are, for convenience, in one casing. B and B_1 are the "calling up" bells. A switch is inserted in the primary circuit of each transformer so that the battery in that circuit is only sending current when the telephone is being used. When a "telephone" is resting on its stand (shown in the upper part of the figure) its weight depresses what we have called the "receiver hook" in the diagram: this breaks the contacts with the top studs b and b_1 and

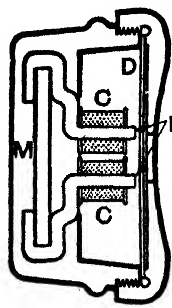
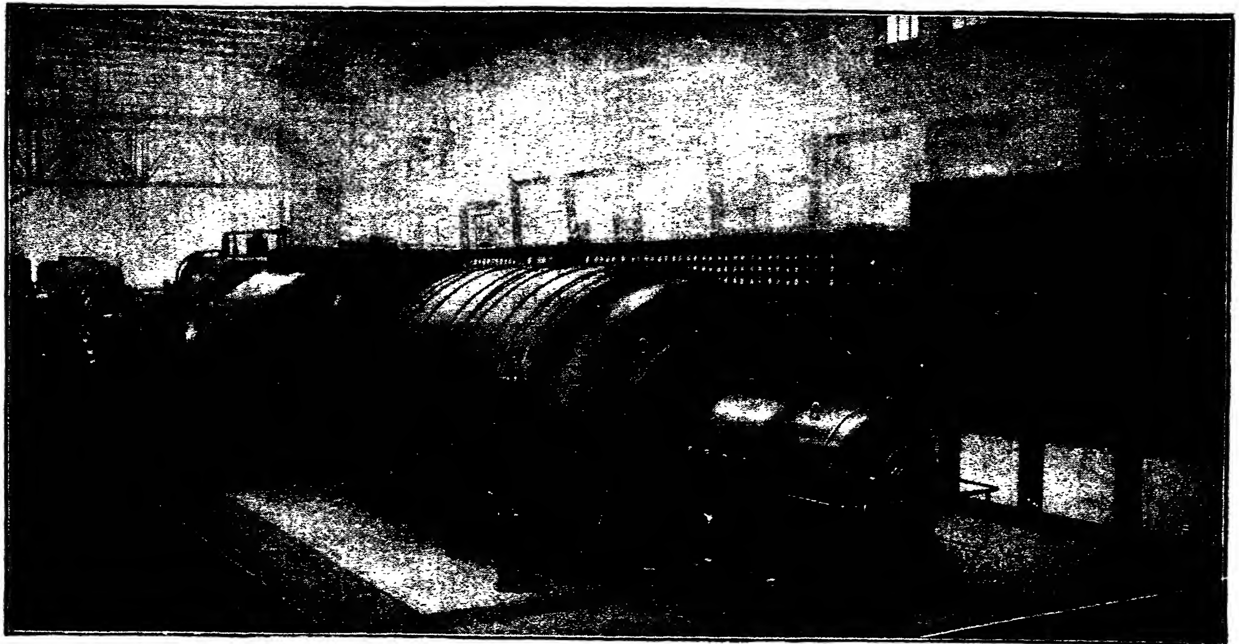


FIG. 561.



30,000 kW. 1500 R.P.M. Turbo-Alternator Set. (Metropolitan Vickers Electrical Co.)

closes the contacts with the lower studs *a* and a_1. If then we imagine the figure to have the telephones on the stands it will be seen that the " speaking circuit " is *not* complete, but the " calling circuit " is ready for action.

Now suppose No. 1 wishes to talk with No. 2. He presses the switch S to the other contact and current flows from X to *a*, then to line, to a_1, S_1, bell B_1, line, and back to X; thus the bell B_1 rings. When the " telephones " are taken from the stands the " hooks " rise, the contacts at *a* and a_1 are broken, and the contacts at *b* and b_1 closed (this is the case shown in the figure), and the speaking circuit is now complete. When No. 1 speaks into MT the varying current induced in the secondary of the transformer on the right passes to line and through the receiver R_1 of No. 2. When No. 2 replies into M_1T_1 the varying current induced in the secondary on the left passes to line and through the receiver R of No. 1. Study the diagram carefully.

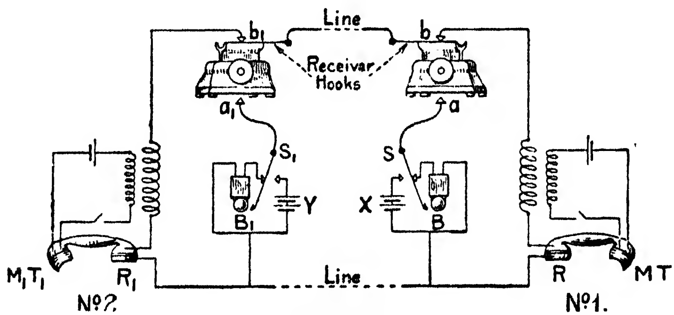


FIG. 562.

Practical telephony *on the commercial scale as it exists to-day* is such a highly technical subject that no understandable explanation which would be of any real value to the student could be given in the space available in this book (and the same applies to practical telegraphy): specific books on the subjects must be referred to.

(4) THE LOUD SPEAKER.—The student is perhaps mainly in touch with the loud speaker as an adjunct to his " wireless " receiver. The history of loud speakers from the early days of wireless—the old *horn type*, the *balanced armature cone type*, the *inductor dynamic type*, to the *moving coil type*—is interesting as noting our progress from the throaty reproduction of the old horn to the excellent reproduction of the moving coil of to-day: the moving coil is in fact the best—if (and only " if ") it is properly " matched " to the output valve of the receiver (see a book on *Wireless*).

As the name suggests, the moving part is a small coil carrying the varying current output of the receiver, and this coil is in the magnetic field of a magnet (Fig. 563). The varying current in the coil in the magnetic field causes a varying movement of the coil which is communicated to the air *via* a parchment diaphragm usually in the form of a cone attached to it, and the sound is reproduced. In one type the magnet producing the field is a permanent magnet, and these are known as *permanent magnet moving coil speakers*; in another type current is taken from the mains to magnetise the magnet, and these are referred to as *electrodynamic moving coil speakers*.

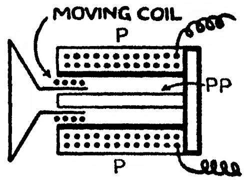


FIG. 563.

2. Alternators and D.C. Generators

Referring again to the rotating coil of Art. 1. page 508, we saw that an induced *alternating* E.M.F. was developed in it which went through a complete cycle during one revolution, the E.M.F. being a maximum when the coil was horizontal, *i.e.* its plane lying along the field, whilst there was zero E.M.F. and reversal when the coil was vertical, *i.e.* its plane at right angles to the field: further, we saw that if the ends P and Q of the coil were connected to two **slip rings** mounted on the axle and rotating with the coil, an alternating current would be delivered to an external circuit joined to the fixed brushes B_1 and B_2. This is the fundamental principle of the **alternator.**

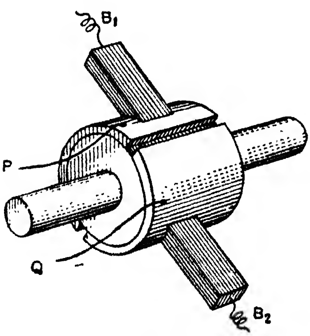


FIG. 564.

To obtain a *direct* or *continuous* current in the external circuit a **commutator** would be used in place of the slip rings. For this simple case, the commutator would consist of one ring split in two halves insulated from each other (Fig. 564). P would be joined to one and Q to the other, and the brushes would be fixed as shown (or radial instead of sloping if they were carbon). It would also be arranged that the gap in the ring would come under the brushes when the coil was in the position of reversal. A little consideration will show that in this case the positive end of the coil (whether it be P or Q) is always

under the same brush and therefore always joined to the same end of the external circuit, so that *the current outside is always in the same direction* (but it will be varying very much in strength—Fig. 566 (*a*)): this is the fundamental principle of the **direct current generator** or **dynamo.**

If, instead of a single coil, two coils at right angles were used, then the E.M.F. in one would be a maximum when it is zero in the other: hence *by suitably connecting them* the induced E.M.F. and the current outside will be more uniform in strength. This will be even better if we use a number of coils evenly spaced, in which case we would use a commutator consisting of a number of bars or segments. In practice, then, the commutator takes the form of a number of bars, insulated from each other and from the shaft, forming a cylinder (Fig. 565). Each coil (or group of coils) is connected to two bars (the method of connecting the coils will be seen presently), and the result is a *strong* (see page 598) and *uniform direct current* in the outside circuit. (Study Fig. 566.)

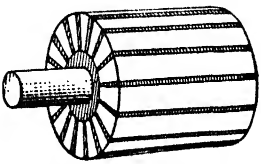


FIG. 565. Principle of Commutator.

3. The Direct Current (D.C.) Generator

The rotating coils are grouped together in what is called the **armature,** and the magnets producing the field are called the **field magnets.** The armature coils are arranged on a cylindrical iron core or *drum* to concentrate the magnetic "lines" into the space where the wires will be cutting them, and the core itself is laminated by being built up of thin circular discs to prevent the flow of eddy currents in it as it rotates (page 490).

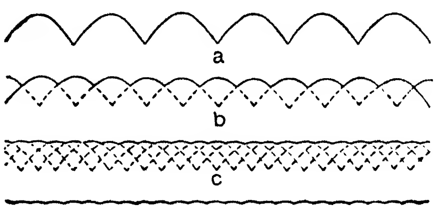


FIG. 566. (a) Single coil; (b) Two coils at right angles; (c) Four coils equally spaced.

(1) ARMATURES.—There are various methods of winding drum armatures, and to illustrate the principle of one method we will take a small machine with only 16 conductors on the armature (Fig. 567). The 16 wires are spaced uniformly along the outside of the iron drum, and cross connexions are made as indicated, the continuous lines denoting the connexions at the front, and the dotted lines those at the back of the armature. The commutator has 8 bars or segments.

From the front end of conductor 1 a connexion is led *via* a commutator segment to the front end of conductor 8. Between 1 and 8 there are *six* conductors; adding one to this we get what is called the *pitch* (seven in this case), and this pitch will be adhered to for the whole winding. Thus the back end of conductor 8 is connected to the back end of conductor 8 + 7 = 15, and the front end of 15 is joined *via* the adjacent segment to the front end of a conductor which is seven ahead of it—viz. to the front end of the conductor 6. The back end of 6 is joined to that of 6 + 7 = 13, and the front of 13 to the front of 4 *via* the next commutator segment. This is repeated, and finally the back end of 10 is joined to the back of 1, and a closed winding is obtained. The brushes make contact as indicated with the commutator bars to which conductors 1 and 8 and conductors 9 and 16 are connected.

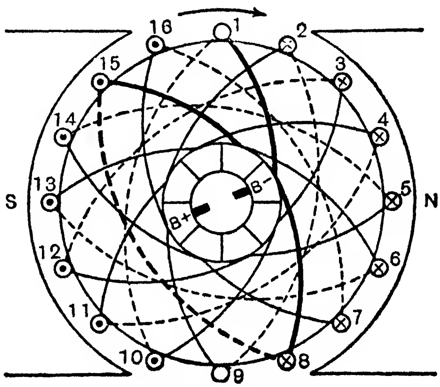


FIG. 567.

An application of Fleming's right-hand rule to the wires along the drum will show that with the armature revolving as indicated, the currents in the conductors on the left are flowing from back to front, whilst those in the conductors on the right are flowing from front to back. Thus *in the armature circuit* the current has two paths, viz.:

(1) B — along 1 — 10 — 3 — 12 — 5 — 14 — 7 — 16 — B +

(2) B — along 8 — 15 — 6 — 13 — 4 — 11 — 2 — 9 — B +,

whilst in the external circuit the current flows from B + to B —. Note that there are two paths in parallel *in the armature* from B — to B +.

Field magnets are either bi-pole or multi-pole. In the former there are two poles N and S as in Fig. 567: in the latter there are any *even* number of poles spaced alternately N and S round the armature. There are two general methods of winding armatures for multipole machines known as **wave winding** and **lap winding**, but in this book we need only take one as an example, and Fig. 568 shows a four-pole drum with 22 conductors and a single wave winding.

The commutator has 11 bars. The front pitch is 5 forward (*i.e.* in the direction of rotation) or + 5, and the back pitch is also 5 forward, or + 5. Starting, say, at the commutator bar B, a front connexion goes to conductor

18; this conductor passes along to the back, and is connected to conductor 13, which is 5 steps ahead of it; this passes along to the front, and is connected (via *a commutator bar on the opposite side of the commutator, in this case, to the starting bar*) to conductor 8, which is 5 steps ahead of it; this passes along to the back and is joined to conductor 3, which is, again, 5 steps ahead; this conductor comes to the front, and is *first connected to the commutator bar next to the one at which we started,* and then to conductor 20. This is repeated until finally we arrive at conductor 1, which brings us down to the bar at which we started, and a closed winding is the result.

By applying the right-hand rule, it will be found that the conductors under the N poles have upward currents, while those under the S poles have downward currents, that the current paths in the armature are:—

—*ve*	9—14—19—2—7—12—17—22—5—10—15—20	To +*ve*
Brush to	4—21—16—11—6—1—18—13—8—3	Brush

Two brush sets are, of course, necessary (p sets may be used, where p = number of poles), but note that they are not diametrically opposite as in the bipolar machine above.

Space does not permit the inclusion of a simple lap winding of a multipolar machine, but the following points of difference may be *just mentioned.* In wave winding the front and back pitches are in the same direction: in lap the front and back pitches are in opposite directions. In single wave winding there are two paths in parallel in the armature whatever the number of poles may be: in single lap winding there are as many paths in parallel in the armature as there are field poles (therefore 4 for a 4-pole machine). In wave winding the pitches *may be* equal: in lap they differ by two. And there are other points of difference. Wave winding is suitable for large voltage, lap winding for large current.

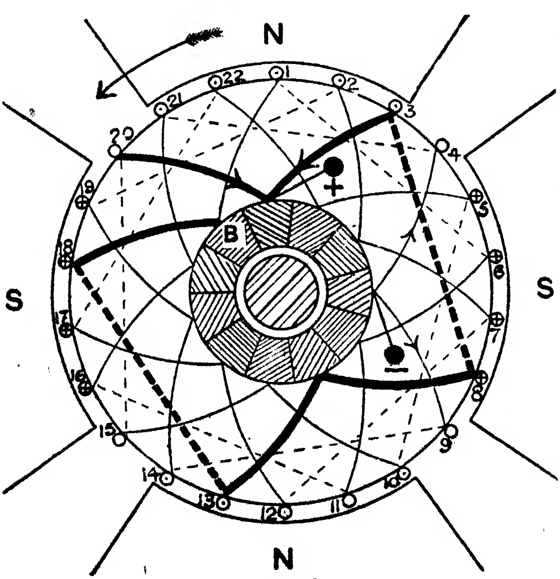


FIG. 568.

(2) FIELD MAGNETS.—These are, of course, *electromagnets*, and in most cases the current used to magnetise them is taken from the machine itself. The whole principle of "self-excitation" is simply this: Owing to the residual magnetism in the fields, the armature, on starting, has a small E.M.F. and current developed in it, and this current, or part of it, passing round the fields strengthens them so that a larger E.M.F. and current are produced, and so on; thus the machine quickly builds up. There are three types, viz. **series, shunt,** and **compound** wound machines.

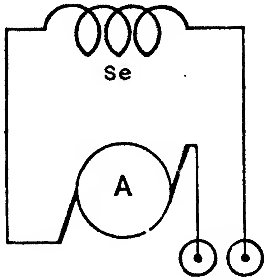


FIG. 569. A = Armature. Se = Field coils.

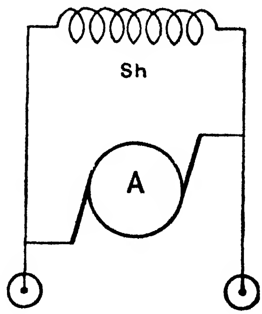


FIG. 570. Sh = Field coils.

(*a*) *Series Wound Machines* (Fig. 569).—In these the field coils consist of *a few turns of thick wire,* and the armature, field coils, and external circuit are all in series. A point about them is that the voltage at the terminals increases as the current taken from them (*i.e.* the load) increases.

(*b*) *Shunt Wound Machines* (Fig. 570).—In these the field coils consist of *a large number of turns of fine wire* joined as a shunt across the brushes so that they are in parallel with the external circuit. In these the terminal voltage falls somewhat as the current outside increases.

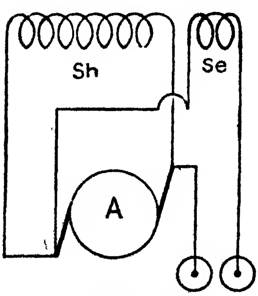


FIG. 571. Sh and Se = Shunt and Series coils on Fields.

(*c*) *Compound Wound Machines* (Fig. 571).—These are a combination of the two preceding types. In Fig. 571 the shunt winding goes from brush to brush, and it is called a *short shunt compound*: sometimes the shunt winding is across both the armature and series coil, in which case it is called a *long shunt compound.* The special point about them is that the voltage at the terminals can be made to be practically constant whatever the current taken.

The above statements can best be realised by examining the *characteristic curves* of the three types, *i.e.* the curves showing the relation between terminal voltage (V) and current output (I). Fig. 572 shows the idea very roughly, and indicates the combination of series and shunt giving the almost level curve of the compound.

Series machines are not much used now, but they are serviceable for cinematograph and search-light purposes. Shunts are extensively used for ordinary lighting and power, and *always for charging accumulators.* Compounds are employed for traction and isolated lighting installations.

(3) AVERAGE E.M.F. OF D.C. GENERATOR. EFFICIENCIES.—Taking first the case of the bi-polar machine of Fig. 567, let ϕ = total magnetic flux through the armature, Z = total number of external wires on the armature, and N = number of revolutions *per second*: then an E.M.F. formula is obtained thus:—

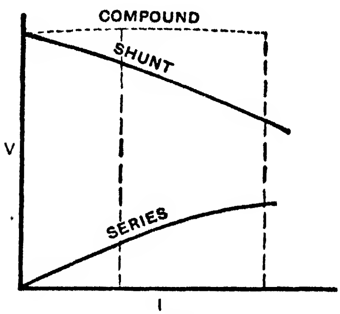


FIG. 572.

Each external wire cuts ϕ lines *twice* in one revolution, *i.e.* each conductor cuts 2ϕ lines per revolution, and therefore $2\phi N$ lines per second; hence:—

E.M.F. for each conductor

$$= 2\phi N/10^8 \text{ volts.}$$

Again, the current has two paths through the armature from brush to brush. Further, each path contains Z/2 conductors in series forming E.M.F., the two paths being arranged in parallel, and just as the combined E.M.F. of two equal batteries in parallel is the same as that of one battery, so the total E.M.F. in this case is that due to Z/2 conductors; hence:—

$$\text{Average E.M.F.} = \frac{2\phi N}{10^8} \times \frac{Z}{2} = \frac{\phi NZ}{10^8} \text{ volts,}$$

$$\textit{i.e. } \text{E.M.F.} = \frac{\text{Flux} \times \text{Revs. per sec.} \times \text{No. of conductors}}{10^8} \text{ volts.}$$

For multipolar machines the E.M.F. is calculated by an extension of the above. Thus it can readily be shown that:—

$$E = \frac{p}{a} \times \frac{\phi NZ}{10^8} \text{ volts,}$$

where p = number of poles, a = number of paths in parallel between brushes, and ϕ = flux from *each* north pole. For the bi-polar $p = 2$ and $a = 2$ and we get the expression above.

In connexion with D.C. generators the following "efficiencies" are employed:—

$$\text{Commercial efficiency} = \frac{\text{Power in the external circuit}}{\text{Power supplied to drive the machines}}.$$

both being expressed in the same units, watts, or H.-P. It is of the order 95 per cent. for a good large machine.

$$\text{Electrical efficiency} = \frac{\text{Power (watts) in external circuit}}{\text{Total power (watts) generated}},$$

the denominator being greater than the numerator because some watts (I^2R) are wasted in the machine.

$$\text{Mechanical efficiency} = \frac{\text{Total power (watts) generated}}{\text{Power (watts) supplied to machine}},$$

or it is equal to commercial efficiency divided by electrical efficiency.

4. The Direct Current Motor

The principle of the D.C. motor is that of the moving coil galvanometer, and it is the converse of the D.C. generator: in the generator we give *motion* to the armature and it delivers a current, whilst in the motor we give the armature a *current* and it is set in motion, which motion, by belting, etc., can be used to drive machinery and the like.

Referring again to Fig. 567, suppose that instead of rotating the armature, the brush B − is joined to the positive and B + to the negative pole of a battery or other source of current, so that current enters at B − and divides flowing through the coils as shown and joining up again at B +. By applying the *left-hand rule* to the wires on the left it will be seen that these are urged *downwards*, whilst the rule, applied to the wires on the right, will show that these are urged *upwards*. Thus we have continuous rotation of the armature in a *counter-clockwise* direction. This is the principle of the **D.C. motor.**

Again this rotating armature of the motor will begin to act as a dynamo, *i.e.* E.M.F.'s will be produced in it. By applying the *right-hand rule* to the wires on the left, which are moving downwards, it will be found that the induced E.M.F. in these is from front to back, *i.e.* the induced E.M.F. in the coils on the left is in the direction *brush* B + *to brush* B −, *i.e.* opposite to the battery pressure supplied to the motor. Similarly, on the right the induced E.M.F. in the wires which are moving upwards is from back to front, *i.e.*, opposite to the battery pressure supplied. In other words, the motor develops a **back E.M.F.** (e) opposite to the applied E.M.F. (E), and, of course, the back E.M.F. is greater the higher the speed. This back E.M.F. is very important in practical motor working. If E be the E.M.F. (volts) applied from the mains (or battery), I the current (amperes) taken by the motor, and e (volts) the back E.M.F., then $I = (E - e)/R$ where R is the resistance (ohms).

When the motor is switched on the speed increases, and therefore e increases, and I, the current from the mains, falls off. This continues until the motor is just taking enough power to drive it at a steady speed according to the " load " on it—the work it is doing.

Every D.C. generator will act as a motor if supplied with current in the way indicated, so that we have *series*, *shunt*, and *compound* machines. The characteristic features of the three types are as follows:—(*a*) *Series motor*. These are suitable for work where a large *starting effort* or *torque* is required, *e.g.* setting heavy trains and trams in motion from rest: they are *not* suitable if the work they are doing, *i.e.* the " load," is likely to be suddenly removed, for if this occurs the motor will " race "—run at an excessive speed—and do damage in consequence. (*b*) *Shunt motor*. These run practically at a constant speed from no load to full load, and are therefore suitable for driving machine tools in workshops, etc., where the load is likely to vary. (*c*) *Compound motor*. These, as used in practice, can exert a powerful starting torque, but they will not race if the load be removed for the shunt coil keeps the no-load speed within safe limits: they are useful for working coal-cutting machines, hoists in mining work, rolling mills, etc.

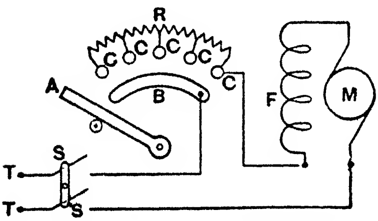


FIG. 573. Starter for Series-wound Motor.

A, Movable arm. B, Contact strip. C C, Contact studs. F, Field winding of motor. M, Armature of motor. R, Starting resistance. S S, Double pole switch. T T, Main terminals.

When a motor is first switched on the back E.M.F. (e) which is due to its rotation is, of course, zero. Hence, since the resistance of the armature is usually low, it might take a current from the mains ($I = (E - e)/R$) large enough to overheat and damage its windings. Thus it is that with large motors, which take some time to run up to full speed, a *starting resistance* is used. The starting switch is so arranged that when it is first closed, there is in the circuit an additional resistance large enough to prevent damage: this resistance is gradually cut out step by step as the starting handle is pushed over and the motor gains speed, until the resistance is all out when the motor is running at the correct speed. A simple type of **starter** for a series motor is shown in Fig. 573 which is self-explanatory.

When a motor is disconnected from the mains, or if the mains supply fails at any time, it should be impossible to start up again without having the starter in circuit. For this reason starters are invariably fitted with a small electromagnet, which is energised by the supply current and attracts and

holds on the revolving arm in the "full on" position. Should the supply cease, the electromagnet loses its magnetism, and a spring attached to the arm at once pulls it to the "off" position. The device is known as a **no-voltage release.** Frequently there is also another device known as an **over-load release,** which causes the motor to be disconnected when the current supply becomes excessive.

In connexion with D.C. motors the following "efficiencies" are employed:—

$$\text{Electrical Effic.} = \frac{\text{Electrical power spent in producing rotation}}{\text{Total electrical power supplied}}$$

$$\text{Commercial Effic.} = \frac{\text{Output (measured on brake)}}{\text{Total power supplied}}.$$

$$\text{Mechanical Effic.} = \frac{\text{Commercial efficiency}}{\text{Electrical efficiency}}.$$

5. The Alternating Current Generator or Alternator

The principle of the alternator was referred to on page 509. In practice, however, the field magnets are sometimes stationary and the armature rotates (as with the D.C. generator), but it is more usual for the armature to be stationary and the fields to rotate: hence to avoid confusion the rotating part is called the **rotor,** and the stationary part the **stator.** The field poles of an alternator must, of course, be magnetised by some D.C. supply—generally by a D.C. generator known as the **exciter.** In Great Britain power stations generate almost exclusively from steam power, and the rotors of large alternators are usually driven directly by steam-turbines: the machines are then called **turbo-alternators.**

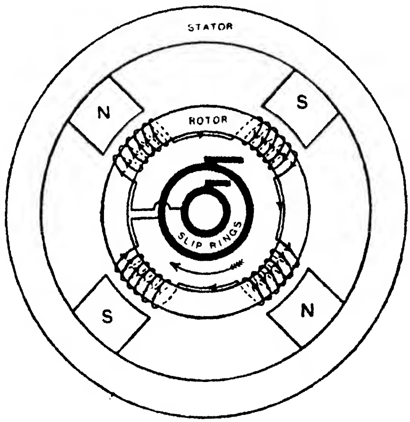


FIG. 574.

The principle of a simple type of small alternator—a four-pole "single phase" machine, as it is called—will be gathered from Fig. 574. For simplicity we have pictured the coils to be on an iron *ring* (in considering the induced E.M.F. only the *outer* wires need be taken into account—the inner wires are not cutting magnetic lines). At any instant a coil will be passing under an

opposite pole to that of its neighbours, so that the E.M.F. induced in it will be in the opposite direction. If, however, the coils be wound in opposite directions and joined as indicated, the extreme ends being connected to two slip rings, they will, at any instant, help each other from one extreme end to the other, but the outside current will be reversed every time the coils pass from pole to pole. Thus the *frequency* will be obtained by multiplying the number of *pairs of poles* by the revolutions per second.

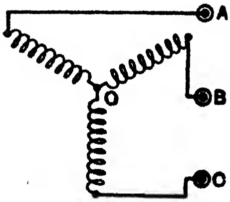


FIG. 575.

It will be noted that in this simple example the "armature" is the *rotor* and the fields the *stator*.

Large alternators, however, consist of a set of rotating field magnets (the rotor) and a stationary group of windings on the armature (the stator). The rotor consists of a heavy fly-wheel, carrying radial pole-pieces. The D.C. current from the exciter is led into the rotor by two slip rings. The stator is a laminated cylindrical structure surrounding the rotor and containing slots on its inner surface parallel to the shaft in which the windings are placed. The ends of the windings are taken to *fixed terminals* on the machine and the currents taken from these to the outside circuit.

In **single-phase alternators** all the conductors are connected in series, and the two ends joined to the two terminals of the machines.

In **two-phase alternators** there are two different sets of conductors, the constituents of one set being spaced half-way between those of the other set, so that the E.M.F. in one set is a maximum when it is zero in the other; thus there is a phase difference of 90°, and there are four terminals to the machine. Instead of a single circuit from the machine carrying A.C. we have two separate circuits carrying A.C. of the same frequency but differing in phase by 90°.

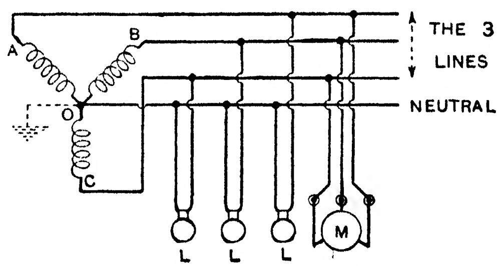


FIG. 576. L = lamps. M = Three-phase motor.

In **three-phase alternators** there are three sets of conductors, arranged so that, while set (1) is producing the maximum E.M.F., set (2) is producing E.M.F. lagging 120° behind set (1), and set (3)

is producing E.M.F. lagging 120° behind set (2), and therefore 240° behind set (1). Instead of six terminals, the starting point of all three windings is usually a common junction; this is often earthed to the shaft of the generator, and the other three ends after forming the windings round the stator are connected to *three terminals* on the machine (Fig. 575). (Such a machine is said to be *star wound.*)

The voltage between any two of the star points (A and B, B and C, C and A) is called the **line voltage,** whilst that between the centre point O and a star point is called the **phase voltage.** The ratio between the two is:—Line voltage = 1·73 × Phase voltage.

In practice to-day A.C. is usually generated and transmitted on the **three-phase system.** Instead of using six wires to transmit the power to where it is wanted a common junction is made as indicated above, and a wire is run from this acting as a common return (Fig. 576) which is usually earthed and referred to as the **neutral.** Our lamps and heating appliances (L) are usually connected *between a line and the neutral* as shown, and the standard voltage between a line and neutral is 230 volts. Consumers are therefore on single-phase, one wire (the neutral) being earthed, and the supply voltage 230. The voltage *between lines* is, however, 230 × 1·73 = 400 volts. Three-phase motors (M) are connected to the three lines (Fig. 576).

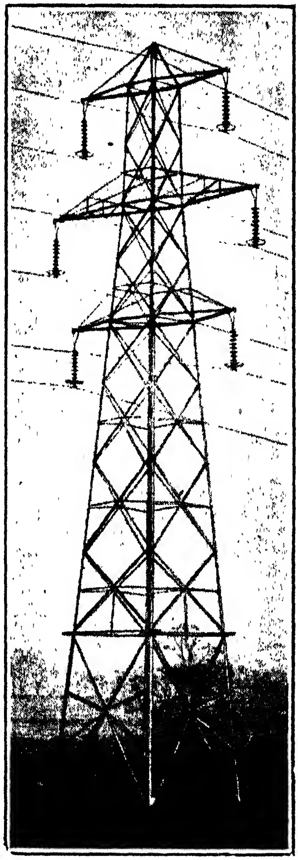


FIG. 577. 132 kV. Double Circuit Straight Line Tower.

6. The Grid

We have seen that every electrical power station has to provide sufficient plant to meet the *maximum* demand of the district it supplies, *plus* extra plant to allow of one or two machines being out of action for ordinary repair, *plus* sufficient to meet emergencies and to avoid serious breakdowns in the supply: in some cases the spare plant carried by a station is enough to supply 35 to 50 per cent. of its peak load. This extra plant is not ideal economically, and of course leads to a higher price to customers for the energy consumed. Further, small and

partially isolated districts get no supply at all owing to the cost to the nearest station of laying down the long cables to the district.

Now suppose that, say, twelve of these individual systems could be linked together by conductors in such a way that any one system could, in case of necessity, get energy from any or all of the other eleven members of the group, or send energy to them. It is clear that the spare plant carried by each could be considerably reduced, for in case of even a serious emergency in one system that station would be able to call upon most of the spare plant in all the other systems of the group. Evidently over that whole interconnected area the electric supply should be carried out more efficiently and economically, and consumers *should* reap the benefit of a smaller charge per unit. Moreover the network of conductors could be used to supply districts previously untouched.

This is one principle underlying the "Grid." The country is divided into nine regional areas or "schemes," and all the important generating stations in each area are interconnected, whilst interconnexions also go to the adjoining areas. It is clear that for transmission in any direction over the long connecting links of the Grid high voltages must be used (page 488). These main circuits (which are three-phase) consist of overhead steel-cored aluminium transmission lines suspended by insulators from steel towers (Fig. 577), and the voltage for them is raised by transformers to 132,000 between conductors. Secondary transmission lines are at 66,000 volts and 33,000 volts. Sub-stations are erected at the district where interconnexion is made with a local supply system, and transformers step-down the high grid voltage to the voltage for that system.

As an example in the *Central England Area or Scheme* which is controlled from Birmingham, nine selected generating stations are interconnected, and there are 18 sub-stations in the area: moreover this area system is connected to those in the adjoining areas by lines to Bedford (South-East England Scheme), Crewe (North-West England Scheme), and Gloucester (South-West England Scheme).

APPENDIX

1. **Solid Angles.**—The expression employed in (26), page 83, for the solid angle subtended at P (Fig. 94) by the circular shell AB may be readily found as follows:—

Consider a sphere and its *circumscribing cylinder* (*i.e.* let the sphere have a cylinder drawn about it, the base diameter, and height of the cylinder, being therefore each equal to the diameter of the sphere), and imagine a series of planes parallel to the ends of the cylinder, such planes dividing the surface of the sphere and the lateral surface of the cylinder into *zones*; *the area of any zone of the sphere is equal to the area of the corresponding zone of the cylinder.*

Now consider Fig. 94, and imagine a sphere with centre P and radius PA. The area of the segment cut off by AB will be equal to that of the corresponding zone of the cylinder. The circumference of this zone is $2\pi\overline{AP}$, *i.e.* $2\pi(x^2 + r^2)^{\frac{1}{2}}$, and its width is $(\overline{AP} - x)$, *i.e.* $(x^2 + r^2)^{\frac{1}{2}} - x$; hence its area (and therefore that of the segment cut off by AB) is $2\pi(x^2 + r^2)^{\frac{1}{2}} \{(x^2 + r^2)^{\frac{1}{2}} - x\}$ Thus—

$$\omega = \frac{\text{Area of segment}}{(\text{Radius})^2} = \frac{2\pi(x^2 + r^2)^{\frac{1}{2}} \{(x^2 + r^2)^{\frac{1}{2}} - x\}}{(x^2 + r^2)}$$

$$= \frac{2\pi\{(x^2 + r^2)^{\frac{1}{2}} - x\}}{(x^2 + r^2)^{\frac{1}{2}}} = 2\pi\left(1 - \frac{x}{(x^2 + r^2)^{\frac{1}{2}}}\right) = 2\pi\,(1 - \cos\theta)..$$

2. **Thermo-electric Diagram.**—The diagram for a few metals is given below. Centigrade temperatures are indicated: for certain calculations these must be converted into Absolute temperatures.

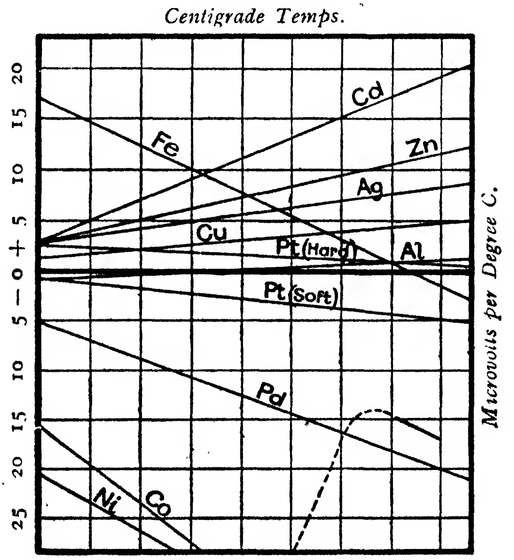


FIG. 578.

3. A Few More Facts about Units.—In dealing with the various *e.s.* and *e.m.* units in preceding pages special attention was drawn to the value (3×10^{10}) which appeared in some form or other in the ratio of the two sets of units. The *theory of units* in detail is outside the scope of this book, but a few notes on the subject will be helpful and interesting.

(1) The three fundamental units in which quantities are expressed are those of *length*, *mass* and *time*, the units generally employed in science being the centimetre, gramme and second respectively: any system built up on these fundamentals is known as an **absolute or C.G.S. system of units.** When we wish to express the idea of mass without implying any particular mass we use the symbol [M]: similarly for length [L] and time [T]. From this we easily deduce the following:—

$$[\text{Velocity}] = \frac{[\text{Distance}]}{[\text{Time}]} = [LT^{-1}]; \quad [\text{Acceleration}] = \frac{[\text{Velocity}]}{[\text{Time}]} = [LT^{-2}].$$

$$[\text{Force}] = [\text{Mass}] \times [\text{Acceleration}] = [MLT^{-2}];$$

$$[\text{Work}] = [\text{Force}] \times [\text{Distance}] = [ML^2T^{-2}].$$

The *powers* to which M, L, and T are raised are usually termed the **dimensions** of the physical quantity expressed: thus we say that the dimensions of force are 1 in mass, 1 in length, and -2 in time.

(2) When we began to build up a system of electrical units, we found that we needed *not only* the units of length, mass and time but **also units of some "*physical property*" of the medium.** In electrostatics (page 160 for example) we had to introduce the dielectric constant K of the medium, whilst in the electromagnetics (page 57 for example) we had to introduce the permeability μ of the medium. (Read again the note at the bottom of page 160.)

Take first the idea of *quantity of electricity* on the electrostatic system, the fundamental equation for which was $F = Q_1Q_2/Kd^2$ (page 160). Clearly from the above we can write:—

$$[\text{Force}] = [Q^2]/[KL^2]; \quad \therefore [Q^2] = [\text{Force}]\,[KL^2] = [MLT^{-2}]\,[KL^2];$$

$$\therefore [Q] = [K^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Now take the idea of *quantity* in the electromagnetic section of our subject. Our starting point was the force between two poles, viz. $F = m_1m_2/\mu d^2$ and as before we have:—

$$[\text{Force}] = [m^2]/[\mu L^2] \text{ which gives as above:—} \quad [m] = [\mu^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}].$$

Again, force on a pole in a field = pole strength × field intensity, from which it follows that: field = force/pole; hence:

$$[H] = [MLT^{-2}]/[\mu^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}]; \quad \therefore [H] = [\mu^{-\frac{1}{2}}M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}].$$

Further the magnetic field due to a current in a coil is $2\pi nI/r$, and as 2, π, and n are merely numbers without dimensions we have :—

$$[H] = [I]/[L]; \quad \therefore [I] = [H]\,[L]; \quad \therefore [I] = [\mu^{-\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}].$$

Finally since quantity = current × time we evidently have:—

$$[Q] = [\mu^{-\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{1}{2}}] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

(3) It is quite reasonable to consider that for *any one particular quantity of electricity* the two expressions obtained above are fundamentally the same. Thus:—

$$[K^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1} = [\mu^{-\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{1}{2}}]; \quad \therefore \left[\frac{1}{\sqrt{K\mu}}\right] = [LT^{-1}].$$

Now $[LT^{-1}]$ is a *velocity* which indicates that although the dimensions of K and μ are unknown these two quantities are so related to each other that the dimensions of $1/\sqrt{K\mu}$ are those of a velocity: remember then that K *and* μ *are not independent of each other but are definitely related as indicated* (see note page 160).

(4) Now let s and m be the numbers representing a certain quantity of electricity measured on the electrostatic and electromagnetic systems. The complete expressions for the quantity are $s[K^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}]$ and $m[\mu^{-\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{1}{2}}]$. But these expressions for one and the same quantity are identical. Hence:—

$$s[K^{\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}] = m[\mu^{-\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{1}{2}}]; \quad \therefore \frac{s}{m}[LT^{-1}] = \left[\frac{1}{\sqrt{K\mu}}\right].$$

In this equation $[LT^{-1}]$ is the unit of velocity: hence:—

$$\frac{s}{m} = \frac{\text{A quantity measured in e.s. units}}{\text{Same quantity measured in e.m. units}} = \frac{1}{\sqrt{K\mu}} = v \text{ cm. per sec.},$$

so that by measuring the same quantity of electricity in e.s. units and in e.m. units the value of $1/\sqrt{K\mu}$, that is of v, can be determined: experiment proves it to be 3×10^{10} cm. per sec., the same as the velocity of light. In fact Maxwell showed that the velocity of an electromagnetic wave in any medium should be equal to $1/\sqrt{K\mu}$ and that in vacuo the value was 3×10^{10} cm. per sec.

Note that since s and m are the *magnitudes* of the same quantity in the two units, their ratio is *the inverse ratio* of the size of the units, *i.e.*

$$\frac{\text{The e.m. unit of quantity}}{\text{The e.s. unit of quantity}} = \frac{1}{\sqrt{K\mu}} = v;$$

$$\therefore \text{The e.m. unit quantity} = v \text{ (the e.s. unit quantity)},$$

and it was stated on page 160 that the e.m. unit quantity was 3×10^{10} e.s. units (this means that $v = 1/\sqrt{K\mu} = 3 \times 10^{10}$ = velocity of light).

(5) If we work out the other electrical and magnetic quantities—potential, capacitance, resistance, etc.—in the same way as we have worked out quantity of electricity above, we will find that the ratio is always $1/\sqrt{K\mu}$, *i.e.* v or v^2 or v^{-1} or v^{-2} where $v = 3 \times 10^{10}$. Thus if *capacitance* be worked out we arrive at the result:—

$$\frac{\text{A capacitance measured in e.s. units}}{\text{Same measured in e.m. units}} = \frac{1}{K\mu} = v^2;$$

$$\therefore \frac{\text{The e.m. unit of capacitance}}{\text{The e.s. unit of capacitance}} = v^2 = (3 \times 10^{10})^2 = 9 \times 10^{20},$$

and it was stated on page 170 that the e.m. unit capacitance was 9×10^{20} e.s. units.

The most convenient method of determining $1/\sqrt{K\mu}$, *i.e.* v experimentally is to measure a capacitance in e.s. and in e.m. units. If a condenser is of accurately known dimensions its capacitance in e.s. units can be calculated, and its capacitance in e.m. units measured by methods already given in previous pages. But there are other methods of carrying out this test.

(6) For a full treatment of the dimensions of all the magnetic and electrical quantities, the ratios of their units in terms of v, methods of determining v etc., see *Advanced Textbook of Electricity and Magnetism*. We quote a few ratios of units: remember that in all cases $v = 1/\sqrt{K\mu} = 3 \times 10^{10}$ cm. per sec., and that the ratio in each case is that of e.m. unit : e.s. unit—

Quantity $= v$	Current $= v$	Capacitance $= v^2$
Potential $= \frac{1}{v}$	Resistance $= \frac{1}{v^2}$	Inductance $= \frac{1}{v^2}$.

(7) The derived practical units are simply related to the absolute *electromagnetic* units by powers of 10: thus the coulomb is $\frac{1}{10}$ or 10^{-1} e.m. unit, the volt is 10^8 e.m. units, the ohm is 10^9 e.m. units, and so on. As these practical units are not easily reproduced with great accuracy as standards for practical purposes the International Conference in 1908 agreed on certain **international units.** These are defined as indicated in preceding pages: they were intended, as already explained, to be practical realisations of the true practical units but they differ slightly. Thus the international ampere = ·99997 true ampere: the international ohm = 1·00052 true ohm: the international volt = 1·00049 true volt.

For methods of determining the relations between the international units and the absolute units, reference may again be made to *Advanced Textbook of Electricity and Magnetism.*

Brief reference may be made to the *principle* of, for example, the Lorenz method of determining the ohm, *i.e.* of finding, say, the length of a column of mercury of 1 sq. mm. cross section and at 0°C which has a resistance of 10^9 C.G.S. electromagnetic units. A circular metal disc is placed at the centre of a solenoid carrying a current, the plane of the disc being at right angles to the magnetic lines, and it is rotated about an axis through its centre, the axis being therefore along the lines. A P.D. is developed between the axis and the circumference which is given by $e = \pi r^2 H/t$ where r = radius of disc, H = field and t = time of rotation. But $H = 4\pi nI$ where n = turns per cm. of solenoid and I = current in e.m. units. Hence $e = 4\pi^2 r^2 nI/t$. The current circuit of the solenoid also includes the resistance R to be measured, and the ends of R are joined through a galvanometer to the rim and axle of the disc in such a way that the P.D. at the ends of R viz. IR *opposes* e. If adjustments are made for no deflection then $RI = 4\pi^2 r^2 nI/t$ and therefore $R = 4\pi^2 r^2 n/t$. Thus R is determined *in absolute units* for r is a length, n is turns per unit length and t a time. We can therefore find the dimensions, etc., of a column of mercury for which R is 10^9 absolute units. There are many details and corrections we have omitted.

In electrical engineering a system of practical units, known as the **metre-kilogram-second (M.K.S.) system,** has crept into use. It uses the metre as length unit and the kilogram as mass unit, and takes the permeability of air to be, not unity, but $1/10^7$.

EXERCISES AND TEST QUESTIONS

Chapter II

(1) What do you understand by the following: Magnetic Poles, Magnetic Fields, Line and Tube of Force, Line and Tube of Induction, Laminated Magnet, Consequent Poles, Molecular Rigidity, Critical Temperature.

(2) Explain briefly the Molecular Theory of Magnetisation and mention any experimental facts which, in your opinion, support the theory. Briefly discuss the extension of the theory indicated by the modern electron theory.

(3) Distinguish between ferromagnetics, paramagnetics, and diamagnetics.

(4) Three circular rings of iron are magnetised, the first by being placed between the poles of a strong horseshoe magnet so that the line joining the poles of the magnet is a diameter of the ring, the second by having one pole of a bar magnet drawn round it several times, the third by having a current carrying coil of wire wrapped round it. Describe the magnetic state of each ring.

Chapter III

(1) The period of vibration of a magnet is 2 seconds. The needle is then broken in exact halves. What is the period of vibration of each half?

(2) Two magnets A and B are in turn suspended horizontally by a vertical wire so as to hang in the magnetic meridian. To rotate A through 45° the upper end of the wire has to be turned once round. To deflect B through the same angle the upper end of the wire has to be turned one and a half times round. Compare the moments of A and B. (Inter. B.Sc.)

(3) Prove that the magnetic force exerted by a short bar magnet at a point A on the line passing through its centre and perpendicular to its axis is the same as the force exerted at a point on the axis the distance of which from the centre of the magnet is $\sqrt[3]{2}$ times the distance of A from the centre.

(4) Two magnets have the same pole strength, but one is twice as long as the other. The shorter is placed " end on " at a distance of 20 cm. from the axis of suspension of a magnetic needle. Where may the other magnet be placed in order that there may be no deflection of the needle? (H.S.C.)

(5) If two short magnets of equal moment are placed with the line joining their centres along the axis of one and perpendicular to the axis of the other, calculate the intensity (magnitude and direction) of the field at the middle point of the joining line. (Inter. B.Sc.)

(6) The moment of a magnet is 1,000. How much work is done in turning it through 90° from the meridian at a place where H = ·16? (Inter. B.Sc.)

(7) Define the pole strength and the magnetic moment of a simple magnet.

A bar magnet, whose magnetic moment is M and length $2l$, is placed with its centre at a distance r from a magnetometer. The direction of the axis is at right angles to the magnetic meridian and passes through the centre of the magnetometer. Find an accurate formula giving the deflection of the magnetometer. If the magnet is turned through one right angle, keeping its centre fixed, find the condition that the magnetometer should be at a neutral point. (Inter. B.Sc.)

(8) Define carefully magnetising force (H), magnetic induction (B), intensity of magnetisation (I), permeability (μ), susceptibility (κ), and establish the relations between these quantities.

(9) What factors determine the time of oscillation of a magnet when free to swing in a horizontal plane? Two bar magnets are bound together side by side and suspended so as to oscillate in an horizontal plane. The time of swing is 12 seconds when like poles are together, and 16 seconds when the direction of one magnet is reversed. Compare the magnetic moments of the magnets. (Inter. B.Sc.)

Chapter IV

(1) Explain how you would compare the moments of two magnets (*a*) by a "torsion" method, (*b*) by the method of oscillations, (*c*) by the method of deflections. How would you compare two magnetic fields?

(2) Describe and explain a method by which the magnetic moment of a steel magnet and the horizontal intensity of the earth's magnetic field may be determined in absolute measure.

(3) A thin uniform magnet 1 metre long is suspended from the N. end so that it can turn freely about a horizontal axis which lies magnetic east and west. The magnet is found to be deflected from the perpendicular through an angle D ($\sin D = \cdot 1$, $\cos D = \cdot 995$). If the weight of the magnet is 10 grm., the horizontal component of the earth's field is ·2 C.G.S. unit, and the vertical component ·4 C.G.S. unit, find the moment of the magnet.

(4) What is meant by a neutral point in a magnetic field? There is found to be a neutral point on the prolongation of the axis of a bar magnet at a distance of 10 centimetres from the nearest pole. If the length of the bar be 10 cm. and $H = 0{\cdot}18$ C.G.S. unit, find the pole strength of the magnet. (H.S.C.)

(5) Describe the principle of measurement employed in the torsion balance. A magnet suspended by a fine vertical wire hangs in the magnetic meridian when the wire is untwisted. If on turning the upper end of the wire half round the magnet is deflected 30° from the meridian, show how much the upper end of the wire must be turned in order to deflect the magnet 45° and 60° respectively. (Inter. B.Sc.)

(6) A small suspended magnet makes 10 oscillations per minute under the influence of the earth's field alone. A bar magnet is brought near it so as not to disturb the *direction* of the pointing of the suspended magnet, but so that the latter now makes 14 oscillations per minute. What would the frequency be if the magnet were now reversed pole for pole? (Inter. B.Sc.)

(7) Calculate the magnetic intensity at a distance d from the centre of a bar magnet of pole strength m and length l small compared with d, (*a*) at a point along the axis of the magnet, (*b*) at a point on the line bisecting the magnet at right angles. How can the result of this investigation be used to test the "inverse square law"? (Inter.B.Sc.)

(8) Explain how the value of the horizontal component of the earth's magnetic field at a given point can be determined. Indicate briefly any corrections which would be necessary in an accurate determination. (Inter. B.Sc.)

(9) Define unit magnetic pole, unit magnetic field, and intensity of magnetisation. Describe fully any method by which you would compare the moments of two bar magnets. (Inter. B.Sc.)

(10) What forces have to be considered when a compass needle is acted on by an "end on" magnet? Explain why the action diminishes rapidly as the distance apart increases. A short magnet 50 cm. to the west of a compass deflects it 45°. Find the moment of the magnet if the earth's horizontal field is ·18 unit. (Inter. B.Sc.)

(11) Define magnetising force, magnetic induction, permeability and susceptibility. Explain how the intensity of magnetisation and the permeability of iron vary with the magnetising force. (Inter. B.Sc.)

(12) Write a short account of the phenomena of hysteresis in ferromagnetic substances and describe a method of taking a material such as iron or steel through a cycle of magnetisation.

Explain fully how the magnetic induction and the intensity of the magnetising field can be determined for each point of the cycle.

Deduce an expression for the amount of energy dissipated in the form of heat in each unit volume of the material during a cycle. (The last part of the question will be understood after reading Chapter XVII.) (B.Sc.)

Chapter V

(1) Explain the following terms:—Conductor, insulator, dielectric, electric field, line of force, tube of induction, and potential.

(2) Write a short essay on "Theories of Electrification."

(3) A hollow metal vessel is insulated and charged to a potential V, and the following operations are successively performed:—(*a*) An insulated metal ball is lowered into the jar without touching it; (*b*) the ball is momentarily earth-connected; (*c*) the jar is momentarily earth-connected; and (*d*) the ball is removed to a distance. State the changes of potential of the jar and the ball at each stage.

(4) Explain how the distribution of a charge on a conductor depends on its shape, and describe how you would verify your statements Explain the action of points in "collecting" the charge from an electrified body. (Inter. B.Sc.)

Chapter VI

(1) Six equal charges are placed at the corners of the base of a hexagonal pyramid. If the slant edge of the pyramid is equal to the diagonal of its base, find the intensity of the field at the apex due to the charges at the base.

(2) An insulated soap bubble 10 cm. in radius is charged with 20 C.G.S. electrostatic units. Taking the atmospheric pressure as 10^6 dynes per sq. cm., find the increase in radius due to the charge.

(3) A circular metal plate A of radius 10 cm. is earthed. At a distance of 1 mm. from it is placed another plate B of the same size, which is insulated and charged with 100 units. Find approximately the charges on the four surfaces and the force on the plate A. (The capacity of a charged circular disc of radius a at a large distance from all other conductors is $2a/\pi$.)

(4) A conducting sphere of diameter 6 is electrified with 105 units; it is then enclosed concentrically within an insulated and unelectrified hollow conducting sphere formed of two hemispheres of thickness $\frac{1}{2}$ and internal diameter 7. The outer sphere is then put to earth; determine the potential of the inner sphere before and after the outer sphere is earth-connected.

(5) Explain the term *electric potential.* If 100 units of work must be done in order to move an electric charge equal to 4 from a place where the

potential is -10 to another place where the potential is V, what is the value of V ?

(6) Show that if the energy in the electrostatic field is regarded as distributed throughout the field the amount of energy per unit volume at any point P is $\frac{kR^2}{8\pi}$, where k is the dielectric constant and R the electric intensity at P.

(7) A, B, C and D are the four corners of a square of 1 cm. side. Charges of $+10$ units are placed at A and C, and a charge of -20 units at B. Calculate (1) the potential, (2) the field at D. (C.G.L.)

(8) A, B and C are three conductors equal in all respects. A is charged, made to share its charge with B and afterwards to share the remainder with C—both B and C being previously without charge. The three are now separately discharged. Compare the quantity of heat resulting from each discharge with what would have been produced by the discharge of A before any sharing of its charge (*i.e.* compare the energies). (Inter. B.Sc.)

(9) Show that the energy required to charge a conductor of capacity C to a potential V is $\frac{1}{2}CV^2$. What becomes of the energy when the conductor is discharged ? (Inter. B.Sc.)

(10) What is meant by the capacitance (capacity) of a conductor ? Find the potential of a charged sphere, and show that its capacitance is equal to its radius. (Inter. B.Sc.)

(11) What is meant by saying that the electric charge on a conductor is 5 E.S. units and its potential 4 E.S. units ?

Point charges of electricity of magnitude 20, 40, -20 and -40 are placed respectively at the corners of a square ABCD of side 20 cm. Find the force on a point charge of 2 units at O the intersection of the diagonals, and the work done in taking the charge from O to E the mid-point of DA. (Inter. B.Sc.)

(12) What is the law of force between two electric charges ? Two pith balls, each weighing 0.05 grm., hang in contact at the end of two parallel silk threads a metre long. To what distance will the balls separate when a charge of 0·5 electrostatic unit is given to each of them ? (Inter. B.Sc.)

Chapter VII

(1) The thickness of the air layer between the two coatings of a spherical air conductor is 2 cm. The conductor has the same capacity as a sphere of 120 cm. diameter. Find the radii of its surfaces.

(2) A sphere of radius 40 millimetres (mm.) is surrounded by a concentric sphere of radius 42 mm., the space between the two being filled with air. What is the relation between the capacity of this system and that of another similar system in which the radii of the spheres are 50 and 52 mm. respectively, and the space between them is filled with paraffin of specific inductive capacity 2·5 ?

(3) Two submarine cables of equal length have conductors whose diameters are 80 and 100 mm., the diameters of the guttapercha coverings being 120 and 180 mm. Determine the relative capacities of the two cables,
$\log 2 = \cdot 30103$, $\log 3 = \cdot 47712$. (C.G.L.)

(4) A condenser A has plates of area 1000, and dielectric of thickness 4; another condenser B has plates, area 800, and the same dielectric of thickness 5. Compare the charges and energies in A and B, when they are connected, A to a source of potential 4, and B to a source of potential 5. (Inter. B.Sc.)

(5) What is the meaning of the term "lines of electric force," and what inference may be drawn from their distribution? How many "lines of force" (approx.) will there be per sq. cm. cross section in the space between the parallel plates, 10 cm. diameter, of an air condenser charged with 250 e.s. units of electricity? (Inter. B.Sc.)

(6) Two conducting discs, one metre in diameter, are placed parallel to one another at a distance apart of one millimetre, and a difference of potential of one volt is maintained between them. Find the charge on the plates, and the attraction between them when they are (*a*) in air, (*b*) in a liquid whose dielectric constant is 2·5. (Inter. B.Sc.)

(7) In an experiment with the torsion balance the lever is deflected through 80°. Find the angle the torsion head must be turned through to reduce the deflection to 45°.

(8) An insulated plate, 10 cm. in diameter, is charged with electricity and supported horizontally at a distance of 1 mm. below a similar plate suspended from a balance and connected to earth. If the attraction is balanced by the weight of one decigram, find the charge on the plate. ($g = 980$ C.G.S.)

(9) How would you compare the capacity of a Leyden jar with that of a standard condenser by some one method? Describe the precautions that must be taken to secure an accurate result. (Inter. B.Sc.)

(10) How can it be shown that inside a hollow closed conductor there is (1) no electrification, (2) no electric intensity? Are there any exceptions to these statements? Prove that in the case of a charge on a spherical surface the absence of any electric intensity inside is a direct consequence of the law of inverse squares. (Inter. B.Sc.)

(11) Describe the attracted disc electrometer, and show how it can be used to measure differences of potential in absolute electrostatic units. Define this absolute unit of potential, and show how the instrument's determinations are in the units specified. If the trap door is 5 cm. in diameter, is 3 mm. from the under plate, and is at 1,200 volts potential difference, what force of attraction would you expect between them? (Inter. B.Sc.)

(12) Show that in Kelvin's attracted disc electrometer the difference of potential between the disc and the plate is $= C\sqrt{8\pi F/S}$, where S is the surface of the disc, C the distance between it and the plate, and F the attraction between them. (Inter. B.Sc.)

(13) Describe some form of electrometer and mention the measurements for which it is particularly suited.

Two parallel plates at a distance of 0·5 cm. are electrified to a P.D. of 1,000 volts. Find the pull per sq. cm. exerted on each plate and how this would be modified if the space between the plates were filled with an insulating oil of S.I.C. = 4. (300 volts = 1 electrostatic unit.) (Inter. B.Sc.)

Chapter VIII

(1) Describe in detail the method of determining the magnetic dip at any point, explaining the reasons for each operation you describe.

(2) Two equal soap bubbles, equally and similarly electrified, coalesce into a single larger bubble. If the potential of each bubble while at a distance from the other was P, show that the potential of the bubble formed by their union is $2P/\sqrt[3]{2}$.

(3) A magnetic needle is suspended on a horizontal axis through its centre of gravity. The horizontal axis is slowly turned round in a horizontal plane.

Describe and explain the position of the needle for various positions of the axis (1) in England, (2) at the Magnetic Equator. (Inter. B.Sc.)

(4) What is the nature of lightning? What should be the arrangement of a good lightning conductor? How should it act? (Inter. B.Sc.)

(5) Define the HORIZONTAL INTENSITY, VERTICAL INTENSITY, and DIP as applied to terrestrial magnetism, and state the relations between them. Explain how you would determine the horizontal intensity at any place. (Inter. B.Sc.)

Chapter IX

(1) Write a short essay on the modern theory of the simple voltaic cell and explain *local action* and *polarisation*.

(2) Explain the construction and action of a zinc|zinc chloride concentration cell.

(3) An electric current is flowing along a wire. You are given a pivoted compass-needle, and are required to find out by its aid which way the current is flowing. How would you proceed (*a*) if the wire in question lies horizontally, (*b*) if the wire runs vertically, (*c*) if the wire is coiled up in a circular coil or open hank? (Inter. B.Sc.)

Chapter X

(1) Explain "current strength," "resistance," and "electromotive force." . Is there any real distinction between "potential difference" and "electromotive force"? If so, explain fully.

(2) Define the absolute electromagnetic and the practical units of current strength, quantity, potential difference, and resistance. Define also erg, joule, watt, kilowatt, Board of Trade unit of electrical energy, temperature coefficient of resistance, and electrochemical equivalent.

(3) State and explain Ohm's Law, and develop the formula for the joint resistance of conductors in parallel.

(4) A circuit is made up of (i) a battery with terminals A, B, its resistance being 3 ohms and its E.M.F. 2·7 volts; (ii) a wire BC, of resistance 1·5 ohms; (iii) two wires in parallel CDF, CEF, with respective resistances 3 and 7 ohms; (iv) a wire FA, of resistance 1·5 ohms. The middle point of the last wire is put to earth. Find the potential at the points A, B, C, F.

(5) The poles of a battery are connected by three pieces of the same copper wire, of lengths 1, 2, and 3 metres respectively, arranged in multiple arc. These are replaced by a piece of wire of the same material, but of double the diameter, and of such a length that the current through the battery is unchanged. Find the length of the wire. (H.S.C.)

(6) Define the electromagnetic units of current, electromotive force, and resistance. Compare the resistances of two copper wires, one 100 cm. long and 0.25 mm. in diameter, the other 75 cm. long and 0.4 mm. in diameter.

(7) Define the terms "one joule," "one kilowatt," "one Board of Trade unit." The price of electrical energy in a certain town is fourpence per Board of Trade unit, and the supply pressure is 200 volts. An electricity energy-meter on this circuit has a resistance of 10,000 ohms in its pressure coil; calculate the cost of the energy wasted in this coil in a quarter of a year. (Take a year as equivalent to 90 full lighting days.)

(8) Two resistances of 5 and 8 ohms respectively are placed in parallel, and in series with this combination is a resistance of 4 ohms. If the potential difference between the extreme terminals is 100 volts, what will be the current in each resistance and the difference of potential between the terminals of each resistance?

(9) In what manner do (*a*) the *conductor resistance* and (*b*) the *insulation resistance* of a uniform electric light cable depend upon the length of the cable? A 55-yd. length of such a cable when tested gave 0·11 ohm and 10,000 megohms respectively for (*a*) and (*b*); find the corresponding values for a mile length of similar cable.

(10) What is a megohm? What is a microhm? If three lengths of a cable, having respectively an insulation resistance of 300, 400, and 500 megohms, be joined in a continuous length, what will be the insulation resistance of the whole cable?

(11) Calculate the resistance of one mile of copper conductor, having a cross-sectional area of ·137 sq. in. The resistance of an inch cube of copper between opposite faces is ·66 microhm.

(12) A glow-lamp takes a current of ·55 ampere when connected to leads at 120 volts pressure. Find (*a*) the hot resistance of the lamp; (*b*) the watts absorbed; (*c*) the energy taken in 1 hour in joules; (*d*) the watt-hours taken in 50 hours; (*e*) the B.O.T. units taken in 100 hours; (*f*) the cost of the energy absorbed at 5d. per unit.

(13) What effect has change of temperature on the resistance of metals, alloys, insulators, and liquids? Define temperature coefficient.

(14) If a cell has an E.M.F. of 1·08 volts and ·5 ohm internal resistance, and if the terminals are connected by two wires in parallel of 1 ohm and 2 ohms resistance respectively, what is the current in each, and what is the ratio of the heats developed in each? (Inter. B.Sc.)

(15) Define resistivity or specific resistance, and apply the definition to find the resistance of a uniform wire of length l, diameter d and resistivity S. Find the length of wire of diameter 0·3 mm. and resistivity $1{\cdot}8 \times 10^{-6}$ ohm-centimetre which would have a resistance of 2 ohms. (Inter. B.Sc.)

Chapter XI

(1) Determine the resistance of a shunt which when joined to a galvanometer of resistance 3,663 ohms will result in $\frac{1}{34}$ of the total current passing through the galvanometer. Determine also (*a*) the joint resistance of the galvanometer and shunt; (*b*) the external resistance which must be added when the shunt is applied, so that the total current may be unaltered.

(2) The resistance of a shunted galvanometer is 75 ohms, that of the shunt being 100. A certain deflection of the galvanometer is obtained when the resistance in the rest of the circuit is 2,000 ohms. Find what additional resistance must be inserted that the galvanometer deflection may remain the same when the shunt is removed. What is the multiplying power of the shunt?

(3) How may the intensity of the magnetic force inside a solenoid be approximately calculated? What is it in one of 300 turns, 15 cm. long, which carries a current of 0·2 ampere? What effect has the diameter of the solenoid? (Inter. B.Sc.)

(4) Explain how the current in a tangent galvanometer properly arranged is proportional to the tangent of the angle of deflection. Describe some form

of tangent galvanometer, and explain how the sensitiveness can be varied by suitably placing a magnet outside a galvanometer. (Inter. B.Sc.)

(5) A galvanometer whose resistance is G is provided with a shunt whose resistance is S. If the current in the circuit is C, prove that the current through the galvanometer will be $\frac{S}{G+S}C$.

The coil of an ammeter has a resistance of 100 ohms, and a difference of potential of 120 millivolts between the terminals gives a deflection from one end of the scale to the other. What resistances must be provided as shunts in order that the instrument may register currents (*a*) from 0 to 5 amperes, (*b*) from 0 to 100 amperes. (Inter. B.Sc.)

(6) On sending a current through a tangent galvanometer, the needle of which is oscillating about a certain mean position, no deflection is produced, but the time of oscillation is reduced. What deductions can be made as to the situation of the galvanometer, and what additional facts would have to be supplied before the current could be calculated from the two times of oscillation. (Inter. B.Sc.)

(7) Give the definitions of an ampere, and an electromagnetic unit of current, and prove the formula giving the number of amperes in a current in terms of the deflection produced in a tangent galvanometer. (Inter. B.Sc.)

(8) A tangent galvanometer having a coil of one turn of 34 cm. radius gives a deflection of 45° with a current of 10 amperes. Calculate the strength of the earth's magnetic field at the centre of the coil. (Inter. B.Sc.)

(9) A sine galvanometer consists of a single coil of 49 turns of wire, the mean radius of which is 20 cm. A current of 0·08 ampere causes such a deflection that the coil has to be turned through 45° to bring the needle to its original position with regard to the coil. Determine the reduction factor of the galvanometer, and the horizontal component of the earth's magnetic intensity. (Inter. B.Sc.)

(10) How would you show by experiment that the magnetic field due to a plane current circuit, at any distance great compared with the dimensions of the circuit, depends not on the form but only on the area and the current, and that it is equal to that of a certain magnet set with axis perpendicular to the plane of the circuit? Show, by considering the case of a plane circular current, that the moment of the equivalent magnet is (area × current). (Inter. B.Sc.)

(11) Describe the construction of the moving coil galvanometer, and explain how, with the addition of a shunt, it can be used as an ammeter for large currents. (B.Sc.)

(12) How does a voltmeter differ from an ammeter in construction and use? A D.C. ammeter (resistance ·02 ohm) has a scale reading up to 5 amperes: how would you use it on a 15-ampere circuit? A D.C. voltmeter (resistance 50,000 ohms) reads up to 50 volts: how would you make it read up to 75 volts with the same scale?

Chapter XII

(1) Two circuits whose resistances are respectively 1 ohm and 10 ohms are arranged in parallel. Compare the amount of current passing through each of these circuits with that through the battery. Compare also the amount of heat developed in the same time in the two circuits.

(2) A wire of resistance *r* connects A and B, two points in a circuit, the resistance of the remainder of which is R. If, without any other change

being made, A and B are also connected by $(n - 1)$ other wires, the resistance of each of which is r, show that the heat produced in the n wires will be greater or less than that produced originally in the first wire according as r is greater or less than $R\sqrt{n}$.

(3) The E.M.F. of a battery is 18 volts and its resistance 3 ohms. The P.D. between its poles when they are joined by a wire A is 15 volts, and falls to 12 volts when A is replaced by another wire B. Compare the resistances of A and B, and the amounts of heat developed in them in equal times.

(4) Determine the final temperature of a copper wire ·166 cm. in diameter through which a current of 10 amperes is flowing. Specific resistance of copper = 1·65 microhms per centimetre cube, emissivity of copper = ·00025, temperature of room = 15° C.

(5) A current of 5 amperes flows for 3 minutes through a wire whose resistance is 2 ohms; given that 1 water-grm.-degree unit = 4·2 joules, find the amount of heat, in water-grm.-degrees units, generated in the wire. (H.S.C.)

(6) If electric energy for heating purposes be supplied at 2d. per unit, find the cost of boiling a quart of water in 5 minutes, and the mean power required. Initial temperature of water 15° C.; efficiency of kettle 90 per cent. (Take 1 quart of water = 2·5 pound)

(7) Define *illumination on a surface, luminous intensity* and *luminous efficiency of a lamp, luminous flux, the lumen, mean horizontal candle-power,* and *mean spherical candle-power.*

(8) Prove that the heat generated in a coil of wire by a current is proportional to the product of the resistance and the square of the current. Explain fully the exact relation between this product and the number of calories generated, the current being expressed in amperes and the resistance in ohms. (Inter. B.Sc.)

Chapter XIII

(1) Develop expressions for v/u and $(v + u)$, where v and u are the velocities of the anions and cations respectively in an electrolytic solution, and explain how the ratio $n : u$ may be determined experimentally.

(2) If 10,000 kilocoulombs are passed through cells arranged in series containing solutions of Cu_2Cl_2, $Hg_2(NO_3)_2$, $CuSO_4$, H_2SO_4, and NaCl, how much copper, mercury, hydrogen, and caustic soda can be obtained? (City and Guilds of London.)

(3) What do you understand by *osmotic pressure, solution pressure,* and *electrode potential,* and describe a method of measuring the electrode potential in the case of a zinc plate immersed in a solution of zinc sulphate.

(4) Twenty-four cells, each of E.M.F. 2 volts and resistance 1 ohm, are arranged in 6 rows, 4 cells in series per row, the 6 rows being in parallel. Find (*a*) the total current, (*b*) the current in each cell, (*c*) the fall in volts per cell, (*d*) the P.D. between the terminals of any one cell, (*e*) the T.P.D. for the battery. External resistance, 10 ohms.

(5) State Faraday's laws of electrolysis, and describe how they are explained on the modern theory of electrolysis. Explain the meaning of the term: electrochemical equivalent of a substance.

Calculate the amount of copper deposited by the passage of 1 ampere for 1 second through a depositing tank, given the weight of hydrogen liberated in electrolysis is $\frac{1}{100000}$ grm. per coulomb. Atomic weight of copper, 63, valency, 2. (Inter. B.Sc.)

(6) How do you account for the fact that an E.M.F. of about $1\frac{1}{2}$ volts is needed to electrolyse water at an appreciable rate? An accumulator 2 volts E.M.F. maintains a current in a circuit of total resistance 2 ohms, an electrolytic cell with back E.M.F. 1·5 volts is then inserted, the resistance being adjusted again to 2 ohms. Compare the currents and the rate of working in the two cases. (Inter. B.Sc.)

(7) A current of electricity driven by an electromotive force of 10 volts traverses a water voltameter in which there is a resistance of 2 ohms and a back electromotive force of 1·5 volts. Calculate the weight of hydrogen, and the number of calories developed in the voltameter per hour, assuming the resistance of other parts to be negligible. (The electrochemical equivalent of hydrogen is 0.000010384 grm. per coulomb, and the mechanical equivalent of one calorie is 42,000,000 $cm.^2$ $grm.^1$ $sec.^{-2}$.) (Inter. B.Sc.)

(8) Describe briefly some form of secondary cell or "accumulator," and give the chemical actions during charge and discharge.

A battery of 55 secondary cells is being charged from a dynamo and the p.d. between its terminals is 127 volts and the charging current is 60 amperes. If the speed of the dynamo increases so that the p.d. between the terminals is 129 volts, the charging current becomes 80 amperes. Find the resistance of the battery and its e.m.f., assuming the dynamo and leads to have negligible resistance and the e.m.f. of the cells to remain constant. (Inter. B.Sc.)

(9) State the law of subdivision of a current in a divided circuit. Explain how you would arrange 36 cells of a battery, each having an internal resistance of 1·6 ohms, so as to send the strongest possible current through an external resistance of 5·6 ohms. (Inter. B.Sc.)

Chapter XIV

(1) What is meant by thermo-electric power, Peltier coefficient, Thomson coefficient, and neutral temperature?

(2) A thermopile is joined up in series with a Daniell's cell and the current allowed to flow for a short time. The thermopile is then removed from the circuit and connected with the terminals of a galvanometer, the needle of which is thereupon considerably deflected but gradually returns to its undisturbed position. Explain this. (B.E.)

(3) State what facts with regard to thermo-electric currents can be established by the aid of a thermo-electric couple.

How may these facts be represented by a diagram? (Inter. B.Sc.)

(4) Suppose that at some point in an electric circuit heat was being developed by the passage of the current. Describe how you would determine whether the heating was due to a resistance or to a thermo-electric (Peltier) effect. (Inter. B.Sc.)

(5) Give a brief account of thermo-electric phenomena and describe how you would use a thermo-couple to measure temperatures 0° C. and 100° C. Illustrate your answer by diagrams. (H.S.C. and Inter. B.Sc.)

Chapter XV

(1) Describe some method of determining accurately the specific resistance of an electrolyte. (B.E. Hons.)

(2) Explain the theory of the Wheatstone Bridge method of comparing resistances. (Inter. B.Sc.)

(3) Describe and explain one method of comparing accurately the electromotive forces of two cells. How are standard cells of constant electromotive force constructed ? (Inter. B.Sc.)

(4) Describe a method of measuring the resistance of a battery.
When two batteries, A and B, are joined in turn to a galvanometer it is found that A gives the greater current; but when another galvanometer is employed B gives the greater current. Explain how this may occur. (Inter. B.Sc.)

(5) What is a potentiometer ? How would you use one to compare the difference of potential between the terminals of a Daniell cell (*a*) when it is on open circuit, (*b*) when the terminals are joined up by a wire of, say, 5 ohms resistance ? (Inter. B.Sc.)

(6) Describe carefully how you would use a potentiometer for measuring currents. How would you adapt it for use with large and small currents respectively ? How would you use it to calibrate a voltmeter ?

(7) Describe (*a*) a method of comparing the capacitances of two condensers, (*b*) a method for the absolute determination of the capacitance of a condenser.

Chapter XVI

(1) What is a magnetic field ? Describe a method of determining the magnetic dip by the revolution of a coil of wire about an axis in its own plane. (Inter. B.Sc.)

(2) Explain what is meant by the coefficient of mutual induction of two coils, and point out the factors upon which it depends. (Inter. B.Sc.)

(3) State two methods by which the direction of the electromotive force induced in a closed circuit which is moved in a magnetic field can be assigned, and show by means of a simple example that the two methods yield the same result.
A bar magnet is suspended by a string attached to the north pole, so that its axis is vertical. A ring of copper wire is allowed to fall over the magnet without touching it, the plane of the ring being horizontal and its centre travelling along the axis of the magnet. Find the direction of the current induced in different positions. (Inter. B.Sc.)

(4) Explain the theory of the induction coil. Compare the currents in the primary and secondary coils, and point out any sources of loss of energy in the apparatus. (Inter. B.Sc.)

(5) Give an account of the phenomena of electromagnetic induction, describing experiments to illustrate them. State Lenz's Law, and point out how it applies to the experiments you have described. (Inter. B.Sc.)

(6) How would you make experiments to show that a current is produced in a closed circuit while the number of tubes of magnetic induction passing through the circuit is changing ? Describe the apparatus used and mode of using it. How would you show that the total flow is proportional to the total change in the number of tubes passing through the circuit ?
How may this result be applied to determine the angle of dip of the earth's magnetic field at a given place ? (Inter. B.Sc.)

(7) Calculate the electromotive force generated, by virtue of the vertical component of the earth's field (which may be taken as $= 0.41$ cm. $^{-\frac{1}{2}}$ grm. $^{\frac{1}{2}}$ sec. $^{-1}$), in the axle of a railway carriage, of length 150 cm., travelling with a speed of 75 kilometres per hour. In what units is your answer given ? How could you observe the existence of this electromotive force ? (Inter. B.Sc.)

(8) A solenoidal coil 70 centimetres in length, wound with 30 turns of wire per centimetre, has a radius of 4·5 centimetres. A second coil of 750 turns is wound upon the middle part of the solenoid. Calculate the coefficient of self-induction of the solenoid, and the coefficient of mutual induction of the two coils. Will the inductance of the solenoid be affected by short-circuiting the ends of the secondary coil? (B.Sc.)

(9) Give the laws for the production of an electromotive force in a conductor moving in a magnetic field.

A telegraph wire 2·5 kilometres long running east and west is part of a circuit whose resistance is 35 ohms. If the wire falls to the ground from a height of 10 metres, find the current in the circuit at the moment that the wire strikes the ground, assuming that the whole wire falls freely at the same time. (g = 980 cm. per sec. per sec. and H = 0·2 C.G.S. units.) (Inter. B.Sc.)

(10) A coil of a single turn of wire in the form of a rectangle of height 15 cm. and width 8 cm. is suspended in a horizontal magnetic field of strength 2·5. Draw a diagram indicating the forces acting on each side of the rectangle, and calculate the couple acting upon it when its plane makes an angle of 45° with the magnetic field and a current of 20 C.G.S. units flows in the wire. (Inter. B.Sc.)

Chapter XVII

(1) In what respects do the magnetic properties of iron and steel differ? Define the terms intensity of magnetisation (I), induction (B), and magnetic force (H). How do you obtain the relation $B = H + 4\pi I$?

(2) Define intensity of magnetisation. Give a short account of the relation between the intensity of magnetisation of iron and the magnetic force acting upon it.

Explain how you would study the matter experimentally in the case of iron given in the form of a long thin rod. (Inter. B.Sc.)

(3) Explain the meaning of *intensity of magnetisation*, *magnetic induction* and *magnetic circuit*. How does the magnetic reluctance of a cylindrical iron rod 35 cm. long compare with that of an air gap 1 mm. wide, assuming both have the same cross-sectional area and that the permeability of the iron is 350 times that of air? (Inter. B.Sc.)

(4) Explain the meaning of the terms *ampere-turn*, *magnetic force or field strength*, *magnetic induction or flux density*. An iron ring of mean circumference 40 cm. is uniformly wound with 800 turns of wire. If the cross-section of the iron is 2 sq. cm., what current will be necessary to produce a total flux of 20,000 lines? What will be the effect of a 2 mm. air-gap in the iron ring? (Take 500 for the permeability of the iron.) (Inter. B.Sc.)

(5) Define magnetic force, H, and magnetic induction, B. Show that the area of the H, B cycle denotes 4π times the energy dissipated per c.cm. of metal during each magnetic cycle. (B.Sc.)

(6) Define the terms *gauss*, *maxwell*, *permeability* and *intensity of magnetisation*. A magnetic field of 50 e.m. units generates a flux of 2500 e.m. units in a long bar of steel. Calculate the permeability and the intensity of magnetisation of the bar, which has a cross-section of 0·25 cm.^{2} (Inter. B.Sc.)

Chapter XVIII

(1) A choke, having an inductance of 0·05 henry and a resistance of 0·5 ohm, is put in series with a non-inductive resistance of 10 ohms, and an alternating sine E.M.F. of 200 volts, of frequency 50, is applied to the

terminals. Find the current in the circuit and the P.D. on the choke and resistance respectively.

(2) An alternating E.M.F. of 200 (virtual) volts and frequency 50 is applied to a circuit of resistance 10 ohms and inductance ·25 henry. Find (*a*) the current, (*b*) the angle of lag, (*c*) the impedance.

(3) A circuit has an inductance of ·02 henry and a capacity of 10 microfarads. Find the frequency so that the impedance may be equal to the ohmic resistance.

(4) A condenser of 4 m.f. capacity is in series with a resistance of 600 ohms and the applied alternating pressure is 1000 volts. Find (*a*) the current, (*b*) the P.D. across the condenser, (*c*) the P.D. across the resistance.

(5) State the law of production of electromotive force in a conductor moving in a magnetic field. Calculate the maximum electromotive force in volts in a coil of 50 turns each of area 80 sq. cm. when making 1,800 revolutions per minute about an axis in its plane, the axis being at right angles to a uniform magnetic field of strength 60 gauss. (Inter. B.Sc.)

(6) Why are very high voltages used in transmitting electrical energy to great distances? Explain the action of the transformers used at the ends of the lines to reduce the voltage to the required amount. (Inter. B.Sc.)

(7) State the laws governing the production of induced currents. A circular coil of 50 turns, of average diameter 20 cm., rotates 10 times per second about an axis perpendicular to a magnetic field of strength 10 units. Calculate the maximum e.m.f. induced in the coil. (Inter. B.Sc.)

(8) Give the laws of production of electromotive force in a coil due to motion in a magnetic field. A coil of 500 turns each of area 100 square centimetres rotates about an axis at right angles to a magnetic field of strength 200 units. If the coil makes 1,200 revolutions per minute, find the average e.m.f. in the coil, and draw a curve showing how the e.m.f. varies during one revolution. (Inter. B.Sc.)

Chapter XIX

(1) State Ohm's law. Air, rendered conducting by X-rays or other agency, does not conform to Ohm's law. Describe carefully, and with the aid of a diagram, how it deviates from it. (Inter. B.Sc.)

(2) What is *ionisation* and how may it be produced in a gas. How may the electrical conductivity of an ionised gas be determined, and what is meant by the saturation current? (B.Sc.)

(3) Describe the construction and action of (*a*) a photo-electric cell, (*b*) a thermionic valve.

(4) Describe the general character of the discharge through a gas contained in a tube as the tube is gradually exhausted. (B.Sc.)

(5) What are cathode rays? Describe briefly how their velocity and e/m have been found. (Inter. B.Sc.)

(6) What is an electron? How can the *mass* and *charge* of an electron be determined?

Chapter XX

(1) Describe and explain how X-rays are produced, giving a sketch of a simple form of X-ray tube, and stating how the chief properties of the rays may be exhibited. (Inter. B.Sc.)

(2) Give an outline account of our present knowledge of the various kinds of radiation from radium. (Inter. B.Sc.)

(3) Give a brief outline of the disintegration theory of the radioactive materials, *e.g.* uranium. (Inter. B.Sc.)

(4) Write an essay on "Modern Ideas on Atomic Structure."

Chapter XXII

(1) Describe and explain the actions of the transmitter and the receiver in a telephone circuit. Sketch and explain a simple house telephone circuit. (Inter. B.Sc.)

(2) A drum armature in a two-pole field contains 150 external conductors and runs at 550 revs. per min. Find the total flux through the armature required to produce an E.M.F. of 115 volts on open circuit.

(3) State (with diagrams) the difference between series, shunt and compound wound machines, and mention their main characteristics.

(4) Develop expressions for the instantaneous value, the maximum value, and the average value of the induced E.M.F. in the case of a coil rotating about a vertical axis in the earth's field.

(5) Give an outline of the action of a dynamo. A dynamo of e.m.f. 80 volts and internal resistance 0·3 ohm has connected directly to its terminals (*i.e.* (i) and (ii) are in parallel with each other), (i) an accumulator battery of e.m.f. 60 volts and internal resistance 1 ohm which is being charged, and (ii) a number of lamps of total resistance 10 ohms, which are being lighted. Find the total current supplied by the dynamo. (B.Sc.)

(6) What do you understand by the back e.m.f. of an electric motor? A shunt-wound D.C. electric motor takes a current of 100 ampères from 200-volt mains. The shunt field coils have a resistance of 0·5 ohm. Find (*a*) the back e.m.f. and hence (*b*) the electrical energy converted into work per second. Verify that (*b*) is the difference between the total power supplied and that turned into heat. (H.S.C.)

ANSWERS

Chapter III. (1) 1 sec. (neglecting thickness). (2) 7 : 11. (4) 25·24 cm. on opposite side of needle. (6) 160 ergs. (7) $\tan\theta = 2\text{M}r/\text{H}(r^2-l^2)^2$. For B position null point $r^3 - 2rl^2 + l^4/r = \text{M/H}$. (9) 25 : 7.

Chapter IV. (3) 205200. (4) $m = 24$. (5) 257·1°; 319·8°. (6) 2 in 1 min. (10) 11250.

Chapter VI. (1) $3\sqrt{3}\,\text{Q}/4d^2$ where d = hexagonal base. (2) $1/(6\pi \times 10^7)$ cm. (3) Charge on A = 0 and $98\frac{2}{3}$; on B = $98\frac{2}{3}$, and $1\frac{1}{3}$; Force on A = 197 dynes. (4) 33; 5. (5) 15. (7) 5·76; 4·14. (8) $\frac{1}{16}$, $\frac{1}{4}$, $\frac{1}{16}$. (11) $2/\sqrt{5}$; $2(\sqrt{5}-1)/\sqrt{5}$. (12) 1·006 cm.

Chapter VII. (1) 10 and 12 cm. (2) 84 : 325. (3) 1 : 1·45 (nearly). (4) 5 : 4; equal. (5) 3·183. (6) (*a*) 6250 e.s. units, 31250 dynes; (*b*) 15625 e.s. units, 195,312·5 dynes. (7) 227·212°. (8) Q = 35 e.s. units. (11) 12,500,000 dynes. (13) 1·769 and 7·076 dynes.

Chapter X. (4) A = − ·25 volt, B = 1·45 volts, C = ·95 volts, F = ·25. (5) $2\frac{2}{11}$ metres. (6) 256 : 75. (7) 34·45 pence (take 90 days). (8) Total current = 14·13 = current in the 4 ohm coil; current in the 5 ohm coil = 8·69; current in the 8 ohm coil = 5·44; P.D. on the 4 ohm coil = 56·52; P.D on the 5 ohm and 3 ohm coils = 43·48 (all approximate). (9) Conductor resistance = 3·52; insulation resistance = 312·5. (10) 127·6 megohms. (11) 17,500 ohms. (12) 218·18 ohms; 66 watts; 237,600 joules; 3300 watt-hours; 6·6 B.O.T. units; 2s. 9d. (14) ·62 amp.; ·31 amp.; Heat 2 : 1. (15) 785·2.

Chapter XI. (1) Shunt 111 ohms, (*a*) 108 ohms, (*b*) 3555 ohms. (2) 6000 ohms, multiplying power 4. (3) H = 5·028. (5) (*a*) ·024 ohms, (*b*) ·0012 ohms. (8) ·185 dynes. (9) Reduction factor ·011312. H = ·1741 dynes. (12) Shunt = ·01 ohm; Series Resistance 25000 ohms.

Chapter XII. (1) $\frac{10}{11}$: $\frac{1}{11}$: 1 and 10 : 1. (3) Resistances 5 : 2; heats 5 : 8. (4) 29·01° C. (5) 2143; 2160 if $·24\text{C}^2\text{R}t$ is used. (6) ·25*d*; 1499 watts (taking 1 quart of water = 2·5 pound, 1 pound = 453·6 grm.).

Chapter XIII. (2) 20641 grm. Hg, 3281 grm. Cu, 103·84 grm. H, 4153 grm. NaOH. (4) $\frac{3}{4}$ amp.: $\frac{1}{8}$ amp.: $\frac{1}{8}$ volt.: $1\frac{7}{8}$ volts.: $7\frac{1}{2}$ volts. (5) ·000315 grm. (6) Current 4 : 1; 16 : 1. (7) ·1588752 grm.; 30964 calories. (8) ·1 ohm.: 121 volts. (9) 3 rows of 12 in series, the 2 rows in parallel.

Chapter XVI. (7) ·0128125, in volts. (8) L = ·05032, M = ·01797 Henrys. (9) ·02 amp. (10) ·4243 dyne cm.

Chapter XVII. (3) Equal. (4) ·795 amp.; 2·7 amp. for same flux. (6) 791 nearly.

Chapter XVIII. (1) 10·537 amp.; 166·7 volt (choke); 105·4 volt (resistance). (2) 2·52 amp.; 82° 45′; 79·3. (3) 356. (4) 1 amp.; 800 volts (nearly); 600 volts (nearly). (5) ·452 volt. (7) 98·7 millivolt. (8) 8 volts.

Chapter XXII. (2) $8·36 \times 10^6$. (5) 21·05 amps. (6) 150 volts, 1500 watts.

INDEX